1 The functions g and h are such that

$$g(x) = \sqrt[3]{2x - 5} \qquad h(x) = \frac{1}{x}$$

(a) Find g(16)

$g(16) = \sqrt[3]{2(16) - 5} = \sqrt[3]{27} = 3$ (1)

3

(1)

(b) Find $hg^{-1}(x)$

Give your answer in terms of x in its simplest form.

find $g^{-1}(x)$:

$g(x) = \sqrt[3]{2x - 5}$

$x = \sqrt[3]{2y - 5}$

$x^3 = 2y - 5$

$x^3 + 5 = 2y$

$y = \frac{x^3 + 5}{2}$

$g^{-1}(x) = \frac{x^3 + 5}{2}$ (1)

$hg^{-1}(x) = \frac{1}{\frac{x^3 + 5}{2}}$ (1)

$= \frac{2}{x^3 + 5}$ (1)

$hg^{-1}(x) = \frac{2}{x^3 + 5}$

(3)

(Total for Question 1 is 4 marks)

2 $f(x) = \sqrt[3]{x}$
$g(x) = 2x + 3$

$h(x) = fg(x)$

Find $h^{-1}(x)$

$h(x) = fg(x)$

$= f(2x+3)$

$= \sqrt[3]{2x+3}$ (1)

$y = \sqrt[3]{2x+3}$

To find the inverse of $h(x)$, we swap the x with y in the equation.

$x = \sqrt[3]{2y+3}$

$(x)^3 = (\sqrt[3]{2y+3})^3$ (1)

$2y + 3 = x^3$

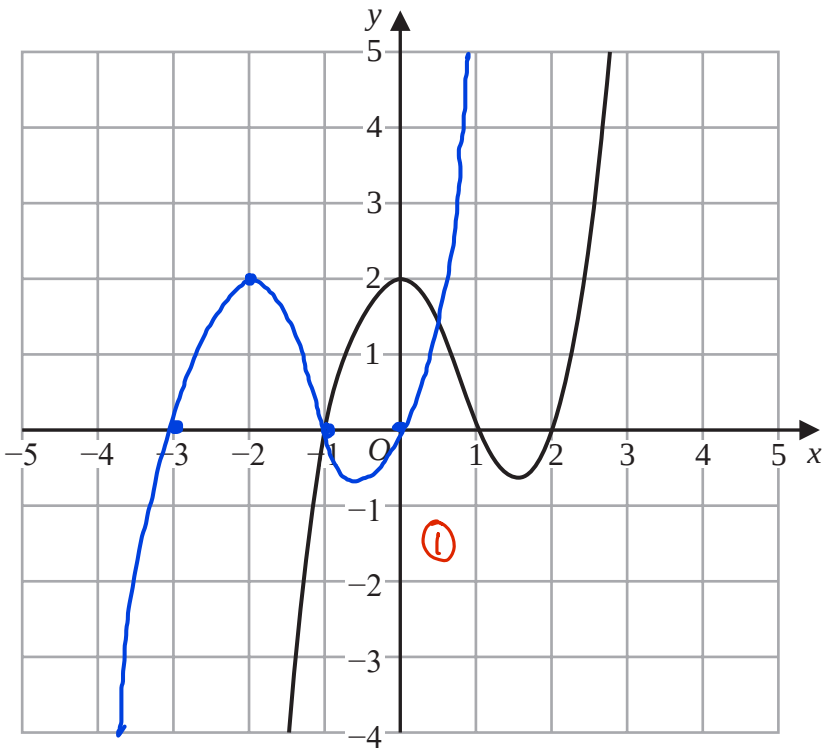
$y = \frac{x^3 - 3}{2}$

$h^{-1}(x) = \frac{x^3 - 3}{2}$ (1)

$h^{-1}(x) =$ $\frac{x^3 - 3}{2}$

(Total for Question 2 is 3 marks)

