

Name: \_\_\_\_\_

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# Higher Unit 16 topic test

Date:

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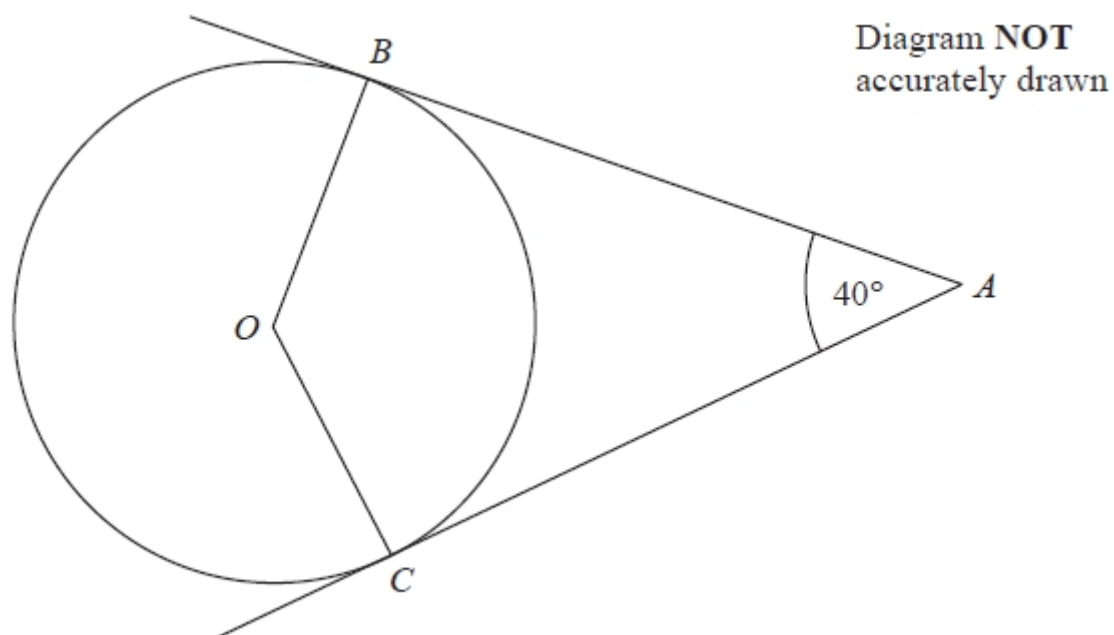
**Time:** 50 minutes

**Total marks available:** 45

**Total marks achieved:** \_\_\_\_\_

## Questions

Q1.



$B$  and  $C$  are points on the circumference of a circle, centre  $O$ .  
 $AB$  and  $AC$  are tangents to the circle.  
Angle  $BAC = 40^\circ$ .

Find the size of angle  $BCO$ .

.....<sup>o</sup>  
(Total for Question is 3 marks)

Q2.

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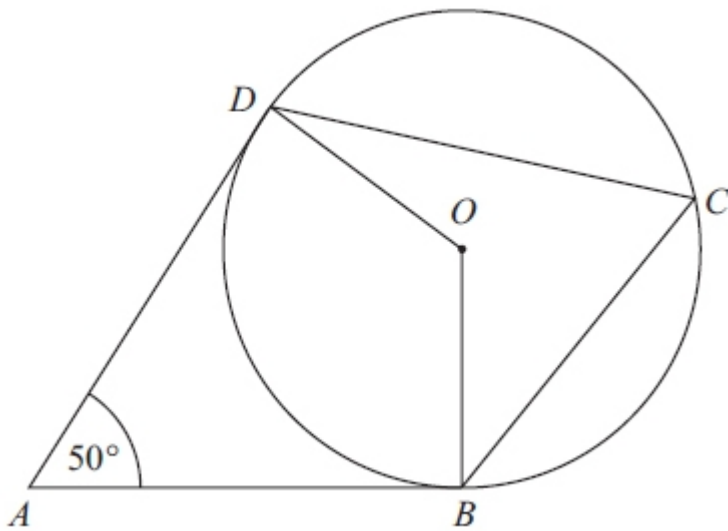


Diagram **NOT** accurately drawn

$B$ ,  $C$  and  $D$  are points on the circumference of a circle, centre  $O$ .  
 $AB$  and  $AD$  are tangents to the circle.

Angle  $DAB = 50^\circ$

Work out the size of angle  $BCD$ .  
Give a reason for each stage in your working.

(Total for Question is 4 marks)

Q3.

