1 (a) Prove that

$$(2m + 1)^2 - (2n - 1)^2 = 4(m + n)(m - n + 1)$$

LHS: $(2m+1)^2-(2n-1)^2$ ↓ difference of two squares

$=(2m+1+[2n-1])(2m+1-[2n-1])$ ①

$=(2m+2n)(2m-2n+2)$ ① ↓ factor out 2 from both brackets

$=2(m+n)\times 2(m-n+1)$

$=4(m+n)(m-n+1)$

$=$ RHS ①

(3)

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4

(b) Is Sophia correct?
You must give reasons for your answer.

Yes, because $2m+1$ and $2n-1$ are both odd numbers and the right-hand side is divisible by 4. ①

(1)

(Total for Question 1 is 4 marks)

2 (a) Factorise $a^2 - b^2$

$(a-b)(a+b)$ (1)

(1)

(b) Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

Let $a^2 = 2^{40}$, $b^2 = 1$

$a = 2^{20}$ $b = 1$

$2^{40} - 1 = (2^{20} - 1)(2^{20} + 1)$ (1)

2^{20} = even number, so $(2^{20} - 1)$ is odd.

$(2^{20} + 1)$ is also odd (1)

∴ Hence, $(2^{20} - 1)(2^{20} + 1)$ is product of two consecutive odd numbers

$(2^{20} - 1)$ and $(2^{20} + 1)$ are consecutive because they are 2 apart. **(2)**

(Total for Question 2 is 3 marks)

3 There are only red counters and yellow counters in bag **A**.

number of red counters : number of yellow counters = 3 : 5

There are only green counters and blue counters in bag **B**.
The number of counters in bag **B** is half the number of counters in bag **A**.

Given that there are x red counters in bag **A**,

use algebra to show that the total number of counters in bag **A** and bag **B** is $4x$

Red counters = x

Yellow counters = $\frac{5x}{3}$

Total counters in bag A = $x + \frac{5x}{3} = \frac{3x + 5x}{3} = \frac{8x}{3}$ (1)

Total counters in bag B = $\frac{8x}{3} \times \frac{1}{2} = \frac{8x}{6} = \frac{4x}{3}$ (1)

Total counters in both bags = $\frac{8x}{3} + \frac{4x}{3}$

$= \frac{12x}{3}$

$= 4x$ (shown) (1)

(Total for Question 3 is 3 marks)