Name: ____

Higher Unit 18 topic test

Date:

Time: 55 minutes

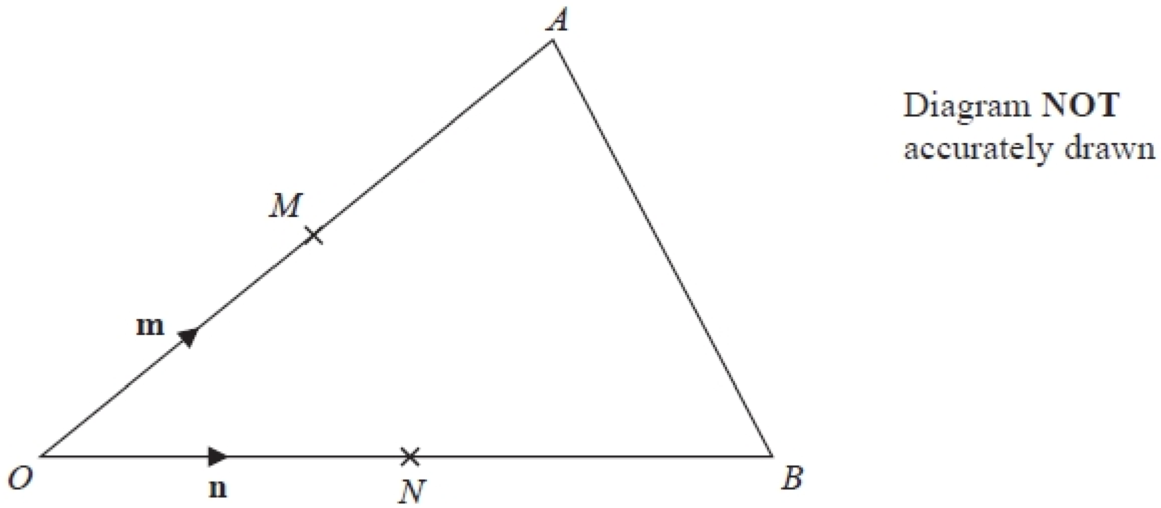
Total marks available: 47

Total marks achieved: ______

Questions

Q1.

*



OAB is a triangle.
M is the midpoint of *OA*.
N is the midpoint of *OB*.

$\overrightarrow{OM}$ = **m**

$\overrightarrow{ON}$ = **n**

Show that *AB* is parallel to *MN*.

(Total for Question is 3 marks)

Q2.

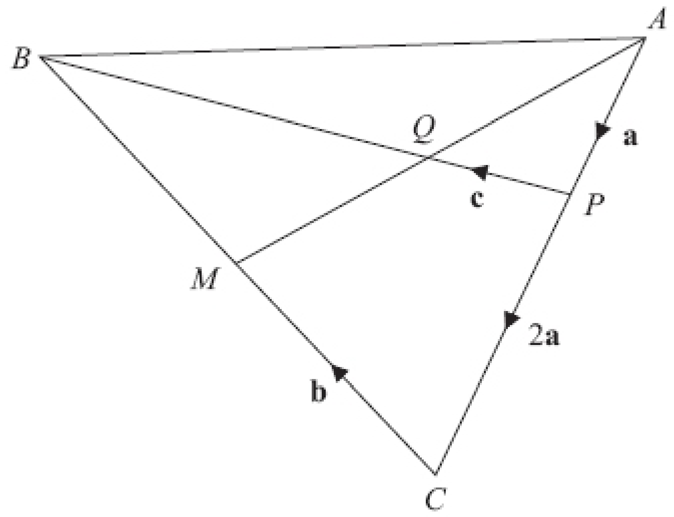


Diagram **NOT** accurately drawn

M is the midpoint of *BC*.
Q is the midpoint of *AM*.

$\overrightarrow{AP} = \mathbf{a}$ $\overrightarrow{PC} = 2\mathbf{a}$ $\overrightarrow{CM} = \mathbf{b}$ $\overrightarrow{PQ} = \mathbf{c}$

(a) Find $\overrightarrow{AM}$ in terms of **a** and **b**.

$\overrightarrow{AM}$ = ...

(1)

(b) Find $\overrightarrow{QB}$ in terms of **c**.

$\overrightarrow{QB}$ = ...