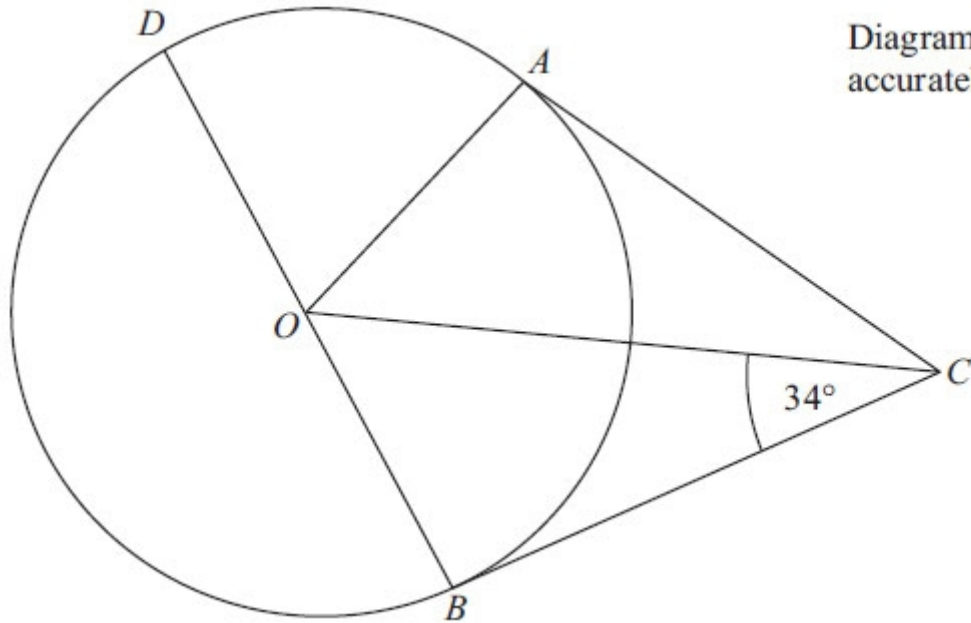


Diagram **NOT**  
accurately drawn

$A$ ,  $B$  and  $D$  are points on the circumference of a circle, centre  $O$ .  
 $BOD$  is a diameter of the circle.  
 $BC$  and  $AC$  are tangents to the circle.  
Angle  $OCB = 34^\circ$ .

Work out the size of angle  $DOA$ .

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(Total for Question is 3 marks)

Q4.

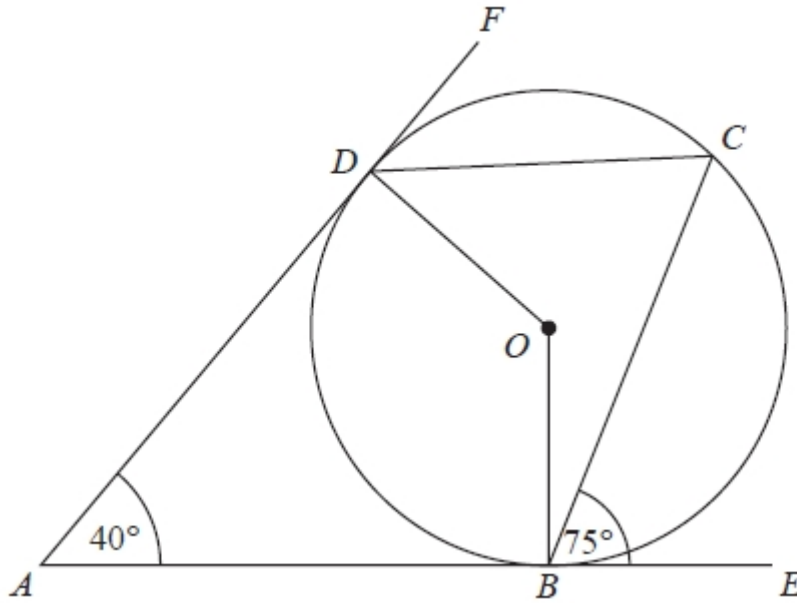


Diagram NOT  
accurately drawn

$B$ ,  $C$  and  $D$  are points on the circumference of a circle, centre  $O$ .  
 $ABE$  and  $ADF$  are tangents to the circle.

Angle  $DAB = 40^\circ$

Angle  $CBE = 75^\circ$

Work out the size of angle  $ODC$ .

.....<sup>o</sup>  
(Total for Question is 3 marks)

Q5.

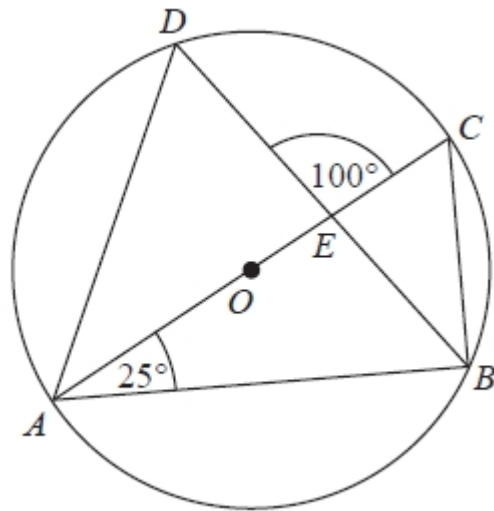


Diagram **NOT**  
accurately drawn

$A$ ,  $B$ ,  $C$  and  $D$  are points on the circumference of a circle, centre  $O$ .  
 $AC$  is a diameter of the circle.  
 $AC$  and  $BD$  intersect at  $E$ .

