1. Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

$(2n+1)^2 - (2n+1)$

$= (2n+1)(2n+1) - (2n+1)$ ①

$= (2n \times 2n) + (2n \times 1) + (1 \times 2n) + (1 \times 1) - (2n+1)$

$= 4n^2 + 4n + 1 + [-1 \times (2n+1)]$

$= 4n^2 + 4n + 1 + [-2n - 1]$

$= 4n^2 + 4n + 1 - 2n - 1$

$= 4n^2 + 2n$ ①

Outside
FOIL
Front
Inside
Last

$2 = 2 \times 1$
$4 = 2 \times 2$
$6 = 2 \times 3$
$8 = 2 \times 4$
$10 = 2 \times 5$

$\boxed{\text{All even numbers have a factor of 2}}$

$4n^2 + 2n$
$= 2(2n^2 + n)$ ①

Since 2 is a factor of $4n^2 + 2$ it is even for all positive integer values of n

2. n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

An even number is always divisible by 2.

$n^2 - 2 - ((n-2)(n-2))$. $n^2 - 2 - (n^2 - 2n - 2n + 4)$

$n^2 - 2 - 1(n^2 - 4n + 4)$. $\cancel{n^2} - 2 - \cancel{n^2} + 4n - 4$ (1)

$4n - 6$. $n^2 - 2 - (n-2)^2 = 4n - 6$.

$2(2n - 3)$. (1)

$n^2 - 2 - (n-2)^2 = 2(2n-3)$. $2(2n-3)$ is a multiple of 2 ∴ $n^2 - 2 - (n-2)^2$ is always an even number. (1)

(Total for Question is 4 marks)

3. Using algebra, prove that $0.1\dot{3}\dot{6} \times 0.\dot{2}$ is equal in value to $\dfrac{1}{33}$

$0.1\dot{3}\dot{6} = 0.136363636\ldots$

$x = 0.13636\ldots$

$10x = 1.36363\ldots$

$100x = 13.63636\ldots$

$1000x = 136.36363\ldots$

$1000x - 10x$

$= 136.36363\ldots$
$- \quad 1.36363\ldots$

135

$990x = 135$

$x = \dfrac{135}{990}$ (1)

$0.\dot{2} = 0.22222\ldots$

$x = 0.2222$

$10x = 2.2222$

$10x - x$

$= 2.2222$
$- \ 0.2222$

2

$9x = 2$

$x = \dfrac{2}{9}$ (1)

$\dfrac{135}{990} \times \dfrac{2}{9}$

$= \dfrac{270}{8910}$

$= \boxed{\dfrac{1}{33}}$ (1)

(Total for Question is 3 marks)

4. $x = 0.4\dot{3}\dot{6}$

Prove algebraically that x can be written as $\frac{24}{55}$

$\frac{432}{990} = \frac{216}{495} = \frac{24}{55}$

$x = 0.4363636\ldots$

$10x = 4.363636\ldots$ ✓

$100x = 43.636363\ldots$

$1000x = 436.363636\ldots$

$1000x - 10x = 436.\dot{3}\dot{6} - 4.\dot{3}\dot{6}$ ✓

$990x = 432$

$\div 990 \quad \div 990$

$x = \frac{432}{990}$

$x = \frac{24}{55}$ ✓

(Total for Question is 3 marks)

5. n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n + 1)$ and $\frac{1}{2}(n + 1)(n + 2)$ is always a square number.

$\frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2)$

$\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}(n^2+2n+n+2)$

$\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}(n^2+3n+2)$

$\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}n^2 + \frac{3}{2}n + 1$ ✓

$n^2 + 2n + 1$

$(n+1)(n+1)$

$(n+1)^2$

Therefore the sum will always be a square number because $\frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2) = (n+1)^2$ ✓

(Total for Question is 2 marks)

6. Prove that the square of an odd number is always 1 more than a multiple of 4

an expression to represent any odd number

$2n+1$ (1)

$(2n+1)^2 = (2n+1)(2n+1)$

odd^2

$= 4n^2+2n+2n+1$ (1)

$= 4n^2+4n+1$ (1)

$= \underbrace{4(n^2+n)}_{\text{a multiple of 4}} \underbrace{+1}_{+1}$ is one more than a multiple of 4 as required (1)

(Total for Question is 4 marks)

7. Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

If n is odd: $n^2 - n = (2n+1)^2 - (2n+1)$

$= 4n^2 + 4n + 1 - (2n+1)$

$= 4n^2 + 2n = \underline{\underline{2(2n^2+1)}}$ multiple of 2 ∴ even.

1

If n is even: $n^2 - n = (2n)^2 - (2n)$

1

$= 4n^2 - 2n = \underline{\underline{2(2n^2-1)}}$ multiple of 2 ∴ even.

$n^2 - n$ is even when n is odd and when n is even.

∴ $n^2 - n$ is never an odd number.

(Total for Question is 2 marks)

8. Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

$2n$ ← even number ← when n is any whole number

$2n+2$ ← 'next' even number

$$(2n)^2+(2n+2)^2 = (2^2n^2)+(2n+2)(2n+2)$$

$$= (2^2n^2)+(4n^2+4n+4n+4)$$

$$= (2^2n^2)+(4n^2+8n+4) \quad (2)$$

$$= 4n^2+4n^2+8n+4$$

$$= 8n^2+8n+4$$

$$= 4(2n^2+2n+1) \quad (1)$$