(4)

(Total for Question is 5 marks)

Q3.

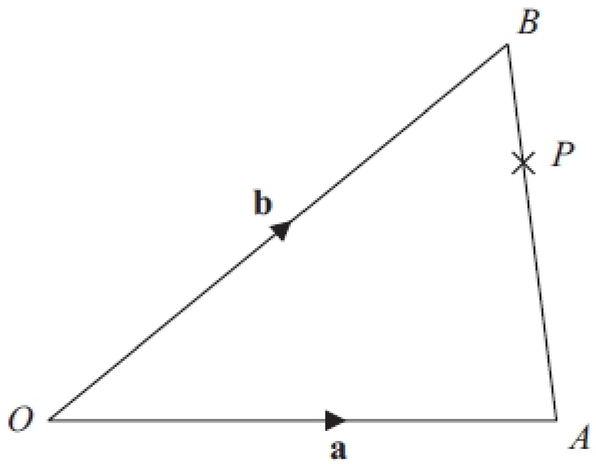


Diagram **NOT** accurately drawn

OAB is a triangle.

$\overrightarrow{OA}$ = **a**

$\overrightarrow{OB}$ = **b**

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

....

(1)

P is the point on *AB* such that *AP* : *PB* = 3 : 1

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**. Give your answer in its simplest form.

....

(3)

(Total for Question is 4 marks)

Q4.

The diagram shows a regular hexagon *OABCDE*.

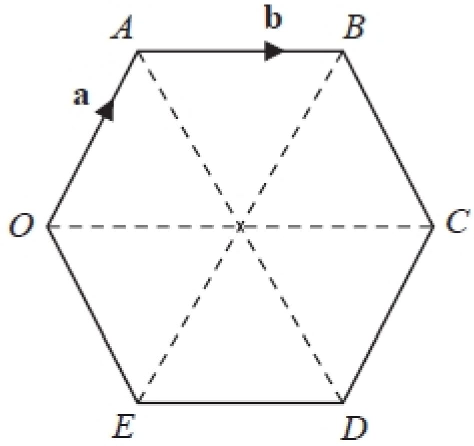


Diagram **NOT** accurately drawn

$\overrightarrow{OA}$ = **a**

$\overrightarrow{AB}$ = **b**

M is the midpoint of *OE*.
N is the midpoint of *AB*.

(a) Find $\overrightarrow{MN}$ in terms of **a** and/or **b**.

$\overrightarrow{MN}$ = ...

(3)

(b) Describe fully what your answer to part (a) shows about the lines *OA* and *MN*.

....

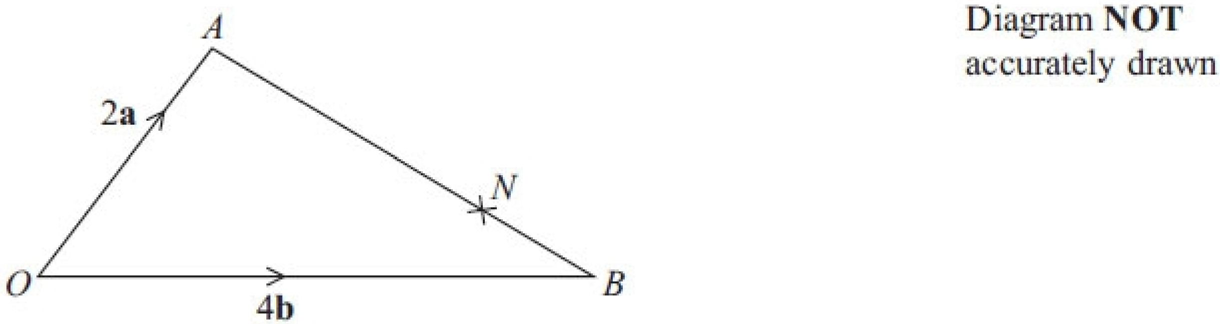
....

(2)

(Total for question = 5 marks)

Q5.

OAB is a triangle.



N is the point on *AB* such that *AN* : *NB* = 3 : 1

$\overrightarrow{OA} = 2\mathbf{a}$
$\overrightarrow{OB} = 4\mathbf{b}$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

....