Angle  $CAB = 25^\circ$   
Angle  $DEC = 100^\circ$

Work out the size of angle  $DAC$ .  
You must show all your working.

.....<sup>o</sup>  
(Total for question = 4 marks)

Q6.

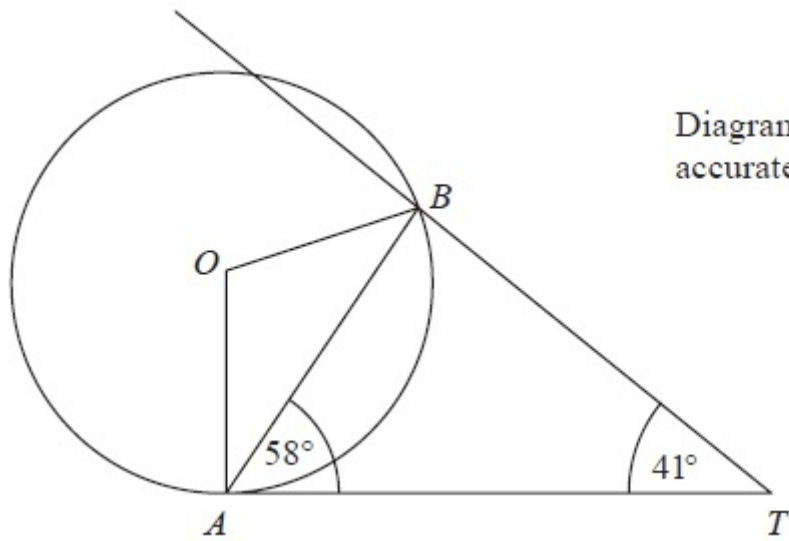


Diagram NOT  
accurately drawn

$A$  and  $B$  are points on the circumference of a circle, centre  $O$ .

$AT$  is a tangent to the circle.

Angle  $TAB = 58^\circ$ .

Angle  $BTA = 41^\circ$ .

Calculate the size of angle  $OBT$ .

You must give reasons at each stage of your working.

(Total for Question is 5 marks)

Q7.

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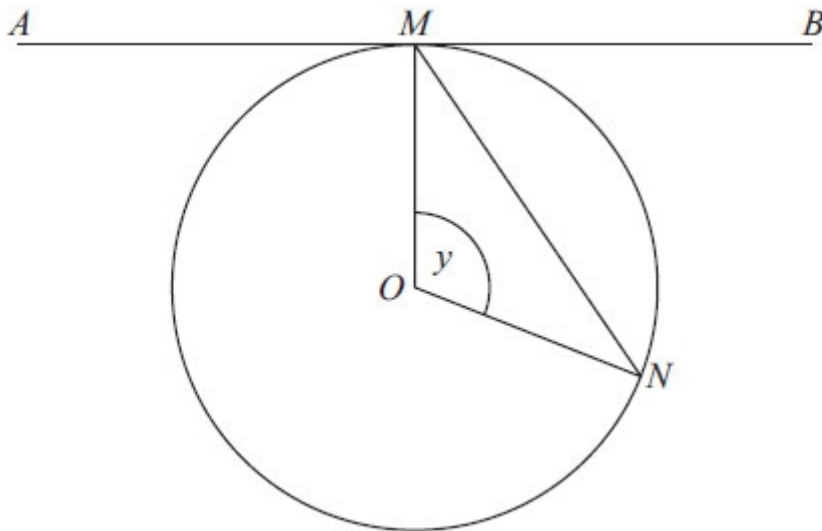


Diagram **NOT**  
accurately drawn

$M$  and  $N$  are two points on the circumference of a circle centre  $O$ .  
The straight line  $AMB$  is the tangent to the circle at  $M$ .

Angle  $MON = y$

Prove that angle  $BMN = \frac{1}{2} y$

(Total for Question is 5 marks)



Q8.

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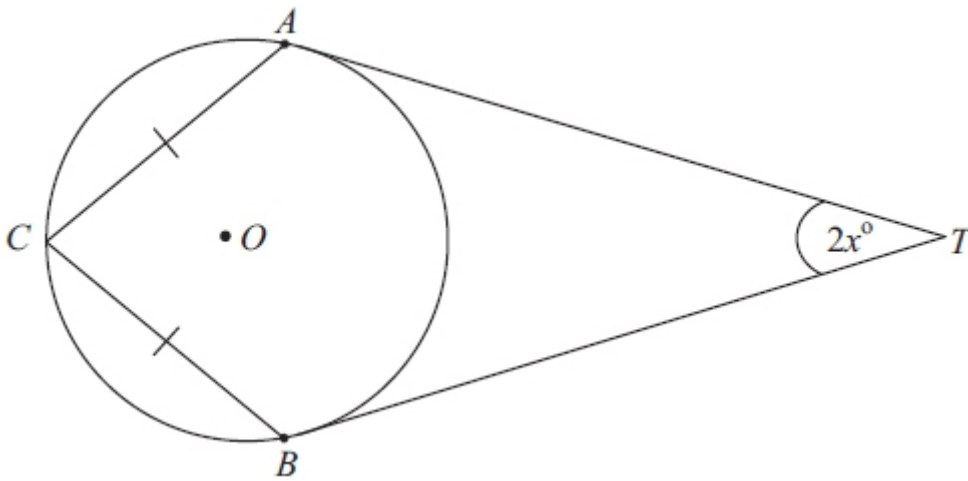


Diagram **NOT** accurately

drawn

$A$ ,  $B$  and  $C$  are points on the circumference of the circle, centre  $O$ .

