Edexcel GCSE

Mathematics

Higher Tier

Number: Ratio

Information for students

The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this selection.

Advice for students

Show all stages in any calculations.
Work steadily through the paper. Do not spend too long on one question.
If you cannot answer a question, leave it and attempt the next one.
Return at the end to those you have left out.

Information for teachers

The questions in this document are taken from the 2009 GCSE Exam Wizard and include questions from examinations set between January 2003 and June 2009 from specifications 1387, 1388, 2540, 2544, 1380 and 2381.

Questions are those tagged as assessing "Ratio" though they might assess other areas of the specification as well. Questions are those tagged as "Higher" so could have (though not necessarily) appeared on either an Intermediate or Higher tier paper.

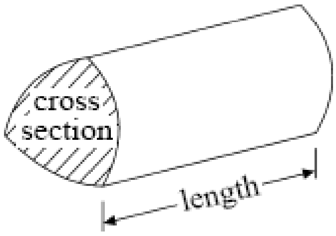
GCSE Mathematics

Formulae: Higher Tier

You must not write on this formulae page.

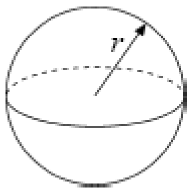
Anything you write on this formulae page will gain NO credit.

Volume of prism = area of cross section × length



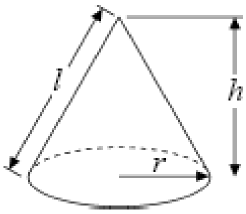
Volume of sphere $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

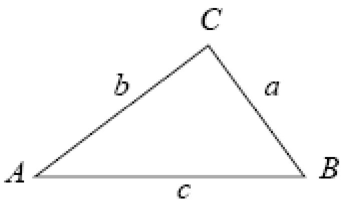


Volume of cone $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl



In any triangle ABC



Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

1. The force, F, between two magnets is inversely proportional to the square of the distance, x, between them.

When $x = 3$, $F = 4$.

(a) Find an expression for F in terms of x.

F = ...

(3)

(b) Calculate F when $x = 2$.

....

(1)

(c) Calculate x when $F = 64$.

....

(2)

(Total 6 marks)

2.

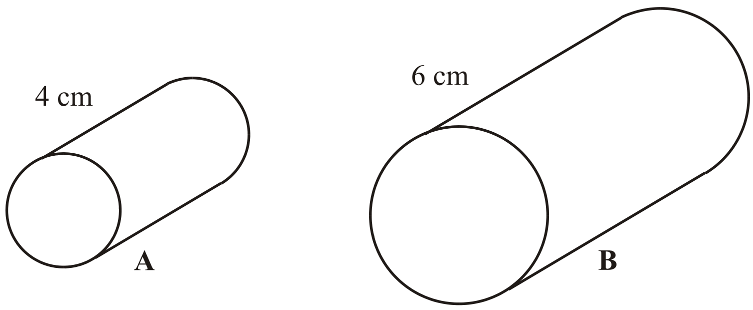


Diagram **NOT** accurately drawn

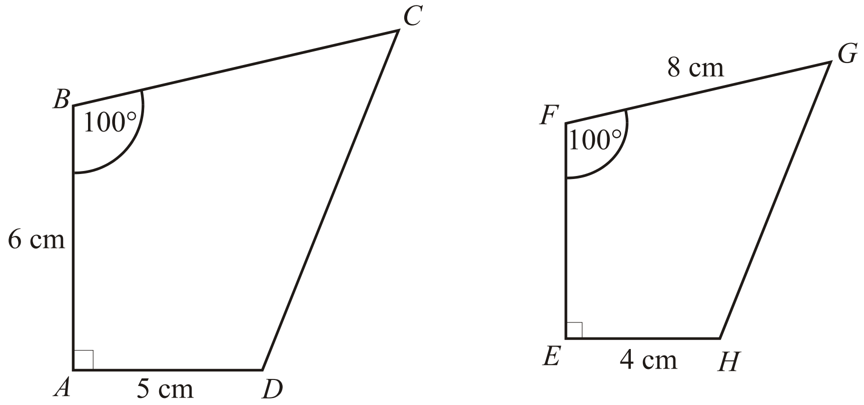
Cylinder **A** and cylinder **B** are mathematically similar.
The length of cylinder **A** is 4 cm and the length of cylinder **B** is 6 cm.
The volume of cylinder **A** is 80 cm^3.

Calculate the volume of cylinder **B**.

………………………… cm^3
(Total 3 marks)

3.

Diagrams **NOT** accurately drawn



Shapes *ABCD* and *EFGH* are mathematically similar.

(i) Calculate the length of *BC*.

BC = ………………… cm

(ii) Calculate the length of *EF*.

