Edexcel GCSE

Mathematics

Higher Tier

Number: Primes, factors, multiples

Information for students

The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 11 questions in this selection.

Advice for students

Show all stages in any calculations.
Work steadily through the paper. Do not spend too long on one question.
If you cannot answer a question, leave it and attempt the next one.
Return at the end to those you have left out.

Information for teachers

The questions in this document are taken from the 2009 GCSE Exam Wizard and include questions from examinations set between January 2003 and June 2009 from specifications 1387, 1388, 2540, 2544, 1380 and 2381.

Questions are those tagged as assessing "Primes, factors, multiples" though they might assess other areas of the specification as well. Questions are those tagged as "Higher" so could have (though not necessarily) appeared on either an Intermediate or Higher tier paper.

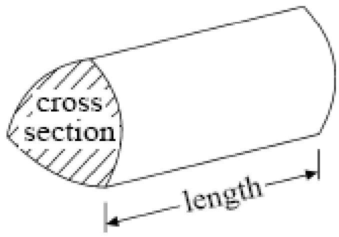
GCSE Mathematics

Formulae: Higher Tier

You must not write on this formulae page.

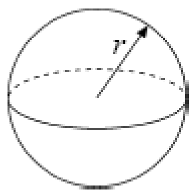
Anything you write on this formulae page will gain NO credit.

Volume of prism = area of cross section × length



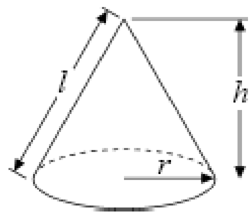
Volume of sphere $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

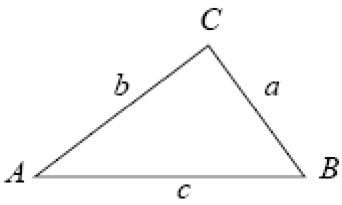


Volume of cone $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl



In any triangle ABC



Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

1. Prove that,

$$(n + 1)^2 - (n - 1)^2$$

is a multiple of 4, for all positive integer values of *n*.

(Total 3 marks)

2. (a) Write down an expression, in terms of *n*, for the *n*th multiple of 5.

....

(1)

(b) Hence or otherwise

(i) prove that the sum of two consecutive multiples of 5 is always an odd number,

(ii) prove that the product of two consecutive multiples of 5 is always an even number.

(5)
(Total 6 marks)

3. The number 40 can be written as $2^m \times n$, where m and n are prime numbers.

Find the value of m and the value of n.

m = ...

n = ...

(Total 2 marks)

4. (a) Write down the value of $4^{\frac{3}{2}}$

....

(1)

(b) Write $\sqrt{8}$ in the form $m\sqrt{2}$, where m is an integer.

....

(2)

(c) Write $\sqrt{50}$ in the form $k\sqrt{2}$, where k is an integer.

....

(2)

(d) Rationalise $\dfrac{1+\sqrt{2}}{\sqrt{2}}$

....

(2)

(Total 7 marks)

5. Martin is organising a summer fair.
He needs bread buns and burgers for the barbecue.

Bread buns are sold in packs. Each pack contains 40 bread buns.
Burgers are sold in packs. Each pack contains 24 burgers.
Martin buys exactly the same number of bread buns as burgers.

What is the least number of each pack that Martin buys?

.... packs of bread buns

…............................. packs of burgers

(Total 3 marks)

6. Tarish says,

‘The sum of two prime numbers is always an even number’.

He is **wrong**.
Explain why.

....

....

(Total 2 marks)

7. (a) Find the Highest Common Factor of 75 and 90.

....
(2)

(b) Find the Lowest Common Multiple of 75 and 90.

....
(2)
(Total 4 marks)

8. p is a prime number not equal to 7

(a) Write down the Highest Common Factor (HCF) of

$$49p \quad \text{and} \quad 7p^2$$

....
(1)

x and y are different prime numbers.

(b) (i) Write down the Highest Common Factor (HCF) of the two expressions

x^2y $\qquad$ xy^2

....

(ii) Write down the Lowest Common Multiple (LCM) of the two expressions

x^2y $\qquad$ xy^2

....

(3)

(Total 4 marks)

9. $A = 2^4 \times 3^2 \times 7$ $\qquad$ $B = 2^3 \times 3^4 \times 5$

A and B are numbers written as the product of their prime factors.