(1)

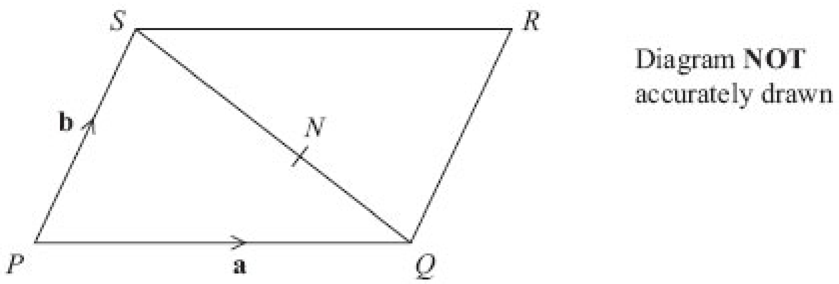
(b) Find $\overrightarrow{ON}$ in terms of **a** and **b**.
Give your vector in its simplest form.

....

(3)

(Total for Question is 4 marks)

Q6.



PQRS is a parallelogram.
N is the point on *SQ* such that *SN* : *NQ* = 3 : 2
$\overrightarrow{PQ} = \mathbf{a}$
$\overrightarrow{PS} = \mathbf{b}$

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$.

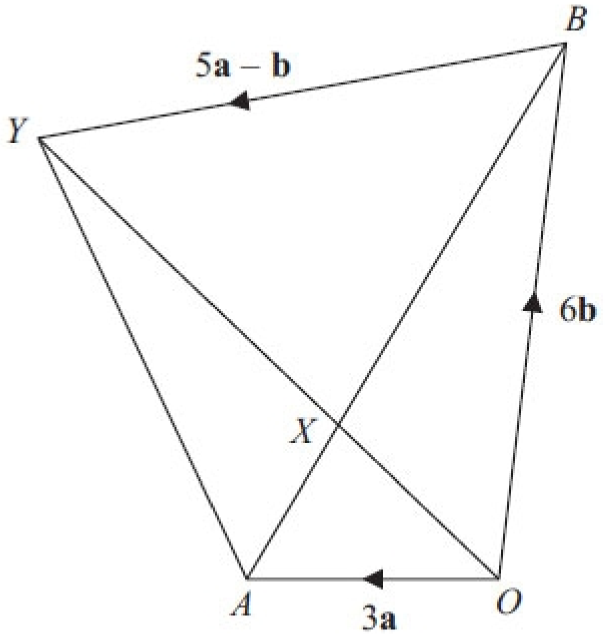
$\overrightarrow{SQ}$ = ...
(1)

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**.

$\overrightarrow{NR}$ = ...
(3)

(Total for Question is 4 marks)

Q7.



OAYB is a quadrilateral.

$\overrightarrow{OA} = 3\mathbf{a}$

$\overrightarrow{OB} = 6\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**.

....

(1)

X is the point on *AB* such that *AX* : *XB* = 1 : 2

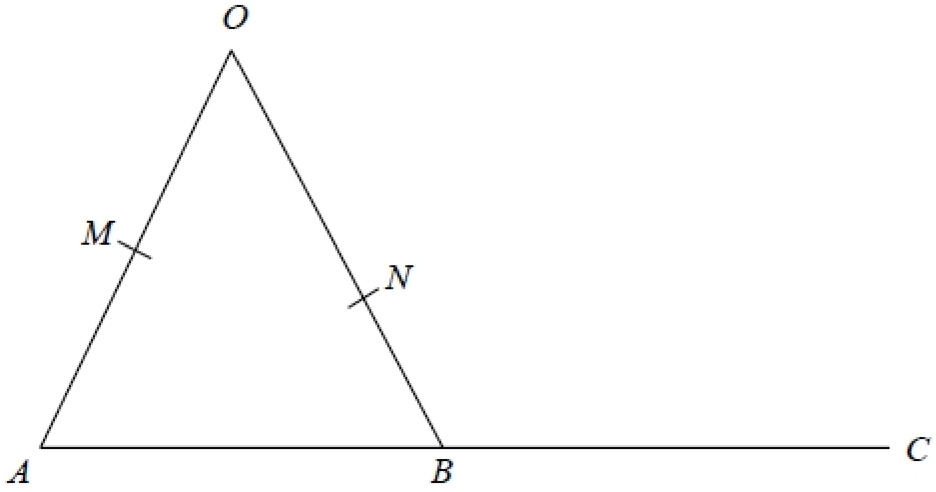
and $\overrightarrow{BY}$ = 5**a** – **b**

*(b) Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$

(4)

(Total for Question is 5 marks)

Q8.