EF = ………………… cm

(Total 5 marks)

4. X and Y are two geometrically similar solid shapes.

The total surface area of shape X is 450 cm^2.
The total surface area of shape Y is 800 cm^2.

The volume of shape X is 1350 cm^3.

Calculate the volume of shape Y.

.... cm^3

(Total 3 marks)

5.

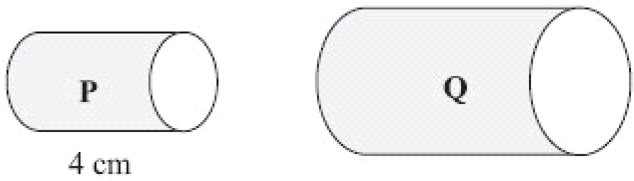


Diagram **NOT** accurately drawn

Two cylinders, **P** and **Q**, are mathematically similar.

The total surface area of cylinder **P** is 90π cm^2.

The total surface area of cylinder **Q** is 810π cm^2.

The length of cylinder **P** is 4 cm.

(a) Work out the length of cylinder **Q**.

…………… cm

(3)

The volume of cylinder **P** is 100π cm^3.

(b) Work out the volume of cylinder **Q**.
Give your answer as a multiple of π.

…………… cm^3
(2)
(Total 5 marks)

6. Jim makes a model of his school.

He uses a scale of 1 : 50

The area of the door on his model is 8 cm^2.

Work out the area of the door on the real school.

.....................................cm^2
(Total 2 marks)

7.

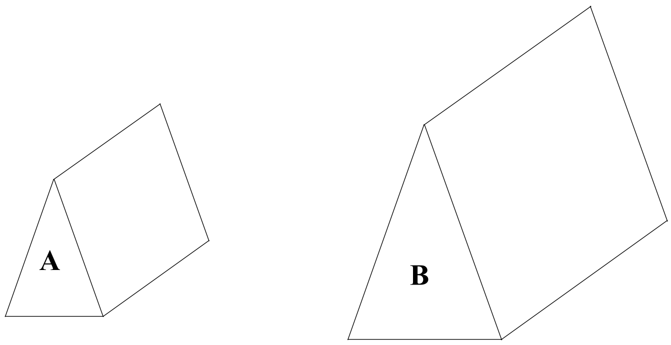


Diagram **NOT** accurately drawn

Two prisms, **A** and **B**, are mathematically similar.
The volume of prism **A** is 12 000 cm^3.
The volume of prism **B** is 49 152 cm^3.
The total surface area of prism **B** is 9728 cm^2.

Calculate the total surface area of prism **A**.

.... cm^2
(Total 4 marks)

8.

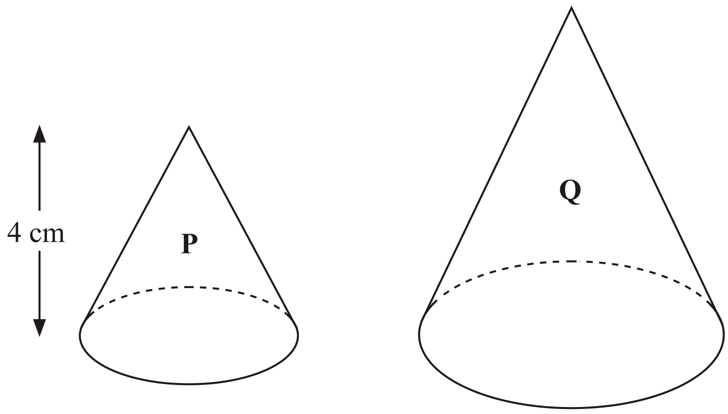


Diagram **NOT** accurately drawn

Two cones, **P** and **Q**, are mathematically similar.
The total surface area of cone **P** is 24 cm^2.
The total surface area of cone **Q** is 96 cm^2.
The height of cone **P** is 4 cm.

(a) Work out the height of cone **Q**.

.... cm

(3)

The volume of cone **P** is 12 cm^3.

(b) Work out the volume of cone **Q**.

.... cm^3

(2)

(Total 5 marks)

9. y is inversely proportional to x^2.

Given that $y = 2.5$ when $x = 24$,

(i) find an expression for y in terms of x

$y =$ ....

(ii) find the value of y when $x = 20$

$y =$ ....

(iii) find a value of x when $y = 1.6$

$x =$ ....

