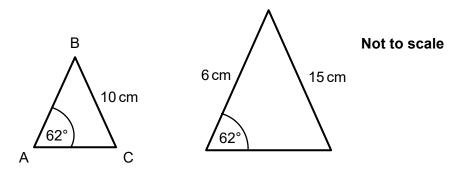


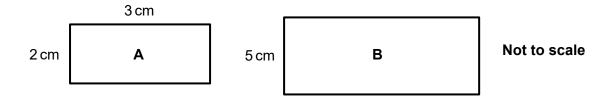


# **OCR 09 Congruence and Similarity (Higher)**

- 1. Calculate  $3\begin{pmatrix} -3\\ 4 \end{pmatrix} + 2\begin{pmatrix} 7\\ 2 \end{pmatrix} \begin{pmatrix} -5\\ -3 \end{pmatrix}$ .
- 2. If  $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$  find  $\mathbf{a} + 2\mathbf{b}$ .
- 3. If  $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$  find  $\mathbf{a} \frac{1}{2}\mathbf{b}$ .
- 4. The two triangles below are similar. Calculate length AB.

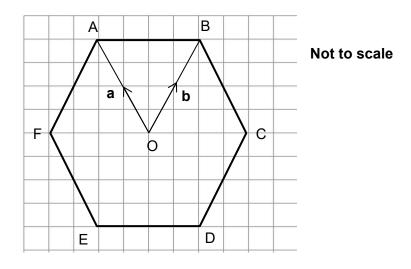


5. Rectangle **A** is similar to rectangle **B**. Calculate the area of rectangle **B**.



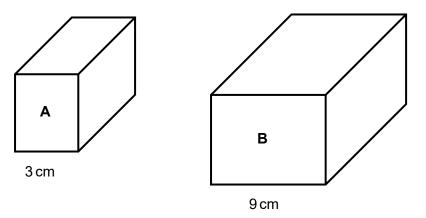


Use the diagram below to answer questions 6 to 8. ABCDEF is a regular hexagon, with centre O.



- 6. Write the vector  $\overrightarrow{AD}$  in terms of **a** and **b**.
- 7. Write the vector  $\overrightarrow{ED}$  in terms of **a** and **b**.
- 8. Write the vector  $\overrightarrow{FC}$  in terms of **a** and **b**.

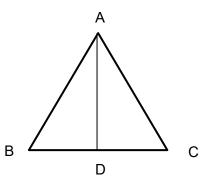
Use the diagram below to answer questions 9 and 10. The diagram shows two mathematically similar solids.



- 9. Solid **A** has a volume of 30 cm<sup>3</sup>. Calculate the volume of Solid **B**.
- 10. Solid **B** has a surface area of 558 cm<sup>2</sup>. What is the surface area of solid **A**?

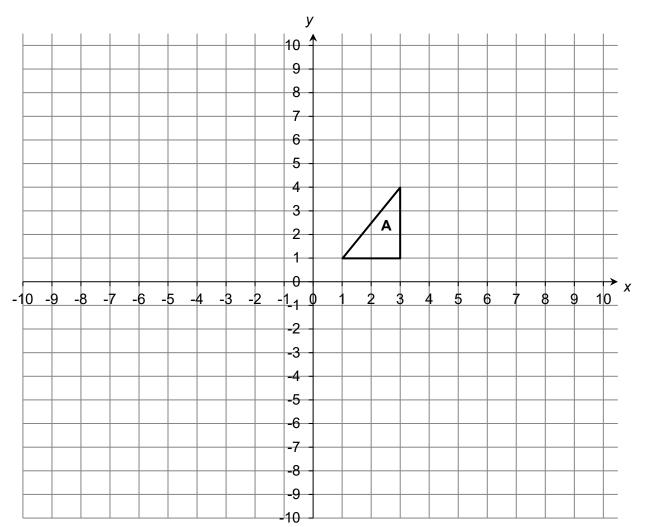


11. ABC is an equilateral triangle. D is the midpoint of BC and AD is perpendicular to BC. Prove that triangles ADC and ADB are congruent.



Not to scale

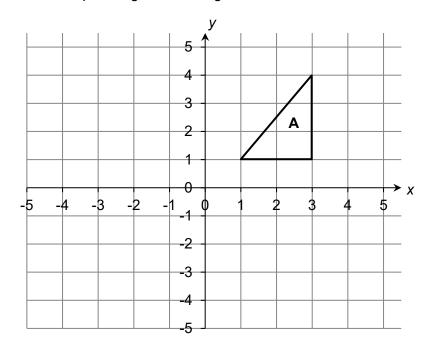




13. On the grid above, enlarge triangle **A** by scale factor -2, centre (0, 0).



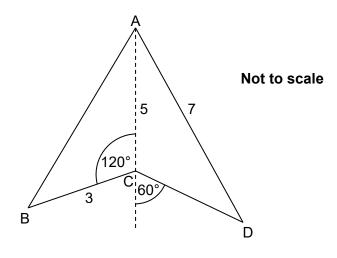
14. Triangle **A** is shown on the grid below. Triangle **A** is reflected in the *y*-axis to give triangle **B**. Triangle **B** is then reflected in the line  $y = -\frac{1}{2}$  to give triangle **C**. Describe the single transformation that maps triangle **A** to triangle **C**.