$TA$  and  $TB$  are tangents to the circle.

$CA = CB$ .

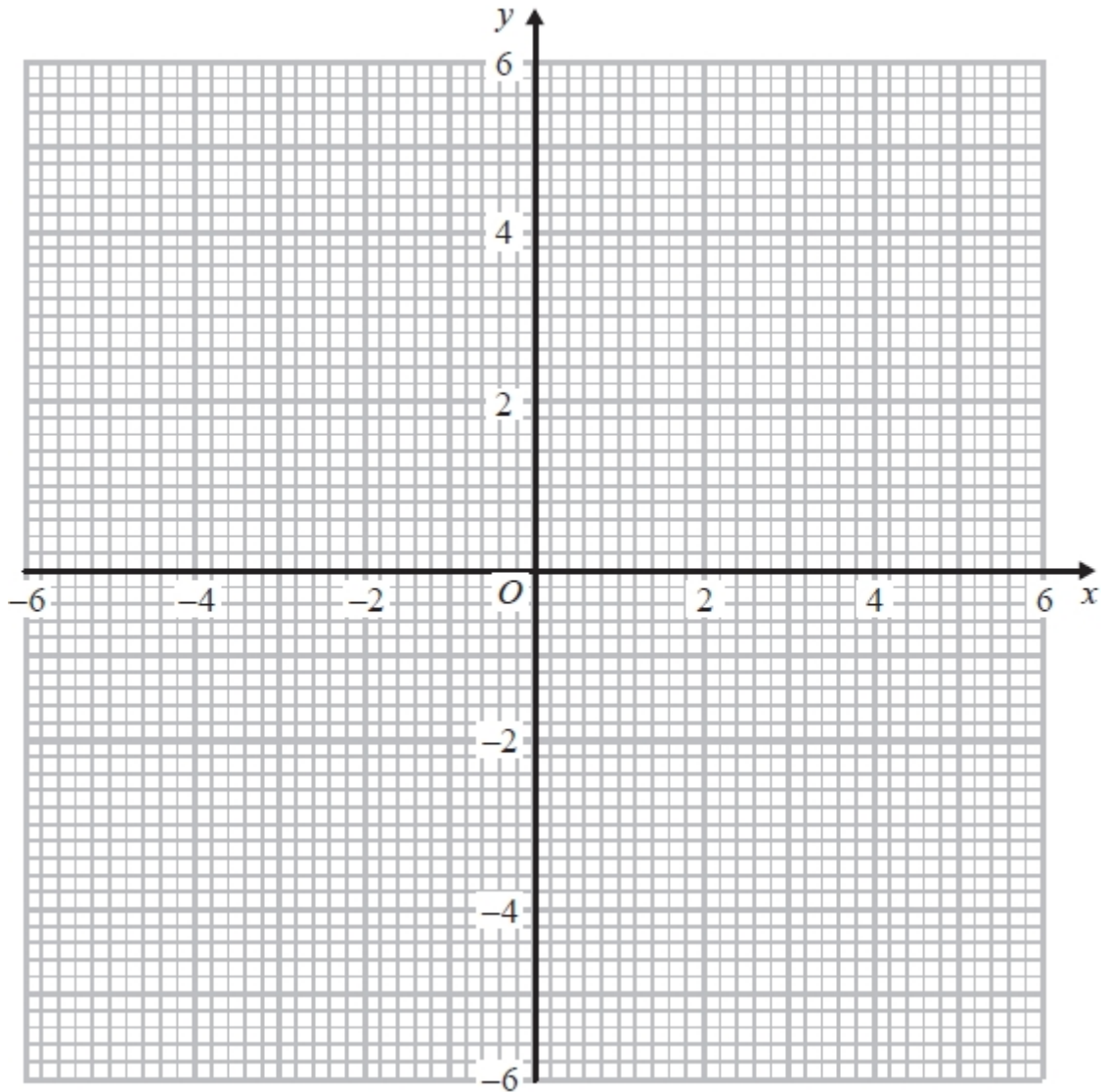
Angle  $ATB = 2x^\circ$ .

Prove that angle  $ACB = (90 - x)^\circ$ .