OMA, *ONB* and *ABC* are straight lines.
M is the midpoint of *OA*.
B is the midpoint of *AC*.

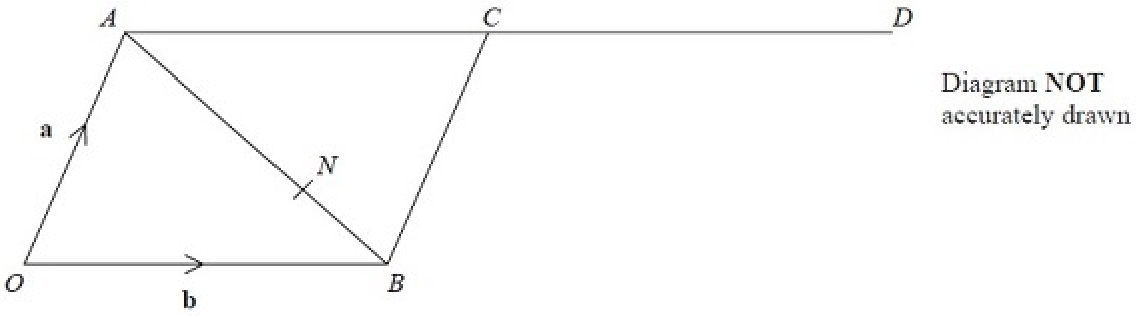
$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{ON} = k\mathbf{b}$ where *k* is a scalar quantity.

Given that *MNC* is a straight line, find the value of *k*.

(Total for question = 5 marks)

Q9.

OACB is a parallelogram.



$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$

The point *N* divides *AB* in the ratio 2:1

(a) Write an expression for $\overrightarrow{ON}$ in terms of **a** and **b**.

....

(3)

*(b) Prove that *OND* is a straight line.

(3)

(Total for Question is 6 marks)

Q10.

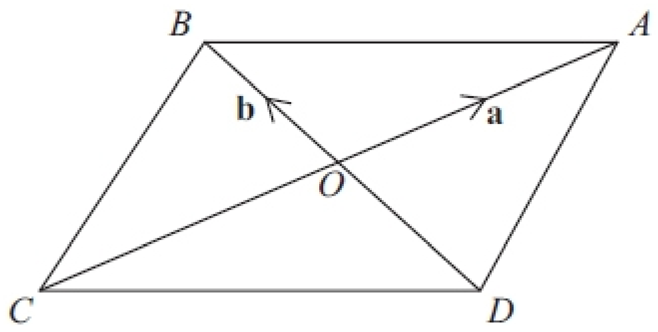


Diagram **NOT** accurately drawn

ABCD is a parallelogram.
The diagonals of the parallelogram intersect at *O*.

$\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**

(a) Write an expression, in terms of **a** and/or **b**, for

(i) $\overrightarrow{CA}$,

. . . .

(ii) $\overrightarrow{BA}$,

. . . .

(ii) $\overrightarrow{BC}$.

. . . .

(3)

X is the point such that $\overrightarrow{OX}$ = 2**a** – **b**

(b) (i) Find an expression, in terms of **a** and **b**, for $\overrightarrow{AX}$.

. . . .

(ii) *B*, *A* and *X* lie on the same straight line.

Explain why.

....

....

....

(3)

(Total for Question is 6 marks)

Examiner's Report

Q1.

Candidates who had some idea of how to find the vectors $\overrightarrow{MN}$ and $\overrightarrow{AB}$ in terms of **m** and **n**, generally scored at least two of the three marks. The third mark was to give a reason based on the forms for $\overrightarrow{MN}$ and $\overrightarrow{AB}$ of why the two lines are parallel. Generally candidates earned the final mark by stating that 2**n** – 2**m** was a multiple of **n** – **m**. In general, notation was poor, with arrows above vectors rarely shown and with underling of **m** and **n** usually absent.

Some candidates did not read the information carefully enough and found that $\overrightarrow{MN}$ and $\overrightarrow{AB}$ were half the values given in the answer. These candidates could score a maximum of two marks.