(Total 6 marks)

01. (a) $F = \dfrac{36}{x^2}$ 3

$F \propto 1/x^2$
$F = k/x^2$
$4 = k / 3^2$
$F = 36/x^2$

M1 for $F = k/x^2$ seen or implied. ($k \neq 1$)
M1 (dep) for subst. or sight of $k = 36$
A1 for $F = 36/x^2$

(b) 9 1

B1 ft for 9 (ft on $F = kx^n$, $n \neq 0$)

(c) $\frac{6}{8}$ 2

$64 = \frac{36}{x^2}$

$x^2 = \frac{36}{64}$

$x = \pm\frac{3}{4}$

M1 for $x^2 = \frac{"36"}{64}$

A1 for $\frac{6}{8}$ *oe (condone* $\pm$*)*

SC: Use of F = kx^2 max M1 M1 A0 B1 ft M0 A0

SC: Use of $F = \frac{k}{\sqrt{x}}$

max M1 M1 A0 B0 M1 ($\sqrt{x} = \frac{4\sqrt{3}}{64}$) A0

[6]

02. 270 3

Sf = $\frac{3}{2}$

Vol = $\left(\frac{3}{2}\right)^3 \times 80$

M1 for $\frac{3}{2}$ *oe (or* $\frac{2}{3}$ *oe or ratio with evidence)*

M1 for $\left(\frac{3}{2}\right)^3 \times 80$ *oe*

A1 cao

[3]

03. (i) 10 5

$8 \times \frac{5}{4}$

B1 for sight of $\frac{5}{4}$ *or* $\frac{4}{5}$ *or 2 or* $\frac{1}{2}$ *oe*
M1 for 8 × 1.25 oe
A1 cao

(ii) 4.8

$6 \times \frac{4}{5}$

M1 for 6 × 0.8 oe
A1 cao

[5]

04. 3200 cm^3 3

SF (length) = $\sqrt{\frac{800}{450}} = \frac{4}{3}$

vol = '$\left(\frac{4}{3}\right)^3$' × 1350

B1 for Sf(length) = $\sqrt{\frac{800}{450}}$ *oe*
M1 for '$\left(\frac{4}{3}\right)^3$' *× 1350 or* '$\left(\frac{4}{3}\right)^3$' $= \frac{vol}{1350}$ *oe*
A1 cao
SC for vol = 2400 give B0 M1 A0

[3]

05. (a) 12 3

$\frac{810\pi}{90\pi}$ or 9
$\sqrt{9}$ or 3

M1 for $\frac{810\pi}{90\pi}$ *or 9 or* $\frac{1}{9}$ *or 1:9 oe*
M1 for $\sqrt{\frac{810\pi}{90\pi}}$ *or* $\sqrt{9}$ *or 3 or* $\frac{1}{3}$ *or* $\sqrt{9} : \sqrt{1}$ *oe*
A1 cao
SC:B1 for answer of 36

(b) 2700π 2

3^3 or 27 or 2700

M1 for "3"^{3} or 27 or $\left(\sqrt{9}\right)^3 : \left(\sqrt{81}\right)^3$ oe or 9^3 or 2700–

A1 cao

[5]

06. 8×50^2

20 000cm^2 2

M1 for 50^2 seen

A1 for 20 000 cm^2 or 2 m^2

[2]

07. $\frac{49152}{12000}$ or 4.096 4

M1 for $\frac{49152}{12000}$ or 4.096 oe

$\sqrt[3]{4.096}$ or 1.6

"1.6"^{2} or 2.56

= 3800

M1 for $\sqrt[3]{4.096}$ or 1.6 oe

M1 for "1.6"^{2} or2.56 oe

A1 for 3800 cao

[4]

08. (a) $\frac{96}{24}$ or 4

$\sqrt{4}$ or 2 = 8 3

M1 for $\frac{96}{24}$ or $\frac{24}{96}$ or 4 or $\frac{1}{4}$ oe

M1 for $\sqrt{\frac{96}{24}}$ or $\sqrt{\frac{24}{96}}$ or $\sqrt{'4'}$ or $\frac{1}{\sqrt{'4'}}$ or 2 or $\frac{1}{2}$ oe

A1 cao

(b) $12 \times 2^3 = 96$ 2

M1 for '2^3' or 8

A1 cao

[5]

09. (i) $y = \frac{1440}{x^2}$ 3

$y = \frac{k}{x^2}$

$k = 2.5 \times 24^2 = 1440$

M1 for either $y \propto \frac{1}{x^2}$ or $y = \frac{k}{x^2}$ seen or implied $k \neq 1$

M1 (dep) $2.5 = \frac{k}{24^2}$ oe or k = 1440

A1 for $y = \frac{1440}{x^2}$ (accept equivalents)

(ii) 3.6 1

$y = \frac{1440}{20^2}$

B1 ft [ft on $y = kn^n$, n integer, $n \neq 0$]

(iii) 30 2

$x = \sqrt{\frac{1440}{1.6}}$

M1 for $x^2 = \frac{"1440"}{1.6}$ or better

A1 cao for 30 or –30

SC: for $y = kx^2$ MIMIAO BIMOAO

SC: for $y = \frac{k}{\sqrt{x}}$ MIMIAO BOMIAO

[6]

01. This question tested a topic, which has regularly appeared in past papers. Such questions have to be read very carefully. It was disappointing to find that a significant number of candidates could not get started.