(Total for Question is 5 marks)

**Q9.**

(a) On the grid, construct the graph of  $x^2 + y^2 = 16$



(2)

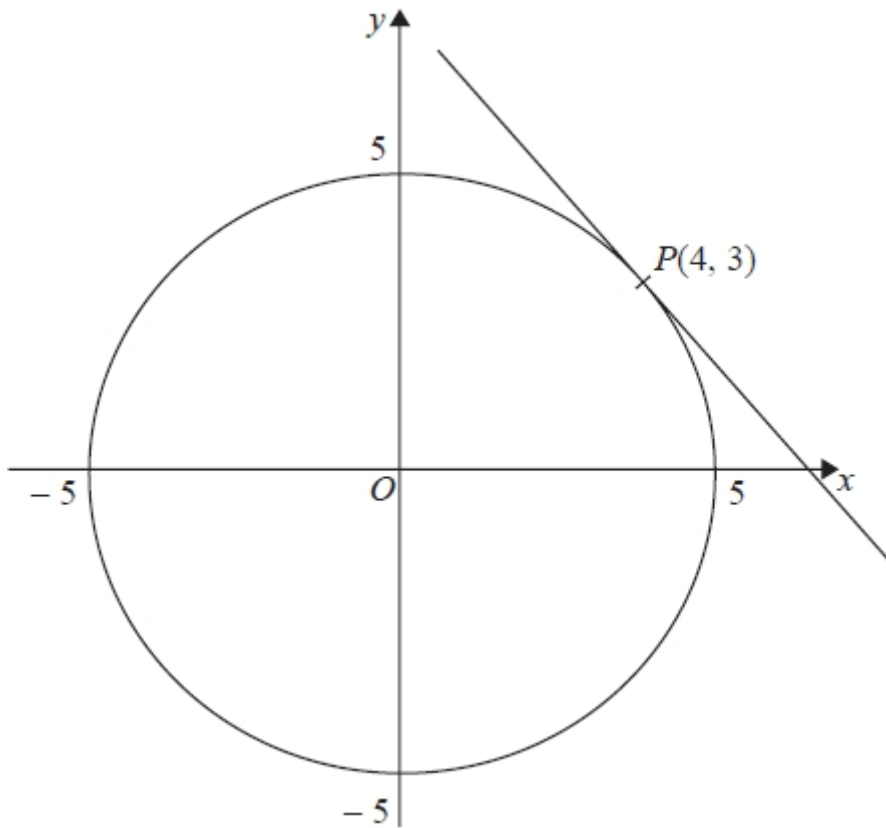
(b) Find estimates for the solutions of the simultaneous equations

$$\begin{aligned}x^2 + y^2 &= 16 \\ y &= 2x + 1\end{aligned}$$

.....  
(3)  
(Total for question = 5 marks)

**Q10.**

Here is a circle, centre  $O$ , and the tangent to the circle at the point  $P(4, 3)$  on the circle.



Find an equation of the tangent at the point  $P$

.....  
**(Total for question is 3 marks)**

**Q11.**

The line  $l$  is a tangent to the circle  $x^2 + y^2 = 40$  at the point  $A$ .  
 $A$  is the point  $(2, 6)$ .

The line  $l$  crosses the  $x$ -axis at the point  $P$ .

Work out the area of triangle  $OAP$ .

.....  
**(Total for question = 5 marks)**

## Examiner's Report

### Q1.

Most students scored a mark for showing that either angle  $OBA$  or angle  $OCA$  was  $90^\circ$ , generally by indicating this on the diagram. However many students lost marks because they did not identify which angle they were finding when doing calculations. It was very common to see  $360 - 90 - 90 - 40 = 140$  without any indication that this was a calculation to find angle  $BOC$ . Just writing 140 in the correct place on the diagram would suffice. Others had no understanding of 3 letter angle notation.  $20^\circ$  was often seen on the diagram for angle  $BCO$  but then  $140^\circ$  was written in the answer space. Students should be encouraged to write all calculated angles in the appropriate space on the diagram as this would greatly increase the number of method marks awarded.

### Q2.

When asked to give reasons in a geometry questions, reasons must be correct and must use correct mathematical language. Reasons given in responses seen to this question were often incomplete or not completely correct. 'Angle between tangent and circle is  $90^\circ$ ' and 'angle at origin is twice the angle at the edge of the circle' are both examples where a communication mark was not awarded as the statements were not accurate enough. It is also important to ensure that the final answer is communicated properly. In this case the value of the angle had to be linked with the angle itself so sight of Angle  $BCD = 65^\circ$  (or similar) was expected rather than just to see a  $65^\circ$  somewhere amongst the candidate's working. Very few candidates used the alternate segment theorem as part of their explanation.

### Q3.

This question testing circle geometry gave a good distribution of marks, with some candidates being able to recognise that the angle between a radius and a tangent is  $90^\circ$ , mostly seen on the diagram. A further small percentage were able to establish, by using a correct method, that angle  $AOC$  or angle  $BOC$  was  $56^\circ$  or that angle  $AOB$  was  $112^\circ$ , while only a quarter could gain all 3 marks for a fully correct solution and identify the answer as  $68^\circ$ .

