1(a). A sequence is defined by the term-to-term rule $u_{n+1} = u_n^2 - 8u_n + 17$.

Given that $u_1 = 4$, find u_2 and u_3.

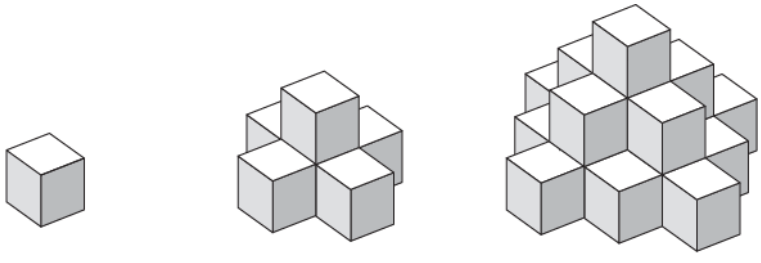
_____ _____ **[2]**

(b). Given instead that $u_1 = 2$, find u_2, u_3 and u_{100}.

_____ _____ _____ **[3]**

2(a). Here is a picture of three towers.

Not all the cubes can be seen in the towers.



Tower 1 Tower 2 Tower 3

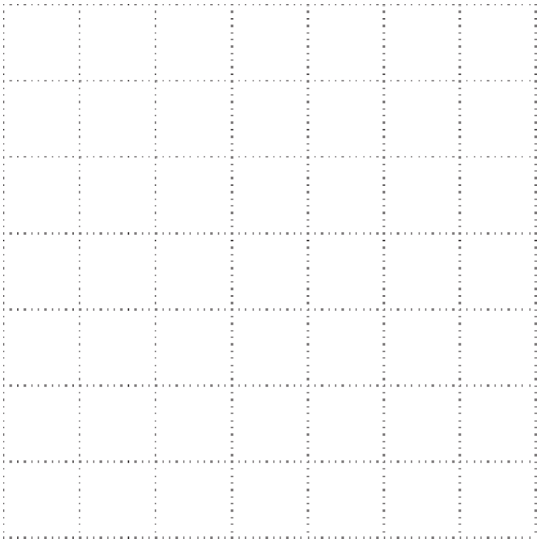
Edith uses 1 cube to build tower 1.

Edith uses 6 cubes to build tower 2. There are 5 cubes on the bottom layer.

Write down the total number of cubes in tower 3.

(a) _____ [1]

(b). Draw a plan view of the arrangement of cubes Edith will use for the bottom layer of tower 4.



[1]

(c). Continue this sequence to show the number of cubes used for the bottom layer of each tower.

Tower 1	Tower 2	Tower 3	Tower 4
1	5	_____	_____

[2]

(d). Find an expression for the number of cubes used in the bottom layer of tower *n*.

(d) _____ [4]

3. This expression can be used to generate a sequence of numbers.

$$n^2 - n + 11$$

(i) Work out the first three terms of this sequence.

_____, _____, _____ [2]

(ii) Show that this expression does not only generate prime numbers.

_____

_____ [2]

4. Here are the first four terms of a sequence.

12 21 30 39

Write an expression for the nth term of this sequence.

_____

[2]

5(a). The nth term of a sequence is $5n + 2$.

Write down the first three terms of this sequence.

_____ _____ _____

[2]

(b). Here are the first four terms of another sequence.

17 14 11 8

Find an expression for the nth term of this sequence.

_____

[2]

6(a). The nth term of a sequence is $n^2 + 5$.

Work out the first three terms of this sequence.

_____ _____ _____ **[2]**

(b). Here are the first four terms of another sequence.

5 11 17 23

Find an expression for the nth term of this sequence.

_____ **[2]**

7(a). The nth term of a sequence is $n(n + 1)$.

Work out the first three terms of this sequence.

_____ _____ _____ [2]

(b). Here are the first four terms of another sequence.

7 4 1 $^{-}2$

Find an expression for the nth term of this sequence.

_____ [2]

8(a). The nth term of a sequence is $6n - 2$.

Find the first three terms of this sequence.

_____ _____ _____ **[2]**

(b). The nth term of another sequence is $5n^2$.

Is the number 1000 a term in this sequence?
Show how you decide.