The better candidates, provided they got the initial algebra correct involving F, x and a k, generally had no problems picking up the marks in parts (a) and (b). Generally only a minority of candidates were able to correctly rearrange the formula to obtain the value for x in a form not involving a square root.

02. Although many candidates were awarded credit for stating/using the correct linear scale factor, it was rare to find a full correct solution to this question on the ratio of volumes being proportional to the ratio of cubes of corresponding lengths. The two most common wrong answers were 120 and 180. A significant minority of those who used the correct method failed to obtain the correct answer because they made the calculation more difficult by cubing the decimal rather than the fraction.

03. **Paper 3**

Few candidates had any understanding of the use of scale factors in this question, nor of how similarity is related to enlargement. In part (a), a few candidates obtained the correct answer due to the way in which the corresponding sides can be intuitively linked. In part (b), it was rare to see any success. It is all too common to see candidates relating the sides by addition and subtraction, rather than multiplication.

Paper 6

This was a slightly unusual question in that it did not deal with similar triangles. Because the two shapes were separate and both in the same orientation most candidates who knew anything about similarity or enlargement were able to gain marks for the question, usually from part (i) as a minimum. A variety of methods was used involving ratios such as *FG/EH* and *BC/AD*. Candidates found *EF* more difficult to establish as this usually involved the ratio 5 : 4

04. There was a great deal of misunderstanding over this question. Of course, it is a standard one of its sort. However, pupils do seem to find the ideas difficult and so it proved on this question. One method which did work successfully is to focus on the two areas and from the fraction $\frac{800}{450} = \frac{16}{9}$. This gives the scale factor for the areas. Square rooting this gives the scale factor of the lengths and cubing this scale factor gives the scale factor for the volumes. Most candidates simply used the ratio of the areas to form a multiplier which was then used directly on the smaller volume to get 2400 cm^3 for the larger one.

05. Specification A

Most candidates attempted this question but only the best were able to achieve full marks. By far the most common answer to part (a) was 36, and to part (b) was 900π.
In part (b), some candidates were able to score a mark for cubing the scale factor they derived in part (a). A small number of candidates calculated the radius of *P* to deduce the radius, and hence the volume, of *Q*. A significant proportion of those candidates getting as far as the volume 2700π did not understand the demand of the question and omitted to include the π in their final answer.

Specification B

The most commonly seen answer to this question was 36 in (a) and 900π in (b). These were both incorrect solutions and occurred when candidates mistakenly used the area scale factor as the length and then volume scale factor. A small minority of candidates were able to often fully correct solutions although the omission of π from the answer to (b) resulted in some candidates failing to gain the final accuracy mark. There was evidence of some poor arithmetic in this question with 810 divided by 90 being evaluated as 90.

06. Specification A

Most candidates did not recognise that this was a question involving similar shapes. The standard solution was to recognise that the area scale factor was the square of the scale given and multiply the area of the door on the model by 502. A few candidates imagined that the model door was a 4cm by 2 cm rectangle and used the 50 scale factor to calculate a real door to be a rectangle 200cm by 100 cm. From these measurements the area of the real door can be calculated by multiplication.

Specification B

Over 80% of candidates gave the incorrect answer of 400cm^2 to this question, using a scale factor of 50 rather than the correct scale factor of 50^2.

07. This type of question is always found difficult by the candidature. Many candidates assume that volume scales in the same way as length and get one mark for comparing volumes. For candidates that are aware of different scale factors, some selected the wrong process - for example, squaring the volume scale factor to get the area scale factor.

08. This question was not answered well. The vast majority of candidates that attempted this question were able to find the scale factor 4 of the enlargement, usually by dividing 96 by 24 or by ratios, but few of these knew how to proceed from this to the linear scale factor 2 in part (a) and the volume scale factor 8 in part (b). Most candidates simply multiplied the height by 4 to get 16cm in part (a), and multiplied the volume by 4 to get 48cm^3 in part (b).

Very few candidates attempted to use the area and volume formulae for a cone.

09. Candidates attempting this question often failed to appreciate the meaning of the word 'inverse'. $y = kx^2$ or $y = \frac{k}{\sqrt{x}}$ were common incorrect starting points for some candidates. Those candidates who started correctly with $y = \frac{k}{x^2}$ frequently went on to score full marks.