Find

(i) the highest common factor of A and B,

....

(ii) the lowest common multiple of A and B.

....

(Total 3 marks)

10. A and B are numbers written as the products of their prime factors.

$A = 3^2 \times 5 \times 7$ $\qquad$ $B = 2 \times 3^3 \times 5^2$

(i) Find the highest common factor (HCF) of A and B.

....

(ii) Find the lowest common multiple (LCM) of A and B.

....
(Total 3 marks)

11. Find the Lowest Common Multiple (LCM) of 42 and 70

....
(Total 2 marks)

01. Either $(n^2 + 2n + 1) - (n^2 - 2n + 1) = 4n$
or $(n + 1 + n - 1)(n + 1 - (n - 1)) = 2n \times 2 = 4n$ 3

B1 + B1 for $(n^2 + 2n + 1) - (n^2 - 2n + 1)$ must have brackets for the 2nd B1
B1 for 4n
Or
B1 for either $(n + 1 + n - 1)$ or $(n + 1 - (n - 1))$
B1 for $(n + 1 + n - 1)(n + 1 - (n - 1))$
B1 for 4n
SC: $n^2 + 2n + 1 - n^2 - 2n + 1 = 4n$ is 2/3

[3]

02. (a) $5n$ 1

B1 cao

(b) (i) $5n + 5(n \pm 1)$

$10n \pm 5$

$5(2n \pm 1)$

Both 5 and $2n \pm 1$ are odd 2

M1 for $5n + 5(n \pm 1)$ or $10n \pm 5$ or for $5(2n \pm 1)$

A1 for stating both 5 and $2n \pm 1$ are odd and odd × odd = odd oe

(ii) $5n \times 5(n \pm 1)$

$25n(n \pm 1)$

25 is odd, one of n or $n \pm 1$ is odd so odd × even × odd = even 3

M1 for $5n \times 5(n \pm 1)$

A1 for realises that one of n and $n \pm 1$ will be even or considers $5n$ or $5(n \pm 1)$ for both odd and even

A1 for establishing correct result oe

(SC if M0, MO awarded in part (b) B1 for using in b(i) or (ii) a numerical argument with more than 2 examples)

(SC for 5n and $5n\pm1$ used B1 in (i) and B1 in (ii) for fully reasoned argument)

[6]

03. $m = 3$

$n = 5$ 2

B1 for 3

B1 for 5

(B2 for $2^3 \times 5$ or $2 \times 2 \times 2 \times 5$)

SC: award B1 only if $m = 3$, $n = 3$, for 8×5 seen

[2]

04. (a) 8 1

B1 cao

(b) $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$ 2

$= 2\sqrt{2}$

B2 cao

(B1 for $\sqrt{4 \times 2}$ or $\sqrt{4}\sqrt{2}$ or $\sqrt{2}\sqrt{2}\ \sqrt{2}$ or $\sqrt{2^3}$)

(Accept 2 on answer line if $2\sqrt{2}$ seen)

(c) $\sqrt{25 \times 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$ 2

$= 5\sqrt{2}$

B2 cao

(B1 for $\sqrt{25 \times 2}$ or $\sqrt{25}\sqrt{2}$ or $\sqrt{5}\sqrt{5}\sqrt{2}$)

(Accept 5 on answer line if $5\sqrt{2}$ seen)

(d) $\dfrac{1+\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}(1+\sqrt{2})}{\sqrt{2} \times \sqrt{2}} = \dfrac{\sqrt{2}+\sqrt{2}\sqrt{2}}{2} = \dfrac{\sqrt{2}+2}{2}$ 2

M1 for × $\sqrt{2}$ top and bottom

A1 cao oe

[7]

05. LCM (40, 24) = 120

Bread buns 120 ÷ 40

Burgers 120 ÷ 24

or

Bread buns: 40 is 2 × 2 × 2 (× 5)

Burgers: 24 is 2 × 2 × 2 (× 3)

Bread buns 3

Burgers 5 3

M1 attempt to find LCM by eg lists of multiples, or summing of 40s and summing of 24s, with at least 3 numbers in each list.

