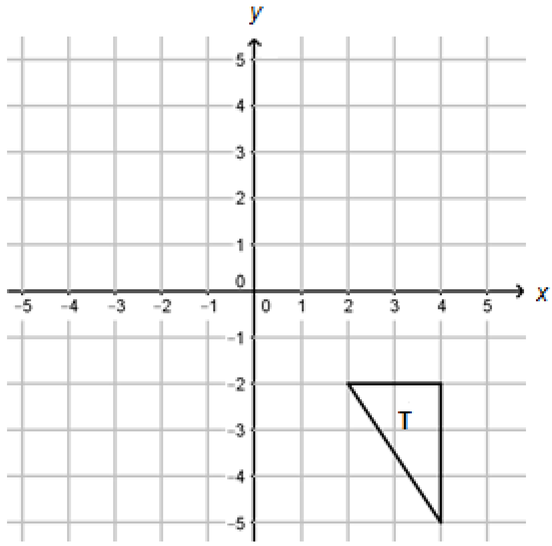
Higher Check In - 9.01 Plane isometric transformations

1. On the grid below, reflect triangle **T** first in the line $x = 1$ and then in the y-axis.



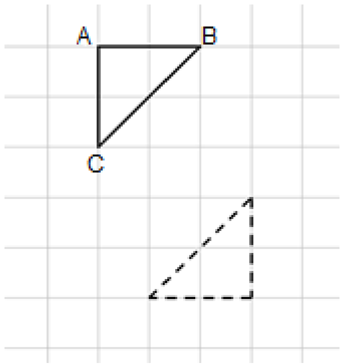
2. Describe the single transformation that is equivalent to the two reflections you carried out in question 1.

3. On the grid above, reflect triangle **T** first in the x-axis and then in the line $y = x$. Describe fully the single transformation that is equivalent to this sequence of reflections.

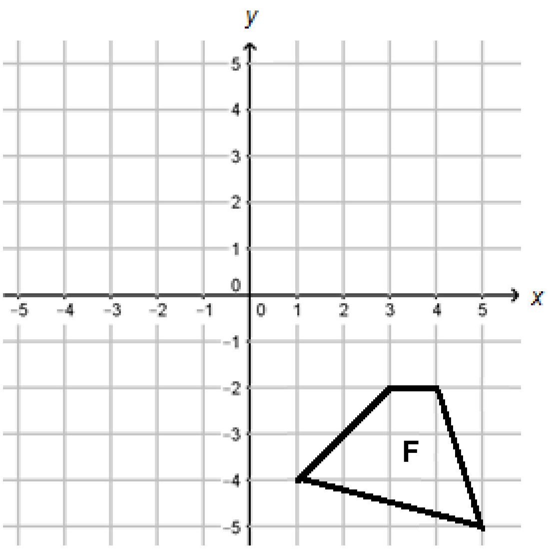
4. What single transformation is equivalent to first translating a shape by $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and then by $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$?

5. What single transformation is equivalent to a clockwise rotation about the origin of 127° followed by an anticlockwise rotation about the origin of 31°?

6. Fully describe a sequence of transformations that would map triangle ABC onto the second triangle.

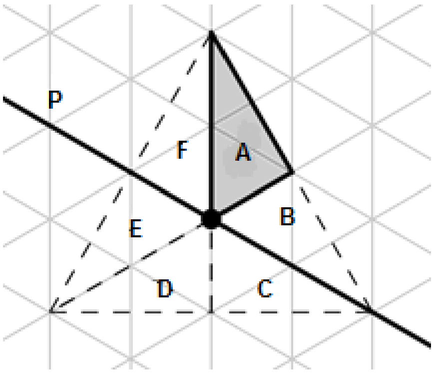


7. When quadrilateral **F** below is reflected in the line $y = -x$, one of its vertices remains in the same place. What are the coordinates of this vertex? Explain why it remains in the same place.



8. Quadrilateral **F** in question 7 is first translated by $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$, then rotated 90° clockwise about (2, 7) and finally reflected in the *x*-axis. Explain how you can work out the area of the transformed quadrilateral without drawing it.

9. The equilateral triangle below can be constructed from a sequence of transformations of the shaded triangle A. Describe how this can be carried out through reflections in the line P or rotations about the centre of the triangle.