Q2.

Part (a) was answered correctly by many candidates. In contrast part (b) was rarely attempted. Working was often unlabelled and difficult to follow so it was virtually impossible to follow the candidates' routes to solving the problem. Many established a first step of correctly identifying *QB* as 2*a* +2*b* − c but then did not know what step to take next.

Q3.

Many candidates have a lack of confidence when it comes to working with vectors and this question was frequently not attempted. Those who did attempt it often gained at least one mark as part (a) was generally answered quite well. In part (b) correct expressions for the vector *AP* were much more common than correct expressions for the vector *BP*. Those trying to use *BP* often failed to recognise that the change in direction required a change of signs. Candidates with some idea of what was required often scored one mark for a suitable 'vector journey' although sign errors were often apparent. Some candidates lost marks by failing to include brackets. Those who scored the first two marks for a correct expression for *OP* were often unable to simplify their answer to gain the final accuracy mark. Misunderstanding of ratios led a considerable number of candidates using ⅓ instead of ¼. Responses to this question were often confused and difficult to follow making the marking of them more challenging for examiners.

Q4.

A significant number of students did not attempt this question. Where it was attempted, responses could only rarely be given any credit. Students showed little understanding of how to approach questions such as this. Vector equations were often not clearly expressed either in terms of directed line segments or in terms of **a** and/or **b**. A small number of students could write down a correct vector expression for $\overline{MN}$ and so gained one mark but they could not usually write this expression accurately in terms of **a** and/or **b**.

Q5.

This question was attempted by most candidates and many were successful. The most common error was to give the vector for *BA* instead of for *AB*.

In Part (b), few candidates started by writing a simple vector equation such as ON = OA + AN, or ON = OB + BN. The biggest difficulty for those who made a serious attempt was getting the direction signs of the vectors correct. Candidates who worked with OA + AN were generally the more successful. Those who chose OB + BN often went on to use NB instead of BN, writing 4b + $\frac{1}{4}$(4b – 2a) instead of 4b + $\frac{1}{4}$(2a – 4b). Some candidates lost the final mark as they were unable to simplify their vector correctly. Other common errors were failing to use brackets appropriately, e.g. writing $\frac{3}{4}$ 4b – 2a instead of $\frac{3}{4}$(4b – 2a), and misinterpreting the ratio and dividing AB into thirds rather than into quarters.

Q6.

A significant proportion of candidates could express ***SQ*** correctly in terms of **a** and **b** though there were a substantial number of candidates who had no idea how to tackle this question. Pythagoras rule, the formula for the area of a triangle and other formulae were used to give incorrect expressions such as a^2 + b2 and ½ab. Some candidates' responses in both parts of the question consisted of numerical ratios. There were some good answers to part (b) of the question but candidates often showed poor communication skills in writing vectors by omitting brackets – for example expressions such as $\frac{2}{5}$**– b + a** + **b** were commonplace**.** Attempting to simplify vector expressions also caused difficulties for many candidates. It would seem that many candidates could benefit from further practice in the manipulation of vectors.

Q7.

The straightforward part (a) of this vector question was correctly answered by only a few of the candidates; when a proof was required in part (b), the percentage of successful candidates dropped even lower.

One mark was awarded to those who could establish that vector *AX* was a third of vector *AB* or that vector *OY* was equal to the sum of vectors *OB* and *BY*.

A further small number gained 3 marks and were able to show that vector *OX* was equal to 2**a** + 2**b** and vector *OY* was equal to 5**a** + 5**b** but were unable to connect the two with a convincing statement of proof as is required in a question testing quality of written communication (QWC).

Q8.
No Examiner's Report available for this question

Q9.

The biggest difficulty for those who made a serious attempt at part (a) was getting the direction signs of the vectors correct. Relatively few candidates chose to write a simple vector equation such as *ON* = *OA* + *AN* or *ON* = *OB* + *BN* as their starting point. Candidates who worked with **a** + 2/3*AB* were generally more successful. Those who started their path with vector **b** frequently used **b** + 1/3*AB* instead of **b** + 1/3*BA*. Difficulty in expanding brackets or omitting brackets altogether prevented some candidates from gaining marks even though their reasoning appeared to be correct.