A1 identify 120 (as LCM)

A1 cao (both)

or

M1 expansion of either number into its prime factors in a factor tree or 8 × 5 or 8 × 3

A1 both expansions correct

A1 cao (both)

SC B2 if answers given the wrong way around

[3]

06. 2 + ‘prime number’ is odd 2

M1 for a counter example showing intent to add 2 and another prime number; ignore incorrect examples
A1 for a correctly evaluated counter example with no examples given that involve either non-primes or incorrect evaluation

Alternative method
B2 for fully correct explanation ‘2 is a prime number, odd + even (or 2) = odd’ oe with no accompanying incorrect statements or examples

*(B1 for ‘2 is a prime number’ **or** recognition that not all prime numbers are odd **or** odd + even (or 2) = odd; ignore incorrect examples or statements)*

[2]

07. (a) 15 2

$75 = 3 \times 5 \times 5$
$90 = 2 \times 3 \times 3 \times 5$

M1 for the 3 prime factors (3, 5, 5) of 75
***OR** the 4 prime factors (2, 3, 3, 5) of 90*
[Alt: M1 for at least 3 factors in each list]
A1 cao

$\text{HCF} = 3 \times 5$

(b) 450 2

Using the above to give
$\text{LCM} = 2 \times 3 \times 3 \times 5 \times 5$

M1 for product of correct factors ($2 \times 3 \times 3 \times 5 \times 5$)
[Alt: M1 for at least 3 multiples in each list]
A1 cao
[SC: B1 for any common multiple of 75 and 90 but not 6750]
[SC: B2 for 15 and 450 reversed]

[4]

08. (a) $7p$ 1

B1

(b) (i) xy 3

B2 for xy
(B1 for any common factor)

(ii) x^2y^2

B1 for x^2y^2

[4]

09. (i) $2^3 \times 3^2$ oe — 1

B1 for $2^3 \times 3^2$ oe

(ii) $2^4 \times 3^4 \times 5 \times 7$ oe — 2

M1 for either 2^4 or 3^4 in a product of factors OR list of at least 3 correct multiples of each of 1008 and 3240
A1 cao
SC: B2 if both answers correct but reversed

[3]

10. (i) $3^2 \times 5$oe — 3

B1 accept as 3 × 3 × 5, 45 or in index form

(ii) $2 \times 3^3 \times 5^2 \times 7$ oe

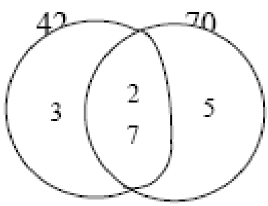
M1 for product that includes both 3^3 (27) and 5^2 (25) or for listing multiples of 315 and 1350
A1 for 9450 oe
(SC B1 for 425250)

[3]

11. 42 = 2 × 3 × 7 — 2
70 = 2 × 5 × 7
= 210

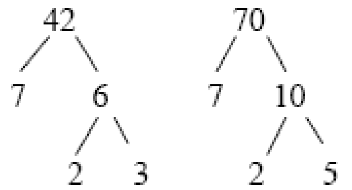
M1 for 2 × 3 × 7 ***or*** *2 × 5 × 7* ***or*** *a list of at least 3 multiples of 42* ***and*** *a list of at least 3 multiples of 70 (condone one error in each list)*
A1 for 210 or an equivalent product, eg. 2 × 3 × 5 × 7
[SC: B1 for any correct common multiple if M0 scored. This could be written as a product, eg.
2 × 3 × 7 × 2 × 5 × 7]

Alternative:



Alternative:
M1 for a fully correct Venn Diagram
A1 for 210 or an equivalent product, eg. 2 × 3 × 5 × 7

Alternative



Alternative:
*M1 for **two** fully correct factor trees*
A1 for 210 or an equivalent product, eg. $2 \times 3 \times 5 \times 7$

*One correct factor tree is **not** enough to justify the award of the method mark. We must see at least one explicit product, eg $2 \times 3 \times 7$*

[2]

01. The response to this question shows that there is a long way to go before a substantial number of candidates can put together a rigorous algebraic proof. Candidates usually fell into the following groups:

Those who substituted $n = 1, 2$ etc and concluded that the answer was always a multiple of 4 based on the evidence of the first two or three results.

Those who tried to expand the brackets but got $n^2 + 1$ and $n^2 - 1$.

Those who expanded the brackets to $n^2 + 2n + 1 - n^2 -2n + 1$and went on to get $4n$.

Those who expanded correctly to $(n^2 + 2n + 1) - (n^2 - 2n + 1)$ and went on to give $4n$.