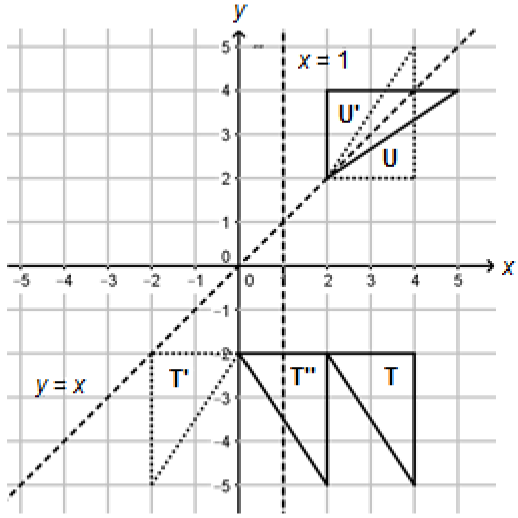
10. The quadrilateral Q has vertices A(2, 5), B(3, 4), C(1, 2), D(1, 4). The quadrilateral is first reflected in the *x*-axis to give quadrilateral R. Quadrilateral R is then reflected in the *y*-axis to give quadrilateral S. Work out the coordinates of the vertices after the second reflection without drawing the transformations.

Extension

In question 3 you reflected a shape in one line and then in a second line. You should have found that this was equivalent to a rotation with its centre at the point where the lines intersect. Investigate whether it is always true that a sequence of two reflections will be equivalent to a rotation about the point of intersection of the mirror lines. Is there a way to predict the angle of rotation? What happens if the lines do not intersect?

Answers

1. The final triangle is marked **T''** below.



2. This is equivalent to a translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$.

3. The final triangle is marked **U'** on the diagram above. The two transformations (**U** and **U'**) are equivalent to rotation about the origin by 90° anticlockwise.

4. Translation by $\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \end{pmatrix} = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$.

5. Clockwise rotation by 96° about the origin.

6. Many solutions are possible. One possible sequence is a reflection in the line BC (or rotation by 180° about the midpoint of BC) followed by a translation of $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

7. The vertex at (2, –2) remains in the same place under this reflection. This is because it is on the mirror line $y = -x$. Any point on a mirror line remains in the same place (the points are called "invariant points") under a reflection.

8. Since reflections, rotations and translations preserve lengths (this is one meaning of the word "isometric" in the title of this test), the area of the transformed quadrilateral will be the same as the area of quadrilateral **F**.

9. Many solutions are possible. One possible sequence is:
   - Rotate triangle A about the centre of the equilateral triangle by 120° clockwise to form triangle C.
   - Rotate triangle A about the centre of the equilateral triangle by 120° anticlockwise to form triangle E.
   - Reflect triangle E in the line P to form triangle F.
   - Reflect triangle C in the line P to form triangle B.
   - Reflect triangle A in the line P to form triangle D.

10. Reflection in the x-axis multiplies each y-coordinate by –1 but does not change the x-coordinate. Reflection in the y-axis multiplies each x-coordinate by –1 but does not change the y-coordinate. Therefore, the vertices will be mapped as follows:
    - A(2, 5) → A''(–2, –5)
    - B(3, 4) → B''(–3, –4)
    - C(1, 2) → C''(–1, –2)
    - D(1, 4) → D''(–1, –4)

Extension

A sequence of two reflections in mirror lines that intersect **will** be equivalent to a rotation about their point of intersection. The angle of rotation is double the angle between the lines. If the two lines do not intersect, it is equivalent to a translation, like in question 1.

GCSE (9–1)

MATHEMATICS

Assessment Objective	Qu.	Topic	R	A	G
AO1	1	Carry out a sequence of reflections			
AO1	2	Describe the single transformation equivalent to reflection in two lines that are parallel			
AO1	3	Carry out and describe a sequence of reflections in two lines that are not parallel			
AO1	4	Describe the single transformation equivalent to two translations			
AO1	5	Describe the single transformation equivalent to two rotations			
AO2	6	Identify and describe a sequence of transformations			
AO2	7	Explain why points are invariant under reflection			
AO2	8	Explain why area is preserved under isometric transformations			
AO3	9	Identify and describe a sequence of transformations			
AO3	10	Solve a problem involving a sequence of transformations			

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