[3]

9(a). Here are the first four terms of a sequence.

7 12 17 22

Write an expression for the nth term of this sequence.

_____ **[2]**

(b). The nth term of another sequence is given by the expression $100 - 8n$.

Write down the first three terms of this sequence.

_____ , _____ , _____ **[2]**

10. The nth term of a sequence is given by $8n - 5$.

(i) Write down the first three terms of this sequence.

(i) _____ _____ _____ [2]

(ii) Is 96 a term in this sequence?
Give a reason for your answer.

_____ because _____ [1]

11(a) Here are the first four terms of a linear sequence, *S*.

.

6 11 16 21

Write down an expression for the *n*th term of the sequence *S*.

_____ **[2]**

(b). An expression for the *n*th term of another sequence, *T,* is $120 - 2n$.

The sequences *S* and *T* have one term with the same value, *v*, in the same position, *n*.

Find the values of *n* and *v* for this term.

n = _____

v = _____ **[4]**

12. Here are the first four terms of a sequence.

16 9 2 -5

Write an expression for the *n*th term of this sequence.

_____ [2]

13(a)

. Here are the first four terms of a sequence.

$$\frac{1}{2} \qquad \frac{4}{3} \qquad \frac{9}{4} \qquad \frac{16}{5}$$

Find the nth term of this sequence.

_____ **[2]**

(b). Here are the first four terms of a quadratic sequence, the nth term of this quadratic sequence is $an^2 + bn + c$.

2 12 28 50

Find the values of a, b and c.

a = _____

b = _____

c = _____

[4]

14(a). Write the next term in each of these sequences.

(i) 1 1 2 3 5 8

(i) ---------- [1]

(ii) 2 4 8 16 32 64

(ii) ---------- [1]

(b). Write an expression for the *n*th term of the sequence below.

15 12 9 6

---------- [2]

15(a)

. A sequence is defined using this term-to-term rule.

$$u_{n+1} = \sqrt{2u_n + 15}$$

If $u_1 = 5$, find u_2.

---------- [1]

(b). Another sequence is defined using this term-to-term rule,

$$u_{n+1} = ku_n + r$$

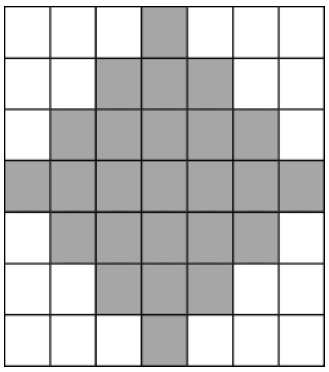
where k and r are constants.

Given that $u_2 = 41$, $u_3 = 206$ and $u_4 = 1031$, find the value of k and the value of r.

k = ----------

r = ---------- [5]

END OF QUESTION PAPER

Question			Answer/Indicative content	Marks	Part marks and guidance	
1	a		1 **nfww** 10 **nfww**	2	**B1** for each	**FT** *their* 'u_2' for u_3
	b		5 **nfww** 2 **nfww** 5 **nfww**	3	**B1** for each	**FT** *their* 'u_2' for u_3
			Total	**5**		
2	a		19	1		
	b			1		
	c		13 25	2	**M1** for 13 or 25	**FT** *their* bottom layer in **(b)** and *their* number of cubes in **(a)**
	d		$2n^2 - 2n + 1$ **oe**	4	**M3** for expression with $2n^2 - 2n$ **oe** or **M2** for expression with $2n^2$ or **M1** for expression with n^2 or first differences	
			Total	**8**		
3		i	11 13 17	2	**B1** for any two correct	
		ii	Any multiple of 11 and its result e.g. 11th term is $121 = 11^2$	2	Accept any correct argument **B1** at least two more evaluated terms	
			Total	**4**		