The latter were in the minority.

02. This question was all about a proof involving sums and products of multiples of 5. Only 5% of the candidature was able to give the rigorous proof that was needed in part (b) though partial credit was awarded to about half of the candidates. Part (a) was answered correctly by 85% of candidates but candidates often then tried unsuccessfully to explain their proof without using the guidance in part (a) and in the stem of part (b).

03. Specification A

Higher Tier

This question was done well by the vast majority of the candidates. Some of those candidates using a factor tree misinterpreted their work to write $m = 2$, $n = 5$ – thus scoring only one of the marks.

Intermediate Tier

Well answered by the majority of candidates. Many candidates drew factor trees, but had difficulty in extracting the right answer, often writing 8 and 5 or 2 and 5.

Specification B

It was pleasing to see many candidates at all levels gaining at least one mark, and often two, in this question. Pairs of 1, 20 and 2, 10 were common wrong answers.

04. Many candidates were able to score some marks in this question. Most errors in part (a) were due to misunderstanding the notation. $4^{\frac{3}{2}}$ was often taken to be $4\frac{3}{2}$ or $4 \times \frac{3}{2}$, and $\frac{11}{5}$, 5.5 and 6 were common incorrect final answers.
In parts (b) and (c), most candidates realised they had to decompose the number in the square root, but many did not know how to do this.
Common errors here were $\sqrt{2 \times 2 \times 2} = 3\sqrt{3}, \sqrt{4 \times 2} = 4\sqrt{2}$ and $\sqrt{25 \times 2} = 25\sqrt{2}$.
In part (d), a significant number of candidates knew that they needed to multiply the top and bottom by $\sqrt{2}$, but most were unable to do this accurately. A common incorrect calculation was $\frac{1+\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1+2}{2} = \frac{3}{2}$, thus failing to notice the implied brackets in the numerator.

05. **Higher Tier**

Most candidates followed the LCM route by listing multiples of 40 and multiples of 24 respectively. They were generally successful and gained full marks. Some candidates followed this route, but overlooked 120 and opted for 240 instead. These gave values of 10 and 6 respectively.

Intermediate Tier

This was a very well answered question, with many candidates getting full marks. Sometimes LCM was used with the numbers given, arriving at the key multiple of 120. More often candidates tackled the question intuitively, arriving at the answers with little working.

06. Many candidates thought that 1 was a prime number. Others had trouble with the word "sum", misinterpreting it as product.

Successful candidates usually offered a correct counter example, frequently 2 + 3 = 5, and often backed this up by a written explanation. On occasions, a correct counter-example worthy of full marks was spoiled by further embellishment including incorrect statements or other examples involving non-primes.

07. **Paper 9**

Many candidates were clearly more comfortable with the concept of HCF rather than LCM. In the main the most common approach was by the listing of factors; incomplete lists often resulting in a HCF of 5 and sometimes 3. This was often the answer given in part (b) also. Attempts to express 75 and 90 as the product of prime factors were seen less often but when seen were usually successful.
Some candidates who understood the concept of LCM failed to arrive at the answer of 450 as a result of arithmetic errors in their listing of multiples.

Paper 10

This question was not well answered by the majority of candidates. Candidates often showed knowledge of factors or multiples but were unable to find the HCF and LCM. The majority of candidates chose to list factors and multiples, very few broke down the numbers into their prime factors.

08. There were a pleasing number of fully correct answers to this question. In part (a) the most common incorrect answer was 7. In (b) the most common incorrect answer for the LCM was x^3y^3.

09. This question was a very good discriminator. Fully correct answers were seen from approximately 12% of candidates. The majority of candidates who answered the question successfully obtained their solution from the product of prime numbers given. Candidates who evaluated the two products were less successful, particularly when finding the LCM of the two numbers.

10. This question was not well understood with many chaotic solutions. About one third of candidates were able to find the HCF but only 20% were able to work out the LCM. Some candidates reversed the answers, some merely calculating the values of the two products, some offering one prime factor for each etc. The most successful way for the average candidate to find the LCM was by listing the multiples.

11. Factor tree methods were the most common employed in attempting to find the LCM. These had to both be correct to gain any credit. Many candidates simply gave a common factor and an answer of 7 was regularly seen, as was the HCF of 14. Candidates choosing to list multiples often made arithmetic addition errors in listing the multiples of 42.