1.

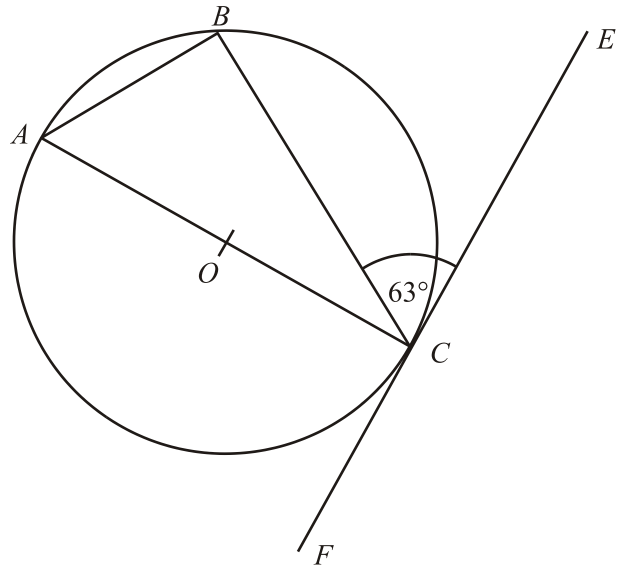


Diagram **NOT** accurately drawn

In the diagram, A, B and C are points on the circle, centre O.
Angle $BCE = 63°$.
FE is a tangent to the circle at point C.

(i) Calculate the size of angle ACB.
Give reasons for your answer.

..............................°

(ii) Calculate the size of angle BAC.
Give reasons for your answer.

..............................°

(Total 4 marks)

2.

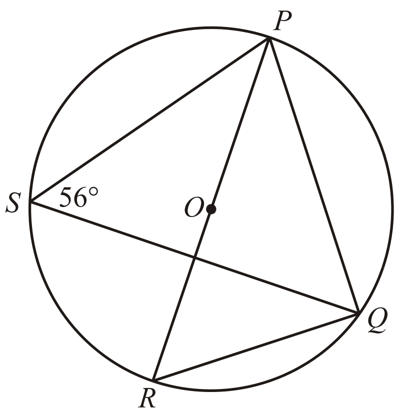


Diagram **NOT** accurately drawn

P, Q, R and S are points on the circumference of a circle, centre O.
PR is a diameter of the circle.
Angle $PSQ = 56°$.

(a) Find the size of angle *PQR*.
Give a reason for your answer.

..................................°

(2)

(b) Find the size of angle *PRQ*.
Give a reason for your answer.

..................................°

(2)

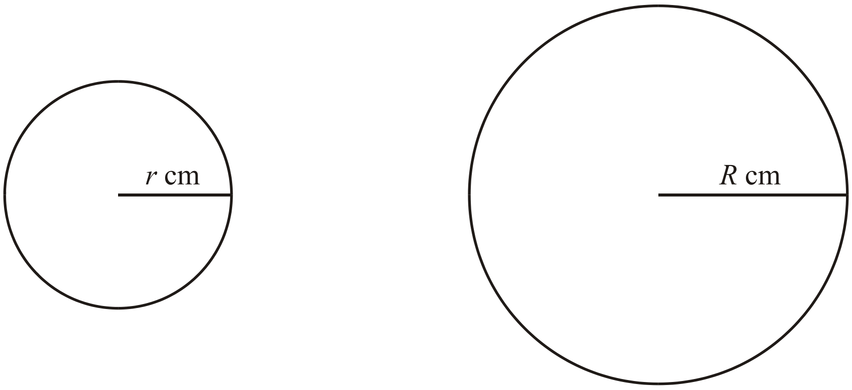
(c) Find the size of angle *POQ*.
Give a reason for your answer.

..................................°

(2)
(Total 6 marks)

3.

Diagram **NOT** accurately drawn



The diagram represents two metal spheres of different sizes.

The radius of the smaller sphere is r cm.
The radius of the larger sphere is R cm.

$r = 1.7$ correct to 1 decimal place.

$R = 31.0$ correct to 3 significant figures.

(a) Write down the upper and lower bounds of r and R.

Upper bound of r = ………………………………

Lower bound of r = ………………………………

Upper bound of R = ………………………………

Lower bound of R = ………………………………

(2)

(b) Find the smallest possible value of $R - r$.

……………………………

(1)

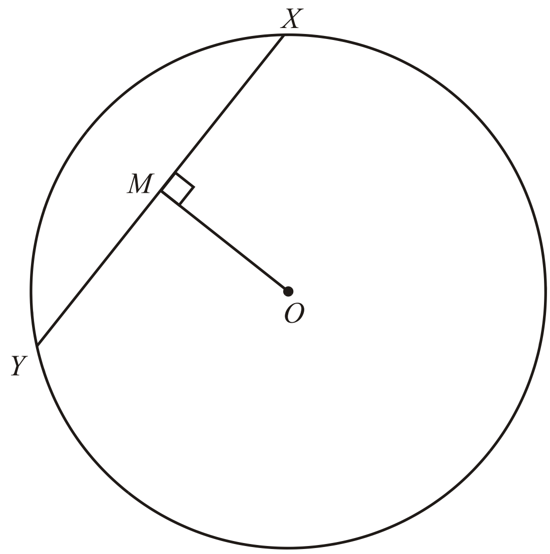
The larger sphere of radius R cm was melted down and used to make smaller spheres of radius r cm.

(c) Calculate the smallest possible number of spheres that could be made.

……………………………

(4)
(Total 7 marks)

4.



X and Y are points on the circle, centre O.
M is the point where the perpendicular from O meets the chord XY.

Prove that M is the midpoint of the chord XY.

(Total 3 marks)

5.

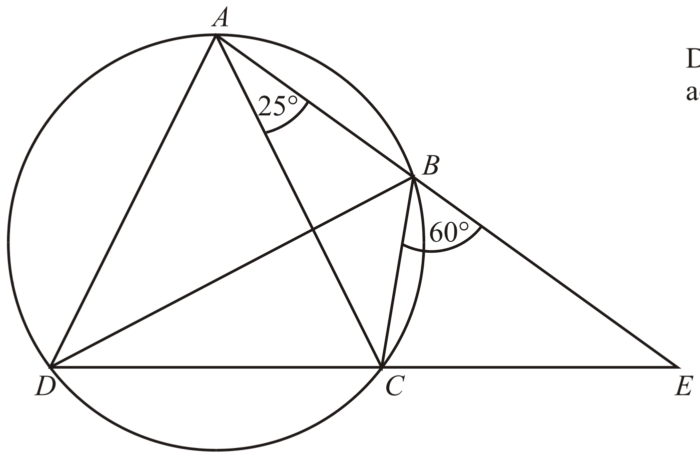


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are four points on the circumference of a circle.
ABE and *DCE* are straight lines.

