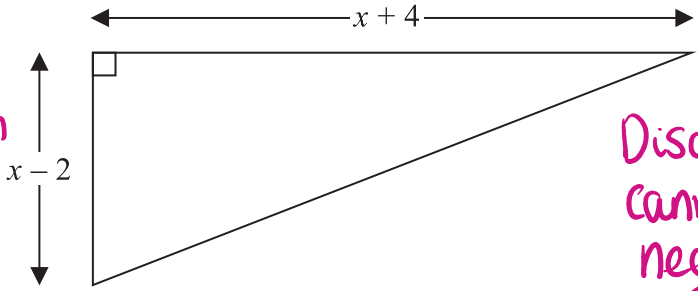
1. The diagram shows a right-angled triangle.

$7+4=11\text{cm}$

$x=7$

$7-2=5\text{cm}$



Discard $x=-9$ since cannot have negative length

All the measurements are in centimetres.

The area of the triangle is 27.5 cm²

Work out the length of the shortest side of the triangle.
You must show all your working.

$$\text{Area of Triangle} = \frac{\text{Base} \times \text{Height}}{2}$$

$$\text{Area} = \frac{1}{2} \times (x-2) \times (x+4) \quad (1)$$

$$\cancel{2} \times \frac{1}{\cancel{2}}(x-2)(x+4) = 27.5 \times 2$$

$$(x-2)(x+4) = 55$$

$$x^2 + 4x - 2x - 8 = 55$$

$$x^2 + 2x - 8 = 55$$

-55 -55

$$x^2 + 2x - 63 = 0 \quad (1)$$

$-7 \times 9 = -63$
$-7 + 9 = 2$

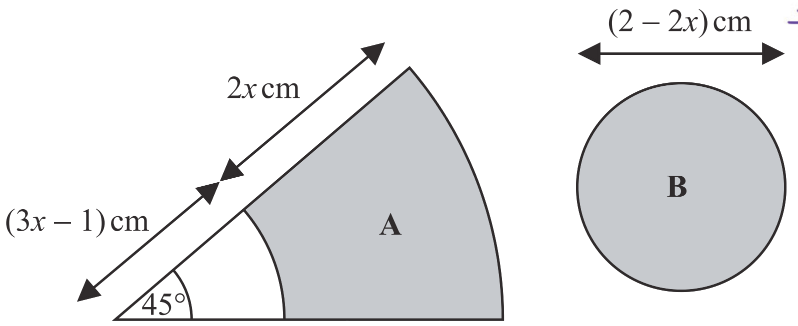
$$(x-7)(x+9) = 0$$

$x - 7 = 0$ $x + 9 = 0$
$x = 7$ or $x = -9$ (1)

(1) 5 cm

2. The diagram shows two shaded shapes, **A** and **B**.

Shape **A** is formed by removing a sector of a circle with radius $(3x - 1)$ cm from a sector of the circle with radius $(5x - 1)$ cm.
Shape **B** is a circle of diameter $(2 - 2x)$ cm.



$\rightarrow r = \frac{1}{2}(2-2x)$

$r = 1 - x$

The area of shape **A** is equal to the area of shape **B**.

$a.s = \frac{\theta}{360} \times \pi r^2$

Find the value of x.
You must show all your working.

Area of shape A = area of sector – cutout.

area of sector $= \frac{45°}{360°} \times \pi \times (5x-1)^2 \text{ cm}^2$

$= \frac{1}{8} \times \pi \times (5x-1)^2$

cutout $= \frac{45°}{360°} \times \pi \times (3x-1)^2 \text{ cm}^2$

$= \frac{1}{8} \times \pi (3x-1)^2$

$(5x-1)(5x-1)$

$25x^2 - 5x - 5x + 1$

$= \frac{1}{8}\pi\left((5x-1)^2 - (3x-1)^2\right)$ ✓1

$(3x-1)(3x-1)$

$9x^2 - 3x - 3x + 1$

$= \frac{1}{8}\pi\left((25x^2 - 10x + 1) - (9x^2 - 6x + 1)\right)$

$= \frac{1}{8}\pi\left(16x^2 - 4x\right)$ ✓2

$(1-x)(1-x)$

$1 - x - x + x^2$

area of B $= \pi(1-x)^2 = \pi(x^2 - 2x + 1)$

$\times 8$ $\frac{1}{8}(16x^2 - 4x)\cancel{\pi} = \cancel{\pi}(x^2 - 2x + 1)$ $\times 8$

$-(8x^2 - 16x + 8)$ $16x^2 - 4x = 8x^2 - 16x + 8$ $-(8x^2 - 16x + 8)$

$8x^2 + 12x - 8 = 0$

$2x^2 + 3x - 2 = 0$ ✓3

$x_1, x_2 = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2 \times 2}$

$= \frac{1}{2}, -2$ ✓4

$x = \frac{1}{2}$ ✓5

(Total for Question is 5 marks)

3. Solve $x^2 = 5x + 24$

$x^2 = 5x + 24$

$x^2 - 5x - 24 = 0$ ①

$(x-8)(x+3) = 0$ ①

When $(x-8) = 0$: $x = 8$

When $(x+3) = 0$: $x = -3$

①

$x = 8,\ x = -3$

(Total for Question is 3 marks)

4. The curve **C** has equation $y = x^2 + 3x - 3$

The line **L** has equation $y - 5x + 4 = 0$

Show, algebraically, that **C** and **L** have exactly one point in common.

If C and L have one point (x, y) in common, they have the same x-value and the same y-value.

Ⓒ $y = x^2 + 3x - 3.$

Ⓛ $y - 5x + 4 = 0 \therefore y = 5x - 4.$

$\therefore x^2 + 3x - 3 = 5x - 4.$

(1)

$x^2 + 3x - 3 = 5x - 4.$

$x^2 - 2x - 3 = -4.$

$x^2 - 2x + 1 = 0.$

(1)

$x^2 - 2x + 1 = 0.$

$(x-1)^2 = 0.$

$\therefore x = 1.$

(1)

(1)

There is only one value of x and so C and L have only one point in common.

(Total for Question is 4 marks)