

1. Prove algebraically that

$(2n + 1)^2 - (2n + 1)$  is an **even number**

for all positive integer values of  $n$ .

$$\begin{aligned}
 & (2n+1)^2 - (2n+1) \\
 &= (2n+1)(2n+1) - (2n+1) \quad \textcircled{1} \\
 &= (2n \times 2n) + (2n \times 1) + (1 \times 2n) + (1 \times 1) - (2n+1) \\
 &= 4n^2 + 4n + 1 + [-1 \times (2n+1)] \\
 &= 4n^2 + 4n + 1 + [-2n - 1] \\
 &= 4n^2 + 4n + 1 - 2n - 1 \\
 &= 4n^2 + 2n \quad \textcircled{1}
 \end{aligned}$$

outside  
↑  
FOIL  
↓ Front ↓ Inside  
Last

$$\begin{aligned}
 2 &= 2 \times 1 \\
 4 &= 2 \times 2 \\
 6 &= 2 \times 3 \\
 8 &= 2 \times 4 \\
 10 &= 2 \times 5
 \end{aligned}$$

All even numbers  
have a factor  
of 2

$$\begin{aligned}
 & 4n^2 + 2n \\
 &= 2(2n^2 + n)
 \end{aligned}$$

Since 2 is a factor  
of  $4n^2 + 2$  it is  
even for all  
positive integer  
values of  $n$

2.  $n$  is an integer greater than 1

Prove algebraically that  $n^2 - 2 - (n - 2)^2$  is always an even number.

An even number is always divisible by 2.

$$n^2 - 2 - ((n-2)(n-2)). \quad n^2 - 2 - (n^2 - 2n - 2n + 4)$$

$$n^2 - 2 - 1(n^2 - 4n + 4). \quad \cancel{n^2 - 2} - \cancel{n^2} + 4n - 4$$

$$4n - 6. \quad n^2 - 2 - (n-2)^2 = 4n - 6.$$

$$2(2n - 3).$$

$$n^2 - 2 - (n-2)^2 = 2(2n-3). \quad 2(2n-3) \text{ is a}$$

multiple of 2  $\therefore n^2 - 2 - (n-2)^2$  is always an even number.

(Total for Question is 4 marks)

3. Using algebra, prove that  $0.\dot{1}\dot{3}\dot{6} \times 0.\dot{2}$  is equal in value to  $\frac{1}{33}$

$$0.\dot{1}\dot{3}\dot{6} = 0.136363636\dots$$

$$x = 0.13636\dots$$

$$10x = 1.36363\dots$$

$$100x = 13.63636\dots$$

$$1000x = 136.36363\dots$$

$$1000x - 10x$$

$$= 136.36363\dots$$

$$- 1.36363\dots$$

$$\hline 135.$$

$$990x = 135.$$

$$x = \frac{135}{990} \quad (1)$$

$$0.\dot{2} = 0.22222\dots$$

$$x = 0.2222\dots$$

$$10x = 2.2222\dots$$

$$10x - x$$

$$= 2.2222$$

$$- 0.2222$$

$$\hline 2$$

$$9x = 2.$$

$$x = \frac{2}{9} \quad (1)$$

$$\frac{135}{990} \times \frac{2}{9}$$

$$= \frac{270}{8910}$$

$$= \boxed{\frac{1}{33}} \quad (1)$$

(Total for Question is 3 marks)

$$\frac{y+x}{y-x} = \frac{y+x}{y-x}$$

$$\begin{aligned} k &: 1 \\ (k+1) &: (k+1) \\ y+x &: y-x \end{aligned}$$

$$\frac{k}{1} = \frac{y+x}{y-x} \quad \checkmark$$

$$k = \frac{y+x}{y-x}$$

$$\times (y-x) \quad \times (y-x)$$

$$k(y-x) = y+x$$

$$\begin{aligned} ky - kx &= y+x \\ + kx &+ kx \end{aligned}$$

$$ky = y+x + kx$$

$$-y \quad -y$$

$$ky - y = x + kx \quad \checkmark$$

$$y(k-1) = x(k+1)$$

$$\div (k-1) \quad \div (k-1)$$

$$y = \frac{x(k+1)}{k-1} \quad \checkmark$$

4.  $x = 0.4\dot{3}\dot{6}$

Prove algebraically that  $x$  can be written as  $\frac{24}{55}$

$$\frac{432}{990} = \frac{216}{495} = \frac{24}{55}$$

$$\begin{aligned} x &= 0.4363636 \dots \\ 10x &= 4.363636 \dots \quad \checkmark \\ 100x &= 43.636363 \dots \\ 1000x &= 436.363636 \dots \end{aligned}$$