Some candidates incorrectly assumed  $OC = BC$  and tried to use an isosceles triangle. Most candidates were not good at naming the angles that they were finding and as a consequence some lost marks by not identifying correctly which angle they were trying to calculate.

### Q4.

There were a variety of methods to complete this problem with its complex configuration. The most common successful approach was to calculate the reflex angle  $BOD$  and the angle at the circumference  $BCD$ , then use the angle sum of a quadrilateral together with angle  $OBC = 15^\circ$ . Other approaches were rare. They included using the alternate segment theorem (although often wrongly applied), or using angle  $BOC = 150^\circ$  and angle  $BOD = 140^\circ$  followed by using angles round the point  $O$  and a suitable isosceles triangle.

In many cases candidates wrote down figures but did not relate them to the angles found. In this case the marks could often not be awarded unless the  $55^\circ$  was given as the answer. Many candidates sensibly put values of angles on the diagram and these were accepted as evidence of correct processes.

### Q5.

There were many attempts at this question where students failed to show any knowledge of circle theorems, but rather made false assumptions about angles in order to provide some basic work. This included assuming there were isosceles triangles, where there were none. Some found  $\angle ABE$  to be  $55^\circ$ , but without the knowledge that  $\angle ABC$  was  $90^\circ$  this got them nowhere useful.

Centres need to remind students that when working with geometry problems they need to either write the angles on the diagram, or if only presented in working, these workings need to clearly show which angles are being worked with.

Few students gained full marks.

### Q6.

Most candidates were able to score some marks in this question. Many were able to find the size of the angle  $\angle OBT$ , but few were able to state all the reasons for their choice of calculations or state them correctly. Furthermore, it was often unclear as to how particular calculations were related to the overall solution of the problem. Candidates should be reminded to clearly identify the angles they are calculating by either an appropriate angle notation (three letter notation here) or by annotating the diagram. A popular incorrect answer involved the erroneous identification of angle  $\angle ABT$  as  $90^\circ$ , ie incorrectly interpreting  $BT$  as a tangent to the circle.

### Q7.

Most candidates gained some credit in this question. Over time the performance has increased, with candidates becoming more aware of the need to give reasons for statements about angles. In this question not only was it necessary to show the  $90^\circ$  angle or state its position on the diagram, but also to justify it as "the angle between a tangent and radius is  $90^\circ$ ". Many candidates also went on to state further properties such as  $\angle OMN$  and  $\angle ONM$  being equal. Further than this required working with algebraic expressions, which was beyond the majority of candidates.

### Q8.

Many candidates gained a mark for correctly identifying a right-angle at  $\angle OAT$  and/or  $\angle OBT$  even if they made no further progress. Others assumed  $\angle CAT$  or  $\angle CBT$  were  $90^\circ$  or even that  $\angle ACB$  was. A variety of proofs were attempted but in this question where marks were awarded for Quality of Written Communication, it was essential that theorems were quoted accurately using correct mathematical language.

### Q9.

There were some excellent solutions to this question showing an accurately constructed circle followed by the plotting of a suitable line and accurate reading off of the solutions of the simultaneous equations. Students who did not see the connection between parts (a) and (b) often began a solution using substitution but they rarely completed the question successfully. They struggled to manipulate the equations correctly. A small but significant group of students found the values of  $x$  but lost a mark because they did not find the corresponding values of  $y$ .

**Q10.**

No Examiner's Report available for this question

**Q11.**

No Examiner's Report available for this question

## Mark Scheme

Q1.

PAPER: 5MB2H_01					
Question		Working	Answer	Mark	Notes
			20	3	M1 for indication that angle between a tangent and radius is 90 (could be seen on the diagram)  M1 for $OAC = 20$ or $AOC = 70$ or $BOC = 140$ or $ABC = ACB$ or $BCA = \frac{180 - 40}{2}$ (= 70) A1 cao