Angle *BAC* = 25°.

Angle *EBC* = 60°.

(a) Find the size of angle *ADC*.

..........................°

(1)

(b) Find the size of angle *ADB*

..........................°

(2)

Angle *CAD* = 65°.
Ben says that *BD* is a diameter of the circle.

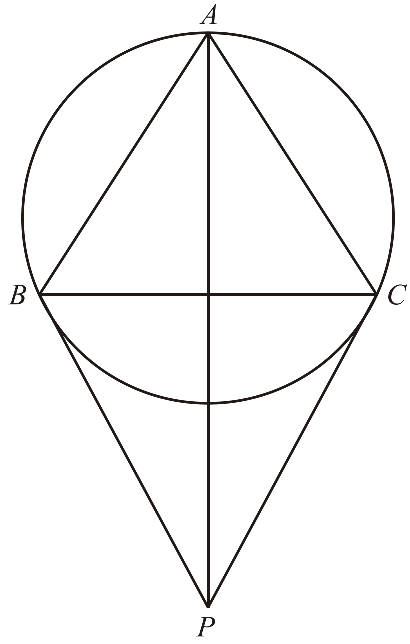
(c) Is Ben correct? You must explain your answer.

....

....

(1)
(Total 4 marks)

6.



A, B and C are three points on the circumference of a circle.
Angle ABC = Angle ACB.
PB and PC are tangents to the circle from the point P.

(a) Prove that triangle APB and triangle APC are congruent.

(3)

Angle $BPA = 10°$.

(b) Find the size of angle ABC.

..............................°

(4)

(Total 7 marks)

7.

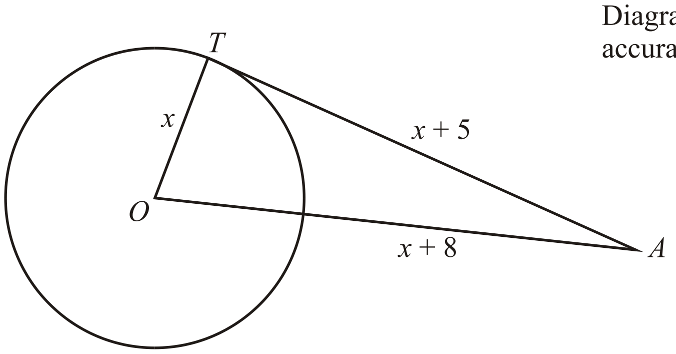


Diagram **NOT** accurately drawn

AT is a tangent at T to a circle, centre O.
$OT = x$ cm, $AT = (x + 5)$ cm, $OA = (x + 8)$ cm.

(a) Show that $x^2 - 6x - 39 = 0$

(4)

(b) Solve the equation $x^2 - 6x - 39 = 0$ to find the radius of the circle.
Give your answer correct to 3 significant figures.

.... cm

(3)

(Total 7 marks)

8.

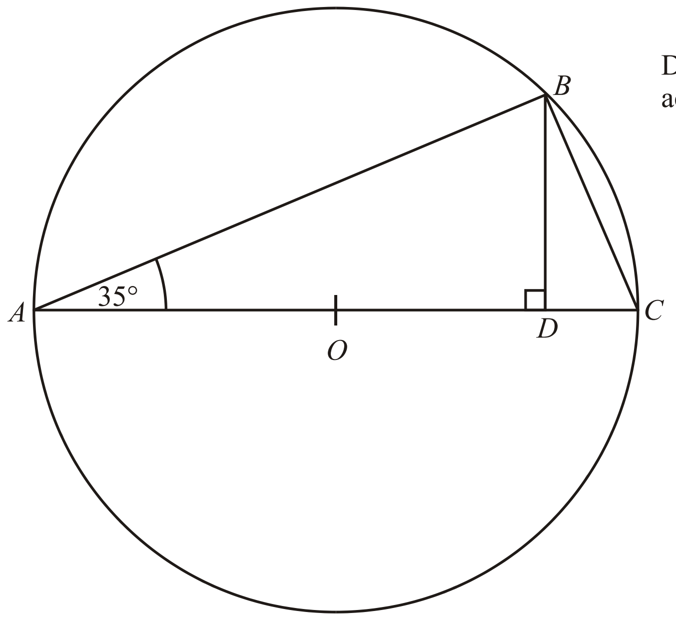


Diagram **NOT** accurately drawn

The diagram shows a circle, centre *O*.

AC is a diameter.
Angle *BAC* = 35°.
D is the point on *AC* such that angle *BDA* is a right angle.

(a) Work out the size of angle *BCA*.
Give reasons for your answer.

..................................°
(2)

(b) Calculate the size of angle *DBC*.

..................................°
(1)

(c) Calculate the size of angle *BOA*.

....................................°

(2)
(Total 5 marks)

9. (a) On the grid below, draw the graphs of

$$x^2 + y^2 = 100$$

and $$2y = 3x - 4$$

(3)

(b) Use the graphs to estimate the solutions of the simultaneous equations

$$x^2 + y^2 = 100$$

and $$2y = 3x - 4$$

....

....

(2)

For all the values of x

$$x^2 + 6x = (x + 3)^2 - q$$

(c) Find the value of q.

$q =$ ....

