1 (a) Express $\frac{x}{x+2} + \frac{2x}{x-4}$ as a single fraction in its simplest form.

$\frac{x}{x+2} + \frac{2x}{x-4}$ (Multiply denominators with each other)

$= \frac{x(x-4)}{(x+2)(x-4)} + \frac{(2x)(x+2)}{(x+2)(x-4)}$ (1)

$= \frac{x^2 - 4x + 2x^2 + 4x}{(x+2)(x-4)}$ (1)

$= \frac{3x^2}{(x+2)(x-4)}$ (1)

$\frac{3x^2}{(x+2)(x-4)}$

(3)

(b) Expand and simplify $(x - 3)(2x + 3)(4x + 5)$

$(x-3)(2x+3)(4x+5)$

$= (2x^2 - 3x - 9)(4x + 5)$ (1)

$= 8x^3 + 10x^2 - 12x^2 - 15x - 36x - 45$ (1)

$= 8x^3 - 2x^2 - 51x - 45$ (1)

$8x^3 - 2x^2 - 51x - 45$

(3)

(Total for Question 1 is 6 marks)

2 Show that $\frac{3x}{x+2} - \frac{2x+1}{x-2} - 1$ can be written in the form $\frac{ax+b}{x^2-4}$ where a and b are integers.

make all terms have a common denominator of $(x+2)(x-2)$

$$\frac{3x(x-2)}{(x+2)(x-2)} - \frac{(2x+1)(x+2)}{(x+2)(x-2)} - \frac{(x+2)(x-2)}{(x+2)(x-2)} \quad (1)\ (1)$$

expand brackets along numerator ↓

$$= \frac{3x^2-6x}{(x+2)(x-2)} - \frac{2x^2+4x+x+2}{(x+2)(x-2)} - \frac{x^2-4}{(x+2)(x-2)}$$

combine onto single fraction ↓

$$= \frac{3x^2-6x-(2x^2+5x+2)-(x^2-4)}{(x+2)(x-2)}$$

simplify terms ↓

$$= \frac{3x^2-6x-2x^2-5x-2-x^2+4}{x^2-4} \quad (1)$$

$$= \frac{-11x+2}{x^2-4} \quad (1)$$

(Total for Question 2 is 4 marks)

3 Solve $\frac{1}{x} - \frac{1}{x+1} = 4$

Give your answer in the form $a \pm b\sqrt{2}$ where a and b are fractions.

$\frac{1}{x} - \frac{1}{x+1} = 4$

$\frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} = 4$

$\frac{x+1-x}{x(x+1)} = 4$ (1)

$\frac{1}{x(x+1)} = 4$

$1 = 4x(x+1)$

$1 = 4x^2 + 4x$

$4x^2 + 4x - 1 = 0$ (1)

$4x^2+4x-1=0$

quadratic formula:

$x = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times -1}}{2 \times 4}$ (1)

$x = \frac{-4 \pm \sqrt{32}}{8}$

$\sqrt{32} = \sqrt{2 \times 16}$
$= \sqrt{2} \times \sqrt{16}$
$= 4\sqrt{2}$

$x = \frac{-4 \pm 4\sqrt{2}}{8}$ (1)

$x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{2}$ (1)

$x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{2}$

(Total for Question 3 is 5 marks)

4 $\frac{2x+3}{x-5} + \frac{x-4}{x+5} - 3$ can be written in the form $\frac{ax+b}{x^2-25}$ where a and b are integers.

Work out the value of a and the value of b.
You must show all your working.

$$\frac{2x+3}{x-5} + \frac{x-4}{x+5} - 3$$

give fractions the same denominator

$$= \frac{(2x+3)(x+5)}{(x+5)(x-5)} + \frac{(x-4)(x-5)}{(x+5)(x-5)} - \frac{3(x+5)(x-5)}{(x+5)(x-5)} \quad (1)$$

$$= \frac{(2x+3)(x+5) + (x-4)(x-5) - 3(x+5)(x-5)}{(x+5)(x-5)}$$

combine onto one fraction

$$= \frac{2x^2+3x+10x+15+x^2-4x-5x+20-3x^2+75}{x^2-25}$$

expand all brackets

$$= \frac{4x+110}{x^2-25}$$

simplify

$a =$ 4 (1)

$b =$ 110

(Total for Question 4 is 3 marks)

5 Solve $\frac{1}{x+4} + \frac{3}{2-2x} = 1$

Eliminate the fractions by multiplying by $(x+4)(2-2x)$:

$$\frac{1}{x+4}(x+4)(2-2x) + \frac{3}{2-2x}(x+4)(2-2x) = 1(x+4)(2-2x)$$

$$2-2x + 3(x+4) = (x+4)(2-2x) \quad (1)$$

$$2-2x+3x+12 = 2x-2x^2+8-8x$$

$$14+x = -2x^2-6x+8$$

$$2x^2+7x+6 = 0 \quad (1)$$

Substituting into formula :

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{7^2-4(2)(6)}}{2(2)} \quad (1)$$

$$= \frac{-7 \pm \sqrt{1}}{4}$$

$$= \frac{-7+1}{4} \quad \text{and} \quad \frac{-7-1}{4}$$

$$x = -\frac{3}{2} \quad \text{and} \quad -2 \quad (1)$$

$x = -2$ and $x = -\frac{3}{2}$

(Total for Question 5 is 4 marks)