In part (b), relatively few candidates achieved any marks. Some candidates were able to find a correct expression (usually simplified) for *OD*. A smaller number were able to give a correct expression for *ND*. Often, however, unsimplified or incorrectly simplified answers for *ON* meant that candidates were unable to prove a straight line relationship. Those candidates obtaining all 3 marks were generally very coherent in their justification though few were actually explicit in their recognition of a common point. Some explanations were unclear with candidates mentioning "gradient" or "same amounts of **a** and **b**" rather than stating that one vector was a multiple of the other. Some candidates set about proving *ON* + *ND* = *OD*.

Q10.

Part (a) was usually well answered by those who understand vector notation. The most common error was leaving expression incomplete or ambiguous by not resolving multiple signs, for example –**a**+ -**b**.

In part (b) many candidates gave the correct response of **a-b**. Marks for the explanation were harder to come by. References to parallel lines were needed; evidence of vector notation, for example showing expressions were multiples of each other, provided good evidence of understanding.

Mark Scheme

Q1.

PAPER: 1MA0_1H					
Question		Working	Answer	Mark	Notes
*			Proof	3	M1 for $\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$ (= $\mathbf{n} - \mathbf{m}$) or $\overrightarrow{NM} = \overrightarrow{OM} + \overrightarrow{NO}$ (= $\mathbf{m} - \mathbf{n}$) or $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ (= $2\mathbf{n} - 2\mathbf{m}$) or $\overrightarrow{BA} = \overrightarrow{OA} + \overrightarrow{BO}$ (= $2\mathbf{m} - 2\mathbf{n}$) M1 for $\overrightarrow{MN}$= $\mathbf{n} - \mathbf{m}$ and $\overrightarrow{AB}$= $2\mathbf{n} - 2\mathbf{m}$ oe C1 (dep on M1, M1) for fully correct proof, with $\overrightarrow{AB}$= $2\overrightarrow{MN}$ or $\overrightarrow{AB}$ is a multiple of $\overrightarrow{MN}$ [SC M1 for $\overrightarrow{MN}$ = $0.5\mathbf{n} - 0.5\mathbf{m}$ and $\overrightarrow{AB}$ = $\mathbf{n} - \mathbf{m}$ C1 (dep on M1) for fully correct proof, with $\overrightarrow{AB}$= $2\overrightarrow{MN}$ or $\overrightarrow{AB}$ is a multiple of of $\overrightarrow{MN}$]

Q2.

		Working	Answer	Mark	Notes
	(a)		$3\boldsymbol{a} + \boldsymbol{b}$	1	B1 oe
	(b)		$3\mathbf{c}$	4	M1 for writing *QB* as a combination of vectors eg ½ ('3**a** + **b**') + **b** or 3**a** + 2**b** – ½ ('3**a** + **b**') or 2**a** + 2**b** – **c** oe M1 for equating **a** + **b** = 2**c** or writing *PQ* in terms of vectors **a** and **b** eg −**a** + ½ ('3**a** + **b**') or 2**a** + **b** – ½ ('3**a** + **b**') A1 for (*PQ*=) ½ **a** + ½ **b** **or** (*QB*=) $^3/_2$ **a** + $^3/_2$ **b** or (*QB*=) 2**c**–**a** + **a** + **c** A1 cao NB: accept equivalent expressions throughout

Q3.

Question		Working	Answer	Mark	Notes
	(a)		**b** – **a**	1	B1 for **b** – **a** or –**a** + **b**
	(b)	$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$ $\overrightarrow{AP}$ = ¾ × (**b** – **a**) $\overrightarrow{OP}$ = **a** + ¾ × (**b** – **a**) OR $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$ $\overrightarrow{BP}$ =¼ × (**a** – **b**) $\overrightarrow{OP}$ = **b** + ¼ × (**a** – **b**)	¼(**a** + 3**b**)	3	B1 for ¾ × '(**b** – **a**)' M1 for $(\overrightarrow{OP} =)\ \overrightarrow{OA} + \overrightarrow{AP}$ or $(\overrightarrow{OP} =)\ \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB}$ or **a** ± ¾ × '(**b** – **a**)' A1 for ¼ (**a** + 3**b**) or ¼ **a** + ¾ **b** **OR** B1 for ¼ × '(**a** – **b**)' M1 for $(\overrightarrow{OP} =)\ \overrightarrow{OB} + \overrightarrow{BP}$ or $(\overrightarrow{OP} =)\ \overrightarrow{OB} + \frac{1}{4}\overrightarrow{BA}$ or **b** ± ¼× '(**a** – **b**)' A1 for ¼ (**a** + 3**b**) or ¼ **a** + ¾ **b**

Q4.