(2)

One pair of integer values which satisfy the equation

$$x^2 + y^2 = 100$$

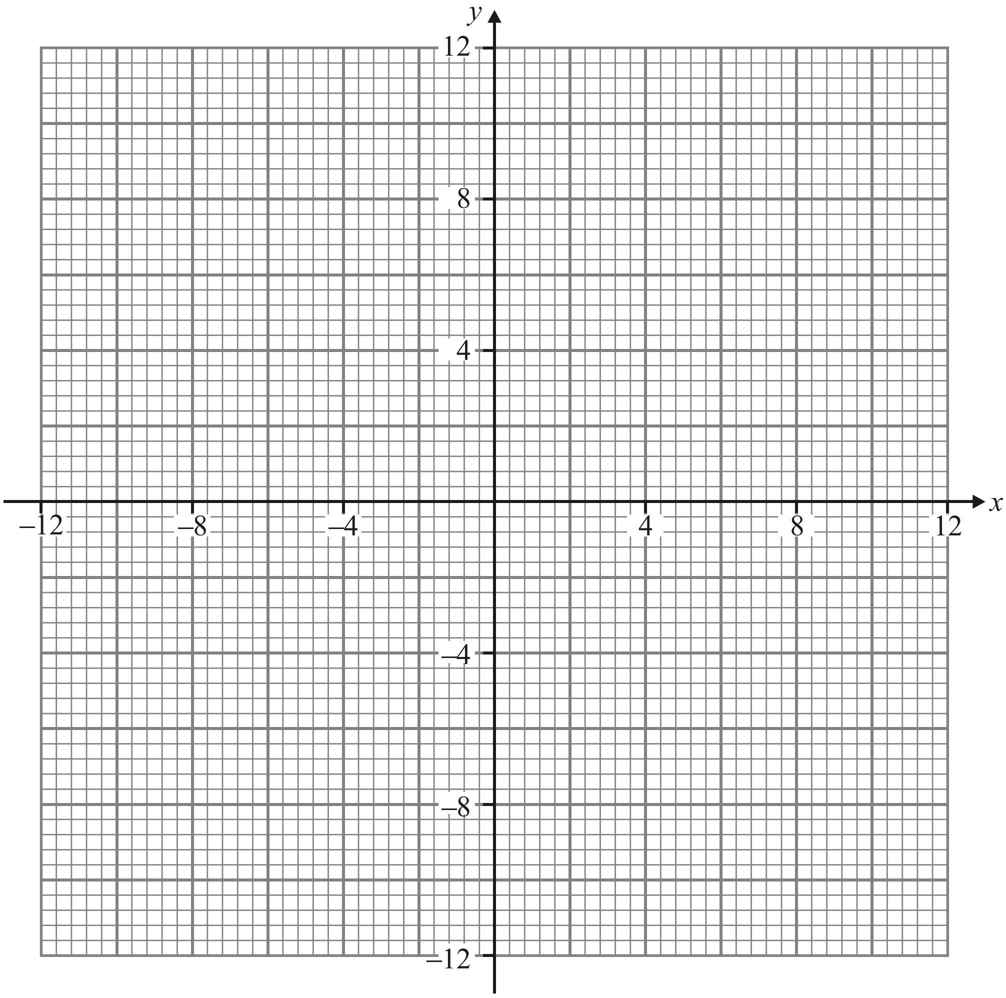
is $x = 6$ and $y = 8$

(d) Find one pair of integer values which satisfy

$$x^2 + 6x + y^2 - 4y - 87 = 0$$

$x =$ ...., $y =$ ....

(3)



(Total 10 marks)

10.

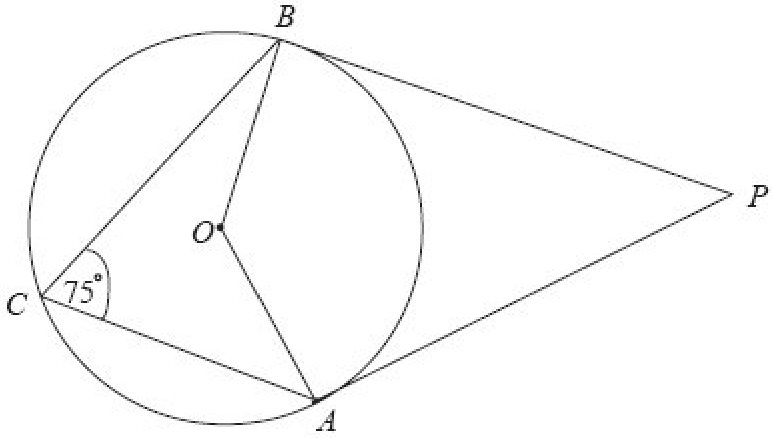


Diagram **NOT** accurately drawn

In the diagram, *A*, *B* and *C* are points on the circumference of a circle, centre *O*.

PA and *PB* are tangents to the circle.

Angle *ACB* = 75°.

(a) (i) Work out the size of angle *AOB*.

……………………°

(ii) Give a reason for your answer.

……………………………………………………………………………………..

……………………………………………………………………………………..

(2)

(b) Work out the size of angle *APB*.

……………………..°

(3)

(Total 5 marks)

11.

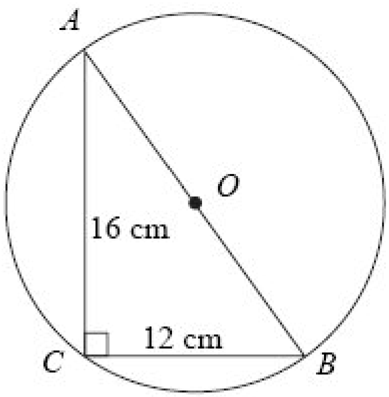


Diagram **NOT** accurately drawn

The diagram shows triangle *ABC* and a circle, centre *O*.

A, *B* and *C* are points on the circumference of the circle.

AB is a diameter of the circle.

AC = 16 cm and *BC* = 12 cm.

(a) Angle *ACB* = 90°.
Give a reason why.

…………………………………………………………………………………………

(1)

(b) Work out the diameter AB of the circle.

………………..cm

(3)

(c) Work out the area of the circle.
Give your answer correct to 3 significant figures.

………………..cm^2

(3)
(Total 7 marks)

12.