$$1000x - 10x = 436.36 - 4.36 \quad \checkmark$$

$$\begin{aligned} 990x &= 432 \\ \div 990 &\div 990 \end{aligned}$$

$$x = \frac{432}{990}$$

$$x = \frac{24}{55} \quad \checkmark$$

(Total for Question is 3 marks)

$$\begin{aligned}
 y &\propto \sqrt[3]{x} \\
 y &= k \times \sqrt[3]{x} \quad \checkmark \\
 \frac{7}{6} &= k \times \sqrt[3]{8} \\
 \frac{7}{6} &= k \times 2 \\
 \div 2 &\quad \div 2 \\
 \frac{7}{12} &= k \quad \checkmark
 \end{aligned}$$

When  $x = 64$ 

$$y = \frac{7}{12} \times \sqrt[3]{64} \quad \sqrt[3]{64} = 4$$

$$y = \frac{7}{12} \times 4$$

$$y = \frac{28}{12}$$

$$y = \frac{7}{3}$$

$$\frac{7}{3} \quad \checkmark$$

5.  $n$  is an integer.

Prove algebraically that the sum of  $\frac{1}{2}n(n+1)$  and  $\frac{1}{2}(n+1)(n+2)$  is always a square number.

$$\begin{aligned}
 &\frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2) \\
 &\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}(n^2 + 2n + n + 2) \\
 &\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}(n^2 + 3n + 2) \\
 &\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}n^2 + \frac{3}{2}n + 1 \quad \checkmark \\
 &n^2 + 2n + 1 \\
 &(n+1)(n+1) \\
 &(n+1)^2
 \end{aligned}$$

Therefore the sum will always be a square number because  $\frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2) = (n+1)^2 \quad \checkmark$

6. Prove that the square of an odd number is always 1 more than a multiple of 4

↙ an expression to represent any odd number

$$2n + 1 \quad (1)$$

$$(2n + 1)^2 = (2n + 1)(2n + 1)$$

odd<sup>2</sup>

$$= 4n^2 + 2n + 2n + 1 \quad (1)$$

$$= 4n^2 + 4n + 1 \quad (1)$$

$$= 4(n^2 + n) + 1$$

a multiple of 4      +1      is one more than a multiple of 4 as required  
(1)

(Total for Question is 4 marks)



7. Given that  $n$  can be any integer such that  $n > 1$ , prove that  $n^2 - n$  is never an odd number.

if  $n$  is odd:  $n^2 - n = (2n+1)^2 - (2n+1)$

$$= 4n^2 + 4n + 1 - (2n + 1)$$
$$= 4n^2 + 2n = 2(2n^2 + 1)$$

①

multiple of 2  $\therefore$  even.

if  $n$  is even:  $n^2 - n = (2n)^2 - (2n)$

$$= 4n^2 - 2n = 2(2n^2 - 1)$$

①

multiple of 2  $\therefore$  even.

$n^2 - n$  is even when  $n$  is odd and when  $n$  is even.  
 $\therefore n^2 - n$  is never an odd number.

(Total for Question is 2 marks)

8. Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

$2n$  ~ even number ~ when  $n$  is any whole number  
 $2n+2$  ~ 'next' even number ~

$$\begin{aligned}
 (2n)^2 + (2n+2)^2 &= (2^2n^2) + (2n+2)(2n+2) \\
 &= (2^2n^2) + (4n^2 + 4n + 4n + 4) \\
 &= (2^2n^2) + (4n^2 + 8n + 4) \quad (2) \\
 &= 4n^2 + 4n^2 + 8n + 4 \\
 &= 8n^2 + 8n + 4 \\
 &= 4(2n^2 + 2n + 1) \quad (1)
 \end{aligned}$$

$y \propto \frac{1}{x^2}$   
 $y = \frac{k}{x^2}$   
 $8 = \frac{k}{2.5^2}$   
 $k = 8 \times 2.5^2$   
 $k = 50$

we want to work out value of  $k$  so sub in info from question

$y = \frac{50}{x^2}$  Sub  $y = \frac{8}{9}$

$$\frac{8}{9} = \frac{50}{x^2}$$

$$x^2 \times \frac{8}{9} = 50$$

$$x^2 \times \frac{8}{9} \times \frac{9}{8} = 50 \times \frac{9}{8}$$

$$x^2 = 56.25$$

$$x = \pm \sqrt{56.25}$$

$x = \pm 7.5$  Since we want negative value  
 $-7.5$