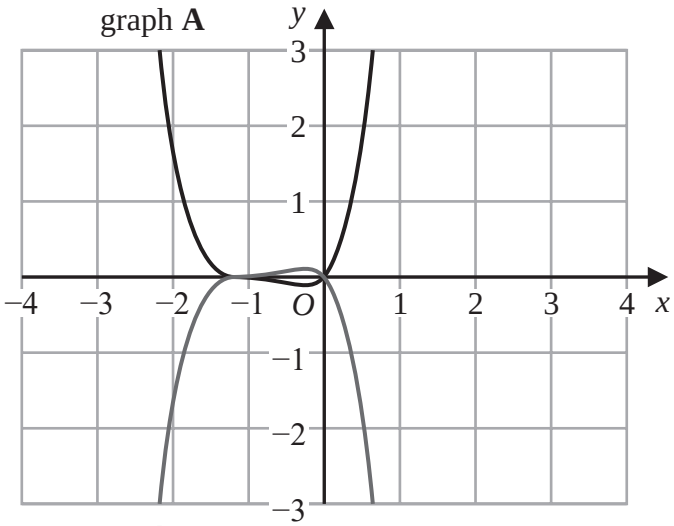
3 The graph of $y = f(x)$ is shown on the grid below.



(a) On the grid above, sketch the graph of $y = f(x + 2)$

translate by vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ (1)

graph **A**



graph **B**

reflection in x-axis

On this grid, graph **A** has been reflected to give graph **B**.
The equation of graph **A** is $y = g(x)$

(b) Write down an equation of graph **B**.

$y = -g(x)$ (1)

(Total for Question 3 is 2 marks)

4 For $x \geqslant 0$, the functions f and g are such that

$$f(x) = 3x + 4 \qquad g(x) = \frac{\sqrt{x} + 2}{5}$$

(a) Find $g^{-1}(x)$

let $y = g(x)$ $\quad y = \frac{\sqrt{x} + 2}{5}$ ✓①

$\times 5 \quad \times 5$

$5y = \sqrt{x} + 2$

$-2 \quad -2$

$5y - 2 = \sqrt{x}$

$(5y - 2)^2 = x$

solve for x

change y to x and x to $g^{-1}(x)$

$(5x - 2)^2 = g^{-1}(x)$

$g^{-1}(x) =$ $(5x - 2)^2$ ✓①

(2)

(b) Solve $gf(x) = 3$

use $g(x) = \frac{\sqrt{x} + 2}{5}$ but substitute $x = 3x + 4$

$\frac{\sqrt{3x+4} + 2}{5} = 3$ ✓①

$\times 5 \quad \times 5$

$\sqrt{3x+4} + 2 = 15$

$-2 \quad -2$

$\sqrt{3x+4} = 13$

$3x + 4 = 169$ ✓①

$-4 \quad -4$

$3x = 165$ so $x = 55$

$\div 3 \quad \div 3$

$x =$ 55 ✓①

(3)

(Total for Question 4 is 5 marks)

5 The functions f and g are given by

$$f(x) = \frac{12}{x+1} \quad \text{and} \quad g(x) = 5 - 3x$$

(a) Find f(–3)

$f(x) = \frac{12}{x+1}$

$f(-3) = \frac{12}{(-3)+1}$ substitute $x = -3$

$= \frac{12}{-2} = -6$ (1)

-6

(1)

(b) Find fg(1)

$fg(x) = \frac{12}{(5-3x)+1}$ (1) substitute g(x) into x

$= \frac{12}{6-3x}$

$fg(1) = \frac{12}{6-3(1)}$ Substitute $x = 1$

$= \frac{12}{3}$

$= 4$ (1)

4

(2)

(c) Find $g^{-1}(4)$

$y = 5 - 3x$ Let $g(x) = y$

$3x = 5 - y$

$x = \frac{5-y}{3}$ Make x the subject

$g^{-1}(x) = \frac{5-x}{3}$ (1) Change y to x

$g^{-1}(4) = \frac{5-4}{3}$ Substitute $x = 4$

$= \frac{1}{3}$ (1)

$\frac{1}{3}$

(2)

(Total for Question 5 is 5 marks)