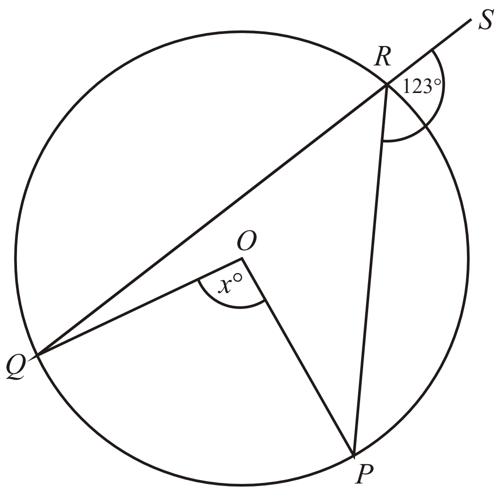


Diagram **NOT** accurately drawn

QRS is a straight line.
QR and *PR* are chords of a circle, centre *O*.
Angle *PRS* = 123°.
Angle *QOP* = *x*°.

Calculate the size of the angle marked *x*°.
Give reasons for your answer.

..................................°

(Total 3 marks)

13.

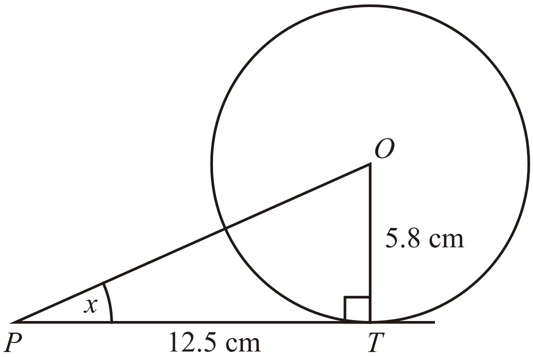


Diagram **NOT** accurately drawn

In the diagram, *T* is a point on a circle, centre *O*.
PT is the tangent to the circle at *T*.

(a) Angle *OTP* is a right angle.
Give a reason why

....

(1)

The radius of the circle is 5.8 cm.
PT = 12.5 cm.

(b) Calculate the size of angle *x*.
Give your answer correct to 1 decimal place.

x =.................... °

(3)

C is the point on the circle where the straight line *OP* crosses the circle.

(c) Calculate the length of *PC*.
Give your answer correct to 3 significant figures.

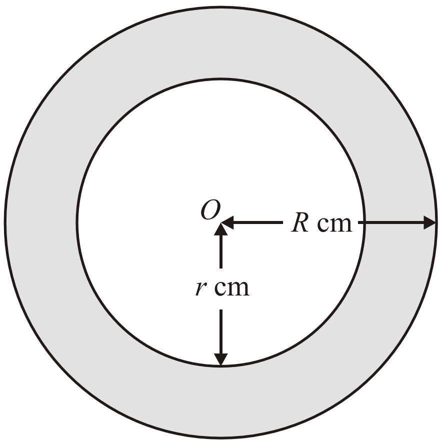
.... cm

(4)

(Total 8 marks)

14. The diagram shows two circles.

Diagram **NOT** accurately drawn



O is the centre of both circles.
The radius of the outer circle is *R* cm.
The radius of the inner circle is *r* cm.
$R = 15.8$ correct to 1 decimal place.
$r = 14.2$ correct to 1 decimal place.

(a) John says that the minimum possible diameter of the inner circle is 28.35 cm. Explain why John is wrong.

....

....

....

(2)

The upper bound for the area, in cm^2, of the shaded region is $k\pi$.

(b) Find the **exact** value of *k*.

k = ...

(4)

(Total 6 marks)

15. The diagram shows some of the markings on a baseball field.

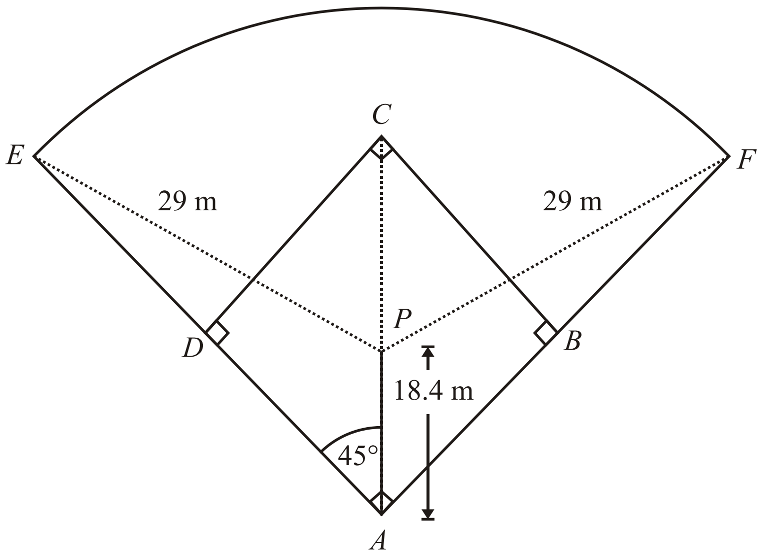


Diagram **NOT** accurately drawn

ABCD is a square.
AC is a diagonal of *ABCD*.
P is a point on *AC*.
ADE and *ABF* are straight lines.

AP = 18.4 m.
Angle *PAE* = 45°.

EF is an arc of the circle, centre *P* and radius 29 m.

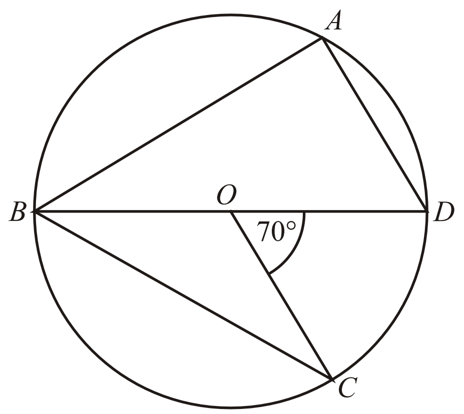
(a) By considering triangle *PAE*, calculate the size of angle *AEP*.
Give your answer correct to 3 significant figures.

.... °
(3)

(b) Calculate the length of the arc *EF*.
Give your answer correct to 3 significant figures.

.... m
(4)
(Total 7 marks)

16. Diagram **NOT** accurately drawn



A, B, C and *D* are points on the circumference of a circle, centre *O*.
BOD is a straight line.
Angle *COD* = 70°

(a) Find the size of angle *BAD*.
Give a reason for your answer.