## Q2.

Question	Working	Answer	Mark	Notes
*	<p> <math>ABO = ADO = 90^\circ</math>            (Angle between tangent and radius is <math>90^\circ</math>)  <math>DOB = 360 - 90 - 90 - 50</math>            (Angles in a quadrilateral add up to <math>360^\circ</math>)  <math>BCD = 130 \div 2</math>            (Angle at centre is twice angle at circumference)         </p> <p> <b>OR</b>  <math>ABD = (180 - 50) \div 2</math>            (Base angles of an isosceles triangle)  <math>BCD = 65</math>            (Alternate segment theorem)         </p>	<p>65°</p>	<p>4</p>	<p>           B1 for <math>ABO = 90</math> <b>or</b> <math>ADO = 90</math> (may be on diagram)            B1 for <math>BCD = 65</math> (may be on diagram)         </p> <p>           C2 for <math>BCD = 65^\circ</math> stated <b>or</b> <math>DCB = 65^\circ</math> stated <b>or</b> angle C = <math>65^\circ</math> stated <b>with</b> all reasons:  <u>angle between tangent and radius is <math>90^\circ</math></u>;  <u>angles in a quadrilateral sum to <math>360^\circ</math></u>;  <u>angle at centre is twice angle at circumference</u>            (accept angle at circumference is half (or <math>\frac{1}{2}</math>) the angle at the centre)            (C1 for one correct and appropriate circle theorem reason)            QWC: Working clearly laid out and reasons given using correct language         </p> <p> <b>OR</b>            B1 for <math>ABD = 65</math> or <math>ADB = 65</math> (may be on diagram)            B1 for <math>BCD = 65</math> (may be on diagram)         </p> <p>           C2 for <math>BCD = 65^\circ</math> stated <b>or</b> <math>DCB = 65^\circ</math> stated <b>or</b> angle C = <math>65^\circ</math> stated <b>with</b> all reasons:  <u>base angles of an isosceles triangle are equal</u>;  <u>angles in a triangle sum to <math>180^\circ</math></u>;  <u>tangents from an external point are equal</u>;  <u>alternate segment theorem</u>            (C1 for one correct and appropriate circle theorem reason)            QWC: Working clearly laid out and reasons given using correct language         </p>

Q3.

	Working	Answer	Mark	Notes
		68	3	<p>M1 for angle <math>OBC = 90^\circ</math> or angle <math>OAC = 90^\circ</math> (may be marked on the diagram or used in subsequent working)</p> <p>M1 for correct method to find angle <math>BOC</math> or <math>AOC</math> or <math>AOB</math></p> <p>e.g. angle <math>BOC = 180 - 90 - 34 (= 56)</math>  or angle <math>AOC = 180 - 90 - 34 (=56)</math>  or angle <math>AOB = 180 - 2 \times 34 (= 112)</math></p> <p>A1 cao</p> <p><b>NB</b> (68 must be clearly stated as an answer and not just seen on diagram)</p>

Q4.

PAPER: IMA0_1H				
Question	Working	Answer	Mark	Notes
		55	3	<p>M1 for angle <math>ABO = 90</math> or angle <math>ADO = 90</math>, or angle <math>OBC = 15</math> or angle</p> <p><math>FDO = 90</math> or angle <math>EBO = 90</math> (could be marked on the diagram)</p> <p>M1 for reflex angle <math>BOD = 360 - (360 - 90 - 90 - 40) (= 220)</math>  or angle <math>BCD = (360 - 90 - 90 - 40) \div 2 (= 70)</math>  or angle <math>BDO</math> or angle <math>DBO = 90 - (180 - 40)/2 (= 20)</math>  or angle <math>BOC = 180 - (15 + 15) (=150)</math></p> <p>A1 cao</p>

Q5.

PAPER: IMA0_2H				
Question	Working	Answer	Mark	Notes
		$35^\circ$	4	<p>M1 for <math>ABC = 90</math></p> <p>M1 for <math>(ACB =) 180 - 90 - 25 (= 65)</math></p> <p>M1 for <math>(DBC =) 180 - '65' - 80 (=35)</math></p> <p>A1 cao supported by working</p> <p>OR</p> <p>M1 for <math>(AOB =) 180 - 2 \times 25 (= 130)</math></p> <p>M1 for <math>(ADB =) 130 \div 2 (=65)</math></p> <p>M1 for <math>(DAC =) 180 - 65 - 80</math></p> <p>A1 cao supported by working.</p>

Q6.

PAPER: 5MB2H 01				
Question	Working	Answer	Mark	Notes
*		113	5	<p>B1 for stating angle <math>\angle TAO = 90</math>  M1 for stating angle <math>\angle OBA</math> or angle <math>\angle OAB = 90 - 58 (=32)</math>  M1 for stating angle <math>\angle ABT = 180 - 58 - 41 (=81)</math> or angle <math>\angle AOB = 180 - 64 (=116)</math>  A1 for 113 clearly identified as the answer  C1 (dep on M1) for correct statements for method used:  angle between <u>tangent</u> and <u>radius</u> = <math>90^\circ</math>  AND at least one of  base <u>angles</u> of an <u>isosceles</u> triangle are <u>equal</u>  sum of <u>angles</u> in a <u>triangle</u> is <u>180</u>  sum of <u>angles</u> in a <u>quadrilateral</u> is <u>360</u></p> <p>NB angles may be seen in diagram</p>

Q7.