Question			Answer/Indicative content	Marks	Part marks and guidance	
4			$9n + 3$	2	**B1** for [+]$9n$	condone use of other letters Examiner's Comments This was well answered. Almost everyone knew that if a series of numbers increase by a given constant then the general term can be written as 'this constant × n' plus 'c'. So $9n + 3$ was seen nearly every time. Just a few got the two numbers the wrong way round giving $3n + 9$.
			Total	2		
5	a		7, 12, 17	2	**B1** for 2 of these in correct position or for 2, 7, 12	Examiner's Comments The first three terms of the sequence were rarely incorrect.
	b		20 – $3n$ as final answer	2	oe; need not be simplified **M1** for –$3n$ or $3n$ oe	condone n3 etc for 3n Examiner's Comments Stating the nth term showed an improvement over past years, with many giving the correct 20 – $3n$, and most reaching $3n$ or better.
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
6	a		6, 9, 14	2	**B1** for two terms correct in the correct position Or **SC1** for 5, 6, 9	
	b		$6n - 1$ as final answer	2	Accept unsimplified **M1** for $6n$ oe soi Or **SC1** for $6x - 1$, $6n$th $- 1$ etc **Examiner's Comments** This sequence question was done well, with errors in finding the terms of the sequence fairly rare in part (a). Finding the nth term of a sequence in part (b) was also usually correct, with some carrying out checks that their expression was correct. A few made the error $n + 6$, often with +6 marked between the terms.	Condone poor notation such as $n6$ etc or $n = 6n - 1$
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
7	a		2, 6, 12	2	**M1** for two correct in the correct positions or for 6, 12, 20 or 0, 2, 6 Examiner's Comments The correct terms 2, 6, 12 were seen often, but sadly so were the terms 2, 5, 10 from using $n^2 + 1$ instead of the given $n(n + 1)$. The second term was also occasionally obtained as 5 from 2 × 3.	
	b		$10 - 3n$ oe	2	Accept unsimplified **M1** for $3n$ or $-3n$ oe soi Or SC1 for $3 - 10x$ oe Examiner's Comments Relatively few gave the correct $10 - 3n$. Although reaching $-3n$ or $3n$ was quite common and gained partial credit, the incorrect $n - 3$ was seen frequently and received no marks.	Condone poor notation such as $n3$ etc or $n = 10 - 3n$
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
8	a		4, 10, 16	2	**B1** for two of these correct and in the correct position or associated in working with correct value of *n;* or B1 for –2, 4, 10 Examiner's Comments The first three terms of the sequence were rarely incorrect.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
	b		no, following work gaining both M marks	3	**M1** for $n^2 = 200$ soi **and M1** for $\sqrt{200}$ or $10\sqrt{2}$ is not an integer, or $\sqrt{200} = 14.1\ldots$ **or** **M1** for $5 \times 14^2 = 980$ **and M1** for $5 \times 15^2 = 1125$ **or** **M1** for one of $5 \times 14^2 = 980$ and $5 \times 15^2 = 1125$ **and M1** for $5 \times 14.1..^2 = 999$ to 1001 or for another trial of 14 to 15, so that the two trials have straddled 1000 **Examiner's Comments** The approach to the AO3 sequence question was fairly evenly divided between those evaluating for $n = 14$ and $n = 15$ (and occasionally something in between), and those attempting to solve an equation. The first method was usually either fully correct or scored zero because $(5n)^2$ had been evaluated instead of $5n^2$, whilst the second method sometimes failed because of the incorrect order of operations being applied or the answer being left as $10\sqrt{2}$ with no recognition that the result needed to be an integer. A few candidates used both methods without indicating their final attempt – sometimes one was correct and the other wrong.	e.g. M2 for '200 is not a square number' ignore subsequent trials once M2 earned
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
9	a		$5n + 2$ as final answer	2	**B1** for $5n$ seen Examiner's Comments Almost all the candidates worked out the term to term rule of +5 but some did not know what to do with it and writing $5n$ was the big step to take. There were some answers of $n + 5$ seen from those who did not know what to do with the 5.	
	b		92, 84, 76	2	**B1** for 92 in correct place or 100, 92, 84 or –92, –84, –76 or two correct Examiner's Comments Some started the sequence from $n = 0$ so giving 100, 92 and 84 as their answer, whilst others gave 92, 192 and 292 by adding on the 100.	
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
10		i	3, 11, 19	2	**B1** for 2 correct in correct position Or **SC1** for $^{-}5$, 3, 11 **Examiner's Comments** Most candidates could find the first three terms of the sequence correctly in part (a)(i) although some started their sequence with $n = 0$ and were only awarded 1 mark. A small minority of candidates treated it as a term-to-term rule and having found the first term as 3 substituted this into the expression to find 19 as the second term.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	No with valid reason	1	Reasons to involve one of: A All numbers are odd or 96 is even B Use of 91 and 99 C Use of 12.625 D Use of 91 and add 8 E Use of 101 and divide by 8 F 96 is a multiple of 8 **Examiner's Comments** In part (a)(ii) many candidates identified that 96 could not be a term in the sequence and a variety of correct explanations were seen. Common explanations involved all of the numbers in the sequence being odd, demonstrating that 101 ÷ 8 was not an integer, showing that the sequence went from 91 to 99 or commenting that 96 was a multiple of 8. Some gave incomplete explanations such as stating that 91 is in the sequence, or that 96 would not give an integer value. Only a very few candidates thought that 96 was in the sequence.	**Exemplar Response** No, all numbers in sequence are odd and 96 is even (1) A No, the sequence is all odd numbers (1) A No, no even numbers in sequence (1) A No, 91 and 99 are in the sequence (1) B No, the sequence is + 8, when we come to 91 we have to plus it with 8, so then it becomes 99 (1) B or D No, because 8 × 12 – 5 = 91 / 8 × 13 – 5 = 99 (1) B No, $8n - 5 = 96$ gives 12.625 which is not an integer (1) C No, 12.625 is not an integer (1) C No, because 101 ÷ 8 = 12.625 (1) C No, because 8 cannot be added to 91 to get 96 (1) D No, 91 is in the sequence and then we have to add 8 not 5 (1) D No, Add 8 to 91 and it gives 99 (1) B or D No, 8 does not go into 101 (1) E No, because 101 ÷ 8 gives a decimal answer (1) E