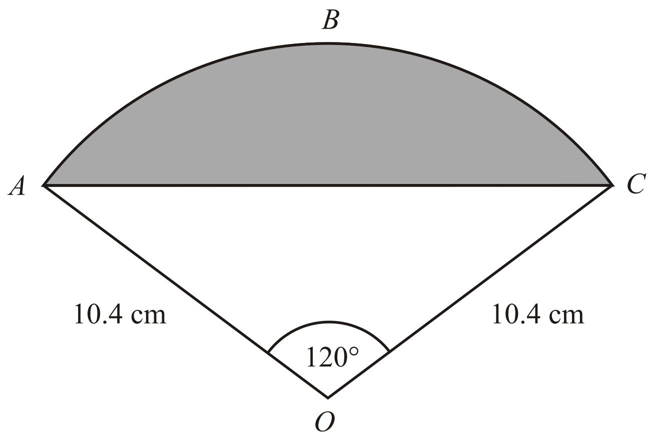
.....................................°
(2)

(b) Find the size of angle *CBD*.
Give a reason for your answer.

.....................................°
(2)
(Total 4 marks)

17.

Diagram **NOT** accurately drawn



The diagram shows a sector *OABC* of a circle with centre *O*.
OA = *OC* = 10.4 cm.
Angle *AOC* = 120°.

(a) Calculate the length of the arc *ABC* of the sector.
Give your answer correct to 3 significant figures.

.....................................cm
(3)

(b) Calculate the area of the shaded segment *ABC*.
Give your answer correct to 3 significant figures.

.....................................cm^2
(4)
(Total 7 marks)

18.

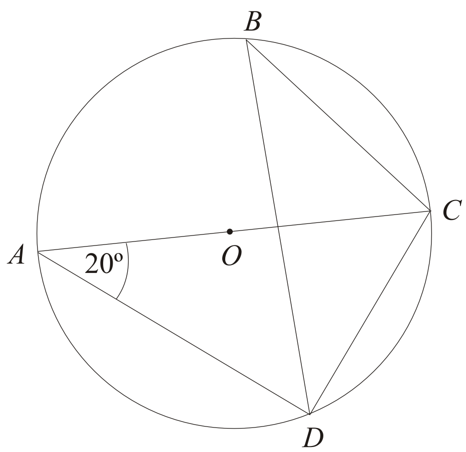


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
AC is a diameter of the circle.

Angle *DAC* = 20º.

(a) Find the size of angle *ACD*.

.... °

(2)

(b) Find the size of angle *DBC*.
Give a reason for your answer.

.... °

....

....

(2)
(Total 4 marks)

19.

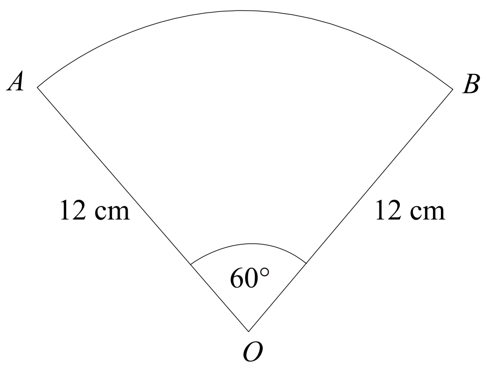


Diagram **NOT** accurately drawn

OAB is a sector of a circle, centre *O*.
Angle *AOB* = 60º.
OA = *OB* = 12 cm.

Work out the length of the arc *AB*.
Give your answer in terms of π.

.... cm
(Total 3 marks)

20.

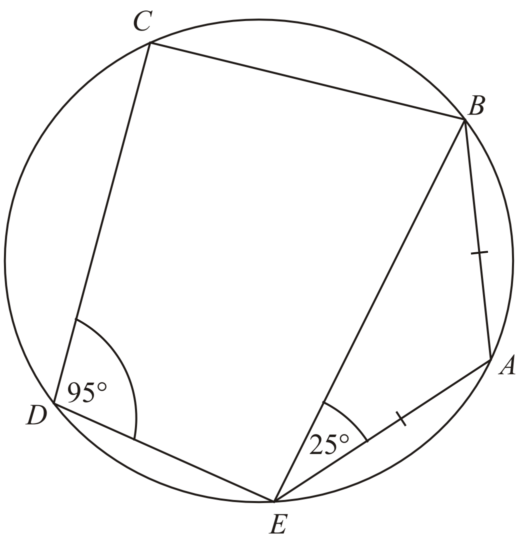


Diagram **NOT** accurately drawn

A, *B*, *C*, *D* and *E* are five points on a circle.
Angle *BEA* = 25° and angle *CDE* = 95°.
AB = *AE*.

(a) (i) Work out the size of angle *BAE*.

.... °

(ii) Give reasons for your answer.

....

....

....

(3)

(b) Work out the size of angle *CBE*.

.... °

(1)

(Total 4 marks)

21.

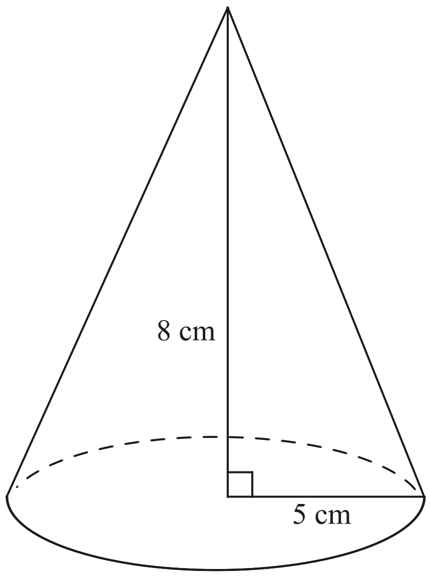


Diagram **NOT** accurately drawn

A cone has a base radius of 5 cm and a vertical height of 8 cm.

(a) Calculate the volume of the cone.
Give your answer correct to 3 significant figures.

.... cm^3

(2)

Here is the net of a different cone.