PAPER: 5MB3H_01					
Question		**Working**	**Answer**	**Mark**	**Notes**
	(a)		$\frac{3}{2}\mathbf{a}$	3	M1 for (eg $\overrightarrow{EO}$ or $\overrightarrow{CB}$ =) **a** – **b** M1 for correct vector expression for $\overrightarrow{MN}$, eg $\overrightarrow{MO}$ + **a** + $\frac{1}{2}$**b** or $\overrightarrow{ME}$ + 2**a** – $\frac{1}{2}$**b** A1 for $\frac{3}{2}$**a** oe
	(b)		*MN* is parallel to *OA* and $\frac{3}{2}$ × length of *OA*	2	B1 for *MN* is parallel to *OA* B1 for *MN* = $\frac{3}{2}$ × length *OA* oe or ft part (a)

Q5.

		Working	Answer	Mark	Notes
	(a)		4**b** – 2**a**	1	B1 for 4**b** – 2**a** oe
	(b)	$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$, 2**a** + ¾ (4**b** – 2**a**) 2**a** + 3**b** – 1.5**a**	3**b** + 0.5**a**	3	M1 for a correct vector for $\overrightarrow{ON}$, e.g. $(\overrightarrow{ON} =)\ \overrightarrow{OA} + \overrightarrow{AN}$, may be written in terms of **a** and **b** e.g. $(\overrightarrow{ON} =)\ 2\mathbf{a} + \frac{3}{4}(4\mathbf{b} - 2\mathbf{a})$ M1 for $(\overrightarrow{AN} =)\ \frac{3}{4}(4\mathbf{b} - 2\mathbf{a})$ oe or $(\overrightarrow{NB} =)\ \frac{1}{4}(4\mathbf{b} - 2\mathbf{a})$ oe or $(\overrightarrow{NA} =)\ \frac{3}{4}(2\mathbf{a} - 4\mathbf{b})$ oe or $(\overrightarrow{BN} =)\ \frac{1}{4}(2\mathbf{a} - 4\mathbf{b})$ oe A1 for 3**b** + 0.5**a**

Q6.

		Working	Answer	Mark	Notes
	(a)		**a - b**	1	B1 for **a - b** oe
	(b)		$\frac{2}{5}$**a** + $\frac{3}{5}$**b**	3	M1 for a correct vector statement ent for $\overrightarrow{NR}$ eg. $(\overrightarrow{NR} =)\ \overrightarrow{NQ} + \overrightarrow{QR}$ or $(\overrightarrow{NR} =)\ \overrightarrow{NS} + \overrightarrow{SR}$ M1 for $\frac{2}{5}$ *SQ* (+ *QR*) or $\frac{3}{5}$ *QS* (+ *SR*) (*SQ*, *QR*, *QS*, *SR* may be written in terms of **a** and **b)** A1 for $\frac{2}{5}$**(a – b) + b** oe or $\frac{3}{5}$**(b – a) + a** oe

Q7.

		Working	Answer	Mark	Notes
	(a)		6**b** – 3**a**	1	B1 for 6**b** – 3**a** oe
	(b)			4	M1 for $\overrightarrow{AX}$ = ⅓ $\overrightarrow{AB}$ or ⅓ '(6**b** – 3**a**)' or ft to **2**b **–** a M1 for $\overrightarrow{OY} = \overrightarrow{OB} + \overrightarrow{BY}$ = 6**b** + 5**a** – **b** (= 5**b** + 5**a**) oe M1 for $\overrightarrow{OX}$ = 3**a** + '2**b** – **a**' = 2**a** + 2**b** oe **Or** $\overrightarrow{OX}$ = 6**b** – ⅔ '(6**b** – 3**a**)' (= 2**a** + 2**b**) oe C1 for ⅖ $\overrightarrow{OY}$ = ⅖×5(**a** + **b**) = 2(**a** + **b**) = $\overrightarrow{OX}$

Q8.