Question			Answer/Indicative content	Marks	Part marks and guidance	
						No, because 101 is not in the 8 times table (1) E No, because 8 × 12 = 96 (1) F [implies multiple of 8] No, because 96 ÷ 8 = 12 (1) F [implies multiple of 8] No, because 96 ÷ 8 = 12 – 5 = 91 (1) F [implies multiple of 8] No, because 96 is in the 8 times table so we don't need to subtract 5 (1) F [implies multiple of 8] No, the difference between the terms are adding 8 (0) [no 91] No, it is because the term closest to 96 is 91 which makes it impossible to be the next term (0) [no 99 or add 8] No, because in this sequence if you add 8 you will not get 96 (0) [no 91] No, never in the " + 8 " sequence (0) [no 91] No, n won't be a whole number (0) [insufficient, need to see 12.625] No, 96 is a factor of 8 (0) [incorrect use of factor]
			Total	3		

Question			Answer/Indicative content	Marks	Part marks and guidance	
11	a		[S =] $5n + 1$ **oe**	2	**M1** for $5n$ **soi** **Examiner's Comments** Most candidates gave the correct expression for the nth term of the sequence. Partially correct expressions such as $5n + 6$ or $5n - 1$ were sometimes seen and only a very small number of candidates gave incorrect answers such as $n + 5$.	Condone any alternative letters in place of n and S but eg $n = 5n + 1$ scores **M1** only Condone $n5$ for $5n$ for 2 or 1 marks

Question			Answer/Indicative content	Marks	Part marks and guidance	
	b		$n = 17$ $v = 86$	4	**B3** for $n = 17$ **or** $v = 86$ or 17^{th} term or value 86 stated OR Algebraic method: **M1** for $120 - 2n =$ *their algebraic* '$5n + 1$' **M1** for correct simplification of *their* equation to $an = b$ OR T & I method: **M1** for one correct trial of same integer n in both sequences **M1** for second correct trial of different integer n in both sequences OR Sequence method: **M1** for one sequence continued for at least 10 more terms **M1** for the other sequence continued until a common term is reached AND **B1** for *their v* resulting from correct substitution of *their* answer for n into $120 - 2n$ or $5n + 1$ or *their* '$5n + 1$' **soi** **Examiner's Comments** Candidates who reached the correct answer had used a variety of methods including algebraic solution, trial and improvement and continuation of the two sequences. The most common method was continuation of the sequences and where this was done methodically it often resulted in the correct	Method marks may only be awarded in one of the three alternatives, mark to candidate's advantage Correct or FT their linear expression in n only eg **M2** scored for $7n = 119$ Correct or FT *their* expression from (a) For this method, sequence must be correct **not** FT their (a) Condone one wrong value in **only** one sequence *Their n* must be positive integer Correct substitution seen or implied by correct evaluation of *their v* from *their n*