- 15. A £5 note is a rectangle,  $125 \text{ mm} \times 65 \text{ mm}$ . A £50 note is a rectangle,  $156 \text{ mm} \times 85 \text{ mm}$ . Show that the two notes are not mathematically similar.
- 16. A packing company has boxes in two sizes, small and large. A large box is a cube with volume 216 cm<sup>3</sup>. A small box is a cube with side length 1.5 cm. What is the maximum number of small boxes that will fit in a large box?
- 17. Two shapes A and B are mathematically similar. The area of shape A is 27 cm<sup>2</sup> and the area of shape B is 432 cm<sup>2</sup>. One length of shape A is 5 cm. Find the corresponding length for shape B.
- 18. A coffee shop sells coffee in two different cup sizes, medium and large. The cups are mathematically similar. The large cup holds 490 ml and is 18 cm tall. The medium cup is 12 cm tall. How much does the medium cup hold? Give your answer to the nearest millilitre.
- 19. Two similar shapes have volumes in the ratio 1 : 343. What is the ratio of their surface areas?



20. Quadrilateral ABCD is shown below. AC = 5, AD = 7, BC = 3 and  $BCA = 120^{\circ}$ . Are triangles ACB and ACD congruent?





#### Answers

- 1.  $\begin{pmatrix} 10\\ 19 \end{pmatrix}$
- 2.  $\begin{pmatrix} 10\\1 \end{pmatrix}$
- $3. \quad \begin{pmatrix} 0 \\ 4.5 \end{pmatrix}$
- 4. Scale factor is 1.5 so  $AB = 6 \div 1.5 = 4$  cm.
- 5. Scale factor is 2.5 so missing length in  $\mathbf{B} = 3 \times 2.5 = 7.5$  cm. Area of rectangle  $\mathbf{B} = 7.5 \times 5 = 37.5$  cm<sup>2</sup>.
- 6.  $\overrightarrow{AD} = -2a$
- 7.  $\overrightarrow{ED} = -\mathbf{a} + \mathbf{b}$  or  $\overrightarrow{ED} = \mathbf{b} \mathbf{a}$
- 8.  $\overrightarrow{FC} = 2(\mathbf{b} \mathbf{a})$
- 9. Scale factor of sides is 3, so volume scale factor is 27. Therefore, volume of solid  $\mathbf{B} = 810 \text{ cm}^3$ .
- 10. Scale factor of sides is 3, so area scale factor is 9. Surface area of solid  $\mathbf{A} = 62 \text{ cm}^2$ .
- 11. Triangles are congruent as angles and sides are equal. Can be proved by SSS, SAS, ASA or RHS.
- 12. Triangle at (2, 4),  $(2, -\frac{1}{2}), (-1, -\frac{1}{2}).$
- 13. Triangle at (-2, 2), (-6, -2), (-6, -8).
- 14. Rotation, 180° about  $\left(0, -\frac{1}{2}\right)$ .
- 15. They are not mathematically similar as  $\frac{156}{125} \neq \frac{85}{65}$ .
- 16. Side length of large box  $= \sqrt[3]{216} = 6 \text{ cm}$ Ratio of side lengths  $= 1: \frac{6}{1.5} = 1: 4$ (Ratio of areas  $= 1^2: 4^2 = 1: 16$ ) Ratio of volumes  $= 1^3: 4^3 = 1: 64$ So the maximum number of small boxes that will fit in a large box is 64.

## GCSE (9–1) MATHEMATICS Section Check In

- 17.  $432 \div 27 = 16$ , so the area scale factor is 16. Length scale factor is 4.  $5 \times 4 = 20$ , so missing length is 20 cm.
- 18. Scale factor =  $18 \div 12 = 1.5$ . Volume of medium cup =  $490 \div (1.5^3) = 145.185 = 145$  ml
- 19. Surface area ratio =  $\left(\sqrt[3]{1^2}\right)$  :  $\left(\sqrt[3]{343}^2\right)$  = 1 : 49.
- 20. Using the cosine rule  $a^{2} = b^{2} + c^{2} - 2bc \cos A \text{ so } AD^{2} = CD^{2} + AC^{2} - 2 \times AC \times CD \times \cos ACD$   $7^{2} = CD^{2} + 5^{2} - 2 \times CD \times 5 \times \cos 120^{\circ}$   $49 = CD^{2} + 25 + 5CD$   $CD^{2} + 5CD - 24 = 0$  (CD + 8)(CD - 3) = 0 CD = -8 or CD = 3The length cannot be negative so CD = 3.
  - Length AC is common to both triangles ABC and ADC. Angle ACB = angle ACD. Length BC = length CD. Triangles ABC and ADC are congruent (SAS).

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### GCSE (9-1) MATHEMATICS Section Check In

Assessment Objective	Qu.	Торіс	R	Α	G
AO1	1	Calculate with vectors			
AO1	2	Calculate with vectors			
AO1	3	Calculate with vectors			
AO1	4	Find a length in similar shapes			
AO1	5	Find an area in similar shapes			
AO1	6	Identify a vector			
AO1	7	Identify a vector			
AO1	8	Identify a vector			
AO1	9	Understand relationship between lengths and volumes of similar shapes			
AO1	10	Understand relationship between lengths and surface areas of similar shapes			
AO2	11	Prove that two triangles are congruent			
AO2	12	Enlarge using a fractional scale factor			
AO2	13	Enlarge using a negative scale factor			
AO2	14	Identify a single transformation			
AO2	15	Show that two rectangles are not similar			
AO3	16	Understand relationship between lengths and volumes of similar shapes			
AO3	17	Understand relationship between lengths and areas of similar shapes			
AO3	18	Understand relationship between lengths and volumes of similar shapes			
AO3	19	Find surface area ratio given volume ratio of similar shapes			
AO3	20	Determine if two triangles are congruent			

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