Question	Working	Answer	Mark	Notes
*		Proof	5	<p>B1 for <math>OM = ON</math> or (<math>OMN</math> is) <u>isosceles</u> triangle or <math>\angle OMB = 90^\circ</math> or <math>\angle AMO = 90^\circ</math>  B1 for <math>\angle OMN = \angle ONM</math> or either = <math>\frac{1}{2}(180 - y)</math> oe  B1 for (<math>\angle</math>) <math>\angle BMN = 90 - \frac{1}{2}(180 - y)</math> [if algebraic in <math>y</math>]  C1 for statement angle between <u>tangent</u> and <u>radius</u> = <math>90^\circ</math> (or <u>perpendicular</u> or <u>right angle</u>)  C1 for correct conclusion with <math>\angle BMN</math> stated, accompanied by correct working clearly laid out and in a logical sequence with correct calculations</p> <p>Acceptable alternative:  B1 for angle at circumference = <math>\frac{1}{2}y</math>  B1 for 'angle at centre is twice the angle at the circumference' oe  B1 for angle <math>\angle BMN = \frac{1}{2}y</math>  C1 for statement '<u>alternate segment theorem</u>'  C1 for correct conclusion with <math>\angle BMN</math> stated, accompanied by correct working clearly laid out and in a logical sequence with correct calculations</p>

Q8.

Question	Working	Answer	Mark	Notes
*	$AOT = 90 - x$ (Angle between tangent and radius is $90^\circ$ ) $AOC = 90 + x$ (Tangents from an external point are equal)  $ACB = 2(180 - (90 + x))$ $\div 2 = 90 - x$  Or  Obtuse angle $BOA = 180 - 2x$ (Angle between tangent and radius is $90^\circ$ ) Reflex angle $BOA = 180 + 2x$ (Tangents from an external point are equal) $ACB = (360 - (180 + 2x)) \div 2 = 90 - x$  Alternative method  $AOB = 360 - 2x - 90 - 90$		5	B1 for $AOT = 90 - x$ or $OAT = 90^\circ$ or $OBT = 90^\circ$ (may be shown on diagram) B1 for $AOC = 90 + x$ B1 for completing the proof  C2 for 2 reasons: Angle between tangent and radius is $90^\circ$ <b>and</b> Tangents from an external point are equal. <b>QWC: proof should be clearly laid out with technical language correct</b> [C1 for 1 of: Angle between tangent and radius is $90^\circ$ <b>or</b> Tangents from an external point are equal, <b>QWC: proof should be clearly laid out with technical language correct</b> ]  <b>OR</b> B1 for obtuse angle $BOA = 180 - 2x$ or $OAT = 90^\circ$ or $OBT = 90^\circ$ (may be shown on diagram) B1 for reflex angle $BOA = 180 + 2x$ B1 for completing the proof  C2 for 2 reasons: Angle between tangent and radius is $90^\circ$ <b>and</b> Tangents from an external point are equal. <b>QWC: proof should be clearly laid out with technical language correct</b> [C1 for 1 of: Angle between tangent and radius is $90^\circ$ <b>or</b> Tangents from an external point are equal,

	$=180 - 2x$ (Angle between tangent and radius is $90^\circ$ )  $ACB = (180 - 2x) / 2$ (Angle at the circumference is half angle at the centre)		<p><b>QWC: proof should be clearly laid out with technical language correct]</b></p> <p>Alternative method</p> <p>B1 for <math>AOB = 360 - 2x - 90 - 90</math>          B1 for <math>ACB = (180 - 2x)/2</math>          B1 for completing the proof</p> <p>C2 for 2 reasons:          Angle between tangent and radius is <math>90^\circ</math> <b>and</b>          Angle at the circumference is half angle at the centre  <b>QWC: proof should be clearly laid out with technical language correct</b></p> <p>[C1 for 1 of: Angle between tangent and radius is <math>90^\circ</math> <b>or</b>          Angle at the circumference is half angle at the centre</p> <p><b>QWC: proof should be clearly laid out with technical language correct]</b></p> <p>B3 maybe awarded for a fully correct alternative method.</p>
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Q9.

PAPER: 1MA0 2H				
Question	Working	Answer	Mark	Notes
(a)		Circle drawn	2	B2 fully correct circle drawn (B1 for circle drawn with centre (0,0) or circle drawn with radius 4) OR M1 at least 5 correct points calculated and plotted A1 fully correct circle drawn
(b)		$x = 1.4, y = 3.8$ $x = -2.2, y = -3.4$	3	M1 for $y = 2x + 1$ drawn or for elimination of one variable A1 for one correct pair of values given or for $x = 1.4, -2.2 (\pm 0.2)$ or fit from graph provided 2 marks in (a) A1 for second correct pair of values given ( $\pm 0.2$ ) or fit from graph provided 2 marks in (a)

Q10.

Paper 1MA1: 2H			
Question	Working	Answer	Notes
		$y = -\frac{4}{3}x + \frac{25}{3}$ oe	M1 for method to find gradient of tangent, eg. $-1 \div \frac{3}{4} = -\frac{4}{3}$ M1 for method to find y-intercept using $y = -\frac{4}{3}x + c$ A1 $y = -\frac{4}{3}x + \frac{25}{3}$ oe

Q11.

Question	Working	Answer	Notes
		60	P1 process to start problem eg draw diagram and find gradient of $OA (= 3)$ P1 process to find equation of tangent with $m = -1/3$ P1 process to find x-axis intercept of tangent P1 process to find area of triangle A1 cao