Question			Answer/Indicative content	Marks	Part marks and guidance	
					answer. Many candidates using this method set out their work in a disorganised manner, made arithmetic slips or showed insufficient terms of each sequence leading to incorrect results. The algebraic method was often clearly laid out and correct, although some errors in signs were seen when collecting terms. Candidates using trial and improvement often used insufficient trials to reach a solution or did not use the same value of *n* in both sequences. Some candidates were confused between the term number and the term value and, having reached the correct values, gave *v* as 17 and *n* as 86.	
			Total	6		

Question			Answer/Indicative content	Marks	Part marks and guidance	
12			$23 - 7n$ oe	2	M1 for $7n$ seen Examiner's Comments In this part, was answered less well than the previous parts. Many candidates identified that the common difference was 7 and many of these knew that this meant that there would be a term involving $7n$. However the fact that it was a decreasing sequence made it harder than usual for candidates to identify the correct expression for the nth term. Common incorrect answers were $23n - 7$, $n - 7$, $7n - 16$ and $7n - 9$. There was little evidence that candidates had checked their expression against the numbers in the sequence.	Condone $s = 23 - 7n$, $t_n = 23 - 7n$ for 2 marks But $n = 23 - 7n$ scores M1 only
			Total	2		

Question			Answer/Indicative content	Marks	Part marks and guidance	
13	a		$\frac{n^2}{n+1}$	2	B1 for n^2 or $n + 1$ Examiner's Comments The key to this part was to spot the patterns in the numerator and in the denominator. Many candidates instead tried to find the first and second differences and at this level this will only work for linear and quadratic sequences.	
	b		[a =] 3 [*b* =] 1 [*c* =] –2	4	B2 for [a =] 3 or M1 for second differences = 6 and M1 for revised differences of –1 0 1 2 or B1 for *b* or *c* correct — accept any correct method see notes Examiner's Comments There are two clear methods to answer this part. Many found a second difference of 6 and then a few gave *a* = 3. This method involves learning how to arrive at "revised differences" and then proceed from there, which was less successful. There were a handful of attempts at using simultaneous equations, but these often did not get beyond the formation of equations.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
			Total	6		
14	a	i	13	1	Ignore subsequent terms Examiner's Comments In part (a), the majority were successful. Some did not recognise the Fibonacci sequence in part (i) and common incorrect answers were 11 or 12. The second sequence caused fewer problems.	
		ii	128	1	Ignore subsequent terms	
	b		$18 - 3n$ oe	2	M1 for $-3n + k$ oe or for $mn + 18$ oe ($m \neq 0$) For 2 or M1, condone eg $n = 18 - 3n$ For 2 or M1, condone use of *other* variable instead of n Examiner's Comments Part (b) was also reasonably well answered.	
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance		
15	a		5	1	<u>Examiner's Comments</u> Although the sequence notation was beyond many, the evaluation of u2 using $u_{n+1} = \sqrt{2u_n + 15}$ with u_1 = 5 was only correctly answered by a small minority of the candidates. Only a few candidates realised they were being given information to set up simultaneous equations in part (b), but about half of these candidates did solve these equations correctly.		
	b		(k =) 5 (r =) 1 nfww	5	**B1** for 206 = 41*k + r* and **B1** for 1031 = 206*k + r* and **M1** for 165k = 825 and **A1** for k = 5 or r = 1 If no or partial method shown, allow full marks for final	If **0** scored, allow **SC2** for final correct answers int erchanged Condone attempt to reduce to one variable by sub. or elim. With max of one error	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					answer correct Examiner's Comments Although the sequence notation was beyond many, the evaluation of u2 using $u_{n+1} = \sqrt{2u_n + 15}$ with $u_1 = 5$ was only correctly answered by a small minority of the candidates. Only a few candidates realised they were being given information to set up simultaneous equations in part (b), but about half of these candidates did solve these equations correctly.	
			Total	6		