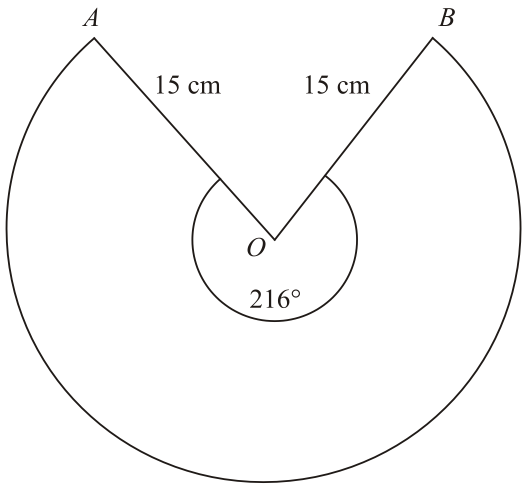


Diagram **NOT** accurately drawn

The net is a sector of a circle, centre *O*, and radius 15 cm.
Reflex angle *AOB* = 216°
The net makes a cone of slant height 15 cm.

(b) Work out the vertical height of the cone.

.... cm

(4)

(Total 6 marks)

22.

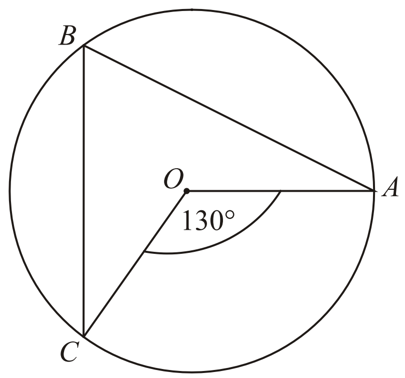


Diagram **NOT** accurately drawn

(a) In the diagram, O is the centre of the circle.
A, B and C are points on the circle.

Angle $COA = 130°$.

(i) Find the size of angle CBA.

....

(ii) Give a reason for your answer.

....

....

(2)

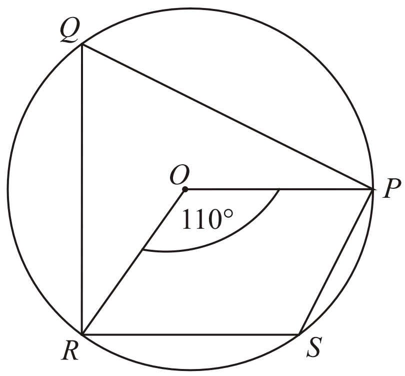


Diagram **NOT** accurately drawn

(b) In the diagram, O is the centre of the circle.
P, Q, R and S are points on the circle.

Angle $ROP = 110°$

Calculate the size of angle RSP.

....

(2)
(Total 4 marks)

23.

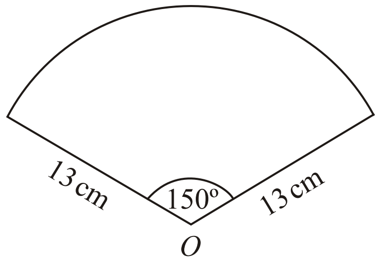


Diagram **NOT** accurately drawn

The diagram shows a sector of a circle, centre *O*.
The radius of the circle is 13 cm.
The angle of the sector is 150°.

Calculate the area of the sector.
Give your answer correct to 3 significant figures.

.... cm^2
(Total 2 marks)

24.

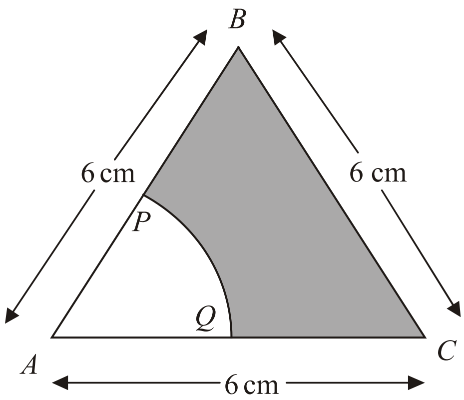


Diagram **NOT** accurately drawn

The diagram shows an equilateral triangle *ABC* with sides of length 6 cm.

P is the midpoint of *AB*.
Q is the midpoint of *AC*.
APQ is a sector of a circle, centre *A*.

Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

.... cm^2
(Total 4 marks)

25. R, S and T are points on the circumference of a circle, centre O.
PS and PT are tangents to the circle.
PSN and $TORN$ are straight lines.
PO are parallel to SR.
$SR = NR$.
Angle OPT = angle OPS.

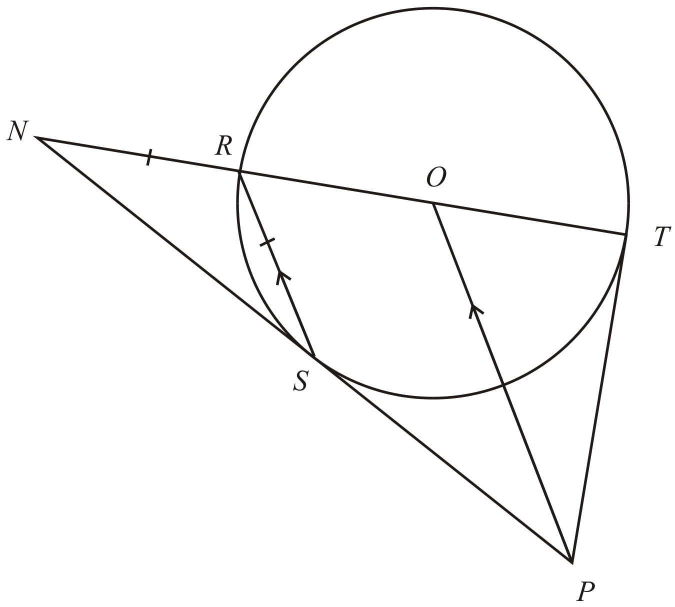


Diagram **NOT** accurately drawn

(a) Work out the size of angle PNT.

..................................°
(3)

(b) Show that $PS = SN$.

(3)
(Total 6 marks)

26.

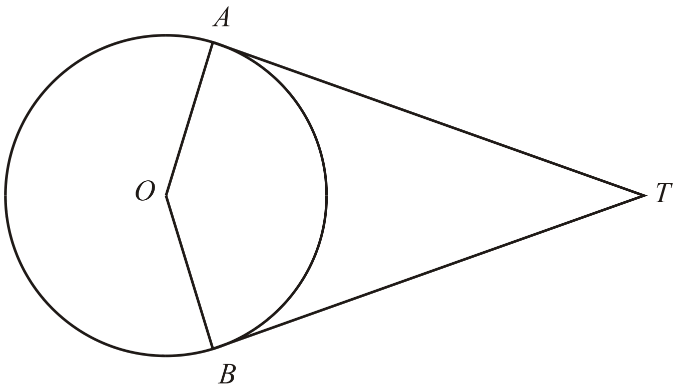


Diagram **NOT** accurately drawn

A and *B* are points on the circumference of a circle, centre *O*.
TA and *TB* are tangents to the circle.

Calculate the size of the angle *ATO* when angle *AOT* = 56°.
Give a reason for each stage in your working.

………………………°
(Total 3 marks)

27.

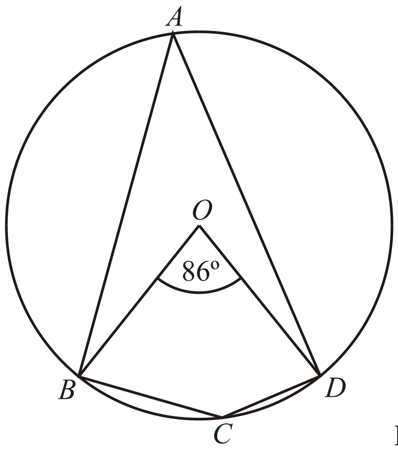


Diagram **NOT** accurately drawn

A,B,C and *D* are points on the circle, centre *O*.
Angle *BOD* = 86º

(a) (i) Work out the size of angle *BAD*.

..........................º

(ii) Give a reason for your answer.

....

....

(2)

(b) Work out the size of angle *BCD*.

..........................º

(1)
(Total 3 marks)

28.

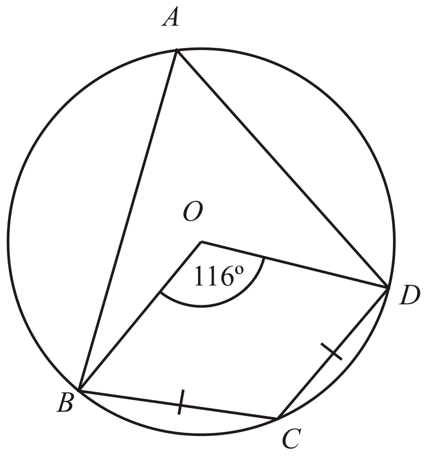


Diagram **NOT** accurately drawn

A,B,C and *D* are points on a circle, centre *O*.
Angle *BOD* = 116°

(a) Calculate the size of angle *BAD*.

..................................°
(1)

BC = *CD*.

(b) Calculate the size of angle *DBC*.

..................................°
(2)
(Total 3 marks)

29.

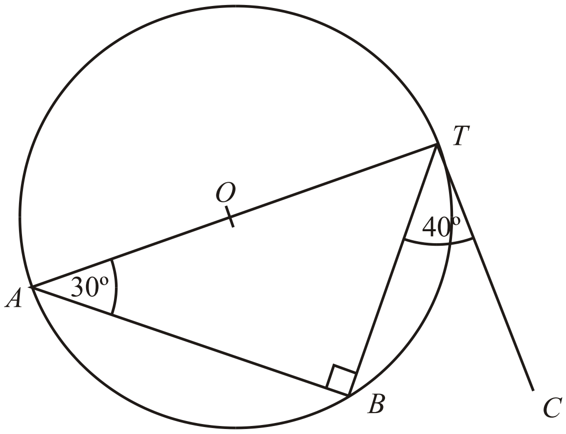


Diagram **NOT** accurately drawn

T, *A* and *B* are points on the circumference of the circle, centre *O*.
AT is a diameter of the circle.
Angle *BTC* = 40°
Angle *TAB* = 30°

Explain why *TC* cannot be a tangent to the circle.

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

(Total 2 marks)

30.

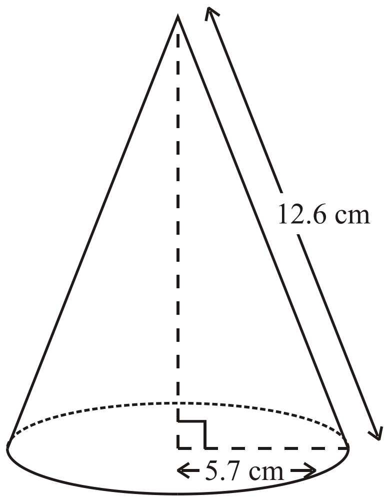


Diagram **NOT** accurately drawn

The radius of the base of a cone is 5.7 cm.
Its slant height is 12.6 cm.

Calculate the volume of the cone.
Give your answer correct to 3 significant figures.

…………………….. cm^3
(Total 4 marks)

31.

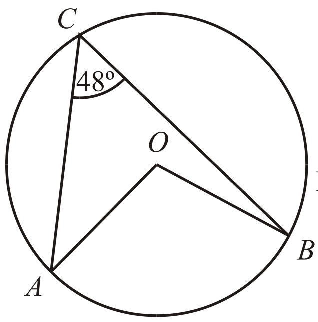


Diagram **NOT** accurately drawn

A, *B* and *C* are points on the circumference of a circle, centre *O*.
Angle *ACB* = 48°

(a) Write down the size of angle *AOB*.
Give a reason for your answer.

..........................°

(2)

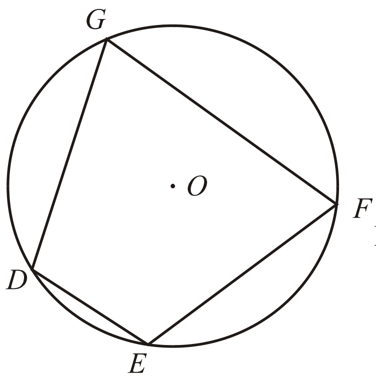


Diagram **NOT** accurately drawn

D, E, F and G are points on the circumference of a circle, centre O.

(b) **Prove** that, in the cyclic quadrilateral $DEFG$,

angle DEF + angle $DGF = 180°$

You may use, without proof, any standard theorem (fact) excluding those that refer to a cyclic quadrilateral.

(3)
(Total 5 marks)

32.

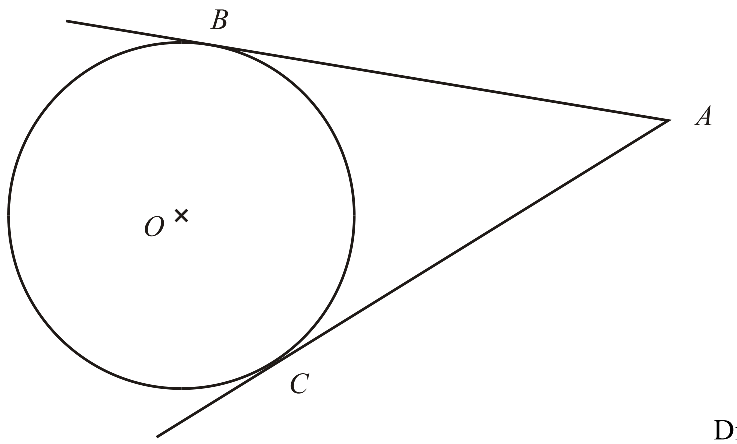


Diagram **NOT** accurately drawn

B and *C* are points on a circle, centre *O*.
AB and *AC* are tangents to the circle.
Angle *BOC* = 108°.

Work out the size of angle *BAO*.
Give a reason for each stage in your working.

..............................°
(Total 3 marks)

33.

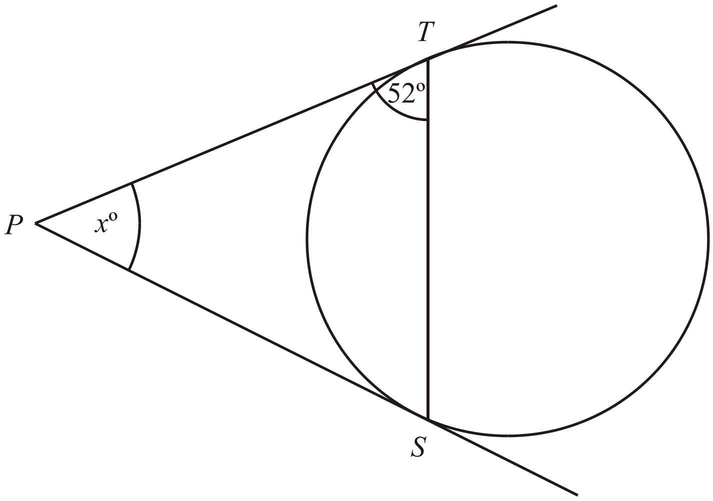


Diagram **NOT** accurately drawn

T and *S* are points on the circumference of a circle.
PT and *PS* are tangents to the circle.
Angle *STP* = 52°.
Angle *TPS* = x°.

(i) Work out the value of x.

x = ...

(ii) Give reasons for your answer.

....

....

....
(Total 2 marks)

34.

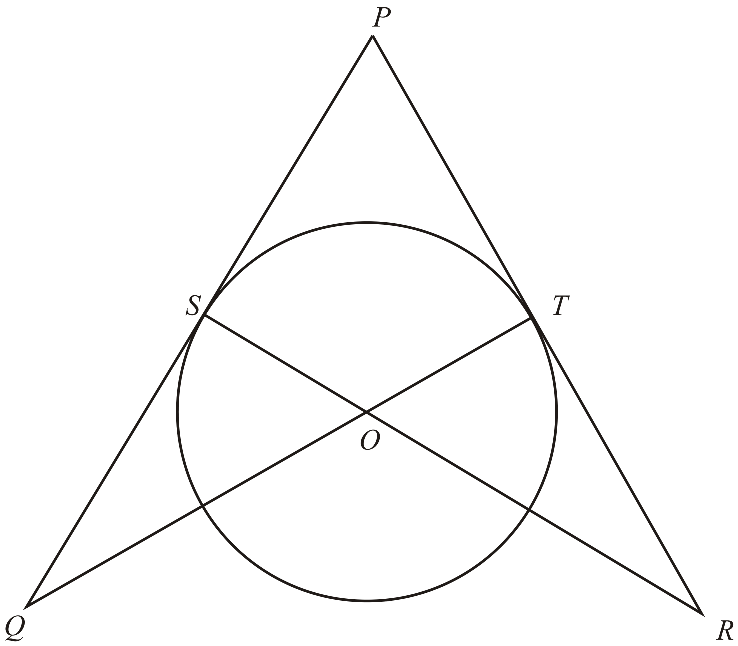


Diagram **NOT** accurately drawn

S and *T* are points on a circle, centre *O*.
PSQ and *PTR* are tangents to the circle.
SOR and *TOQ* are straight lines.

(a) Prove that triangle *PQT* and triangle *PRS* are congruent.

(3)

Asif says that triangle *STQ* and triangle *STR* have equal areas.

(b) Explain why Asif is correct.

....

....

....

....

(2)

(Total 5 marks)

35.

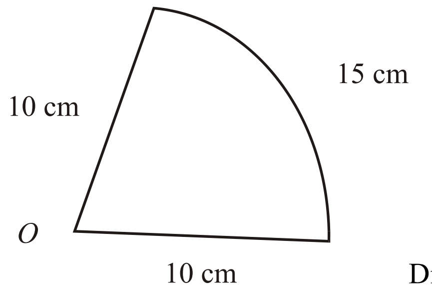


Diagram **NOT** accurately drawn

The diagram shows a sector of a circle, centre *O*, radius 10 cm.
The arc length of the sector is 15 cm.

Calculate the area of the sector.

.... cm^2
(Total 4 marks)

36.

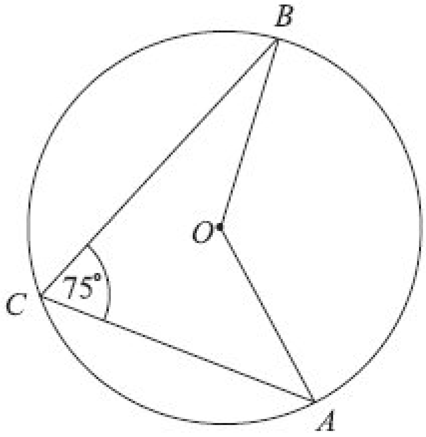


Diagram **NOT** accurately drawn

In the diagram, A, B and C are points on the circumference of a circle, centre O.
Angle $ACB = 75°$.

(i) Work out the size of angle AOB.

…………°

(ii) Give a reason for your answer.