Paper 1MA1: 3H			
Question	Working	Answer	Notes
	$\vec{OM} = 3\mathbf{a}$ $\vec{AB} = 6\mathbf{b} - 6\mathbf{a}$ $\vec{MC} = 3\mathbf{a} + 2(6\mathbf{b} - 6\mathbf{a})$ $= 12\mathbf{b} - 9\mathbf{a}$ $= 3(4\mathbf{b} - 3\mathbf{a})$ $\vec{MN} = k\mathbf{b} - 3\mathbf{a}$ MNC is a straight line so $\vec{MC}$ is a scalar multiple of $\vec{MN}$	4	P1 For process to start e.g. $\vec{OM} = 3\mathrm{a}$ **or** $\vec{MA} = 3\mathbf{a}$ P1 For process to find $\vec{AB}$ (=6**b** – 6**a**) P1 For process to find $\vec{MC}$ (=3**a** + 2(6**b** – 6**a**) **and** $\vec{MN}$ (= k**b** –3**a**) P1 For correct process to find k e.g. 3k**b** - 9**a** = 12**b** - 9**a** A1

Q9.

PAPER: 1MA0_1H					
Question		Working	Answer	Mark	Notes
	(a)	$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ $\overrightarrow{ON} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB}$ $\overrightarrow{ON} = \mathbf{a} + \frac{2}{3}(-\mathbf{a} + \mathbf{b})$ $= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ OR $\overrightarrow{ON} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA}$ $\overrightarrow{ON} = \mathbf{b} + \frac{1}{3}(-\mathbf{b} + \mathbf{a})$ $= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$	$\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$	3	M1 for correct vector equation involving $\overrightarrow{ON}$, eg. $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$, may be written, partially or fully, in terms of **a** and **b**, e.g. ($\overrightarrow{ON} =$) $\mathbf{a} + \frac{2}{3}\overrightarrow{AB}$ M1 for showing answer requires $\overrightarrow{AN} = \frac{2}{3}\overrightarrow{AB}$ or $\overrightarrow{BN} = \frac{1}{3}\overrightarrow{BA}$ A1 $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ oe
	(b)	$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AC} + \overrightarrow{CD}$ $= \mathbf{a} + \mathbf{b} + \mathbf{b}$ $= \mathbf{a} + 2\mathbf{b}$ $\overrightarrow{OD} = 3(\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b})$ $\overrightarrow{OD} = 3\overrightarrow{ON}$	Proof	3	M1 for a correct vector statement for $\overrightarrow{OD}$ or $\overrightarrow{ND}$ in terms of **a** and **b**, e.g. $\overrightarrow{OD} = \mathbf{a} + \mathbf{b} + \mathbf{b}$ oe or $\overrightarrow{ND} = \frac{2}{3}(-\mathbf{b} + \mathbf{a}) + \mathbf{b} + \mathbf{b}$ oe A1 for correct and fully simplified vectors for $\overrightarrow{ON}$ (may be seen in (a)) **and** for $\overrightarrow{OD}$ (= $\mathbf{a} + 2\mathbf{b}$) or $\overrightarrow{ND}$ (= $\frac{2}{3}\mathbf{a} + \frac{4}{3}\mathbf{b}$) C1 (dep on A1) for statement that $\overrightarrow{OD}$ or $\overrightarrow{ND}$ is a multiple of $\overrightarrow{ON}$ (+ common point)

Q10.

Question		Working	Answer	Mark	Notes
	(a)	i *CA* = 2*OA* ii *BA*=*BO*+*OA*= –**b**+**a** iii *BC*=*BO*+*OC*= –**b**–**a**	2**a** **a** – **b** –**a** – **b**	3	B1 for 2**a** oe B1 for **a** – **b** oe B1 for –**a** – **b** oe
	(b)	i *AX* = *AO* + *OX* = – **a** + 2**a** – **b** = **a** – **b** ii *AX* = *BA* so *AX* is parallel to *BA*; *A* is on both *AX* and *BA*, so *B*, *A*, *X* are all on a straight line	**a** – **b** explanation	3	M1 for *AX* = *AO* + *OX* A1 for **a** – **b** oe B1 (dep on M1) for explanation eg *BX* = *BO* + *OX* = –**b** + 2**a** – **b** = 2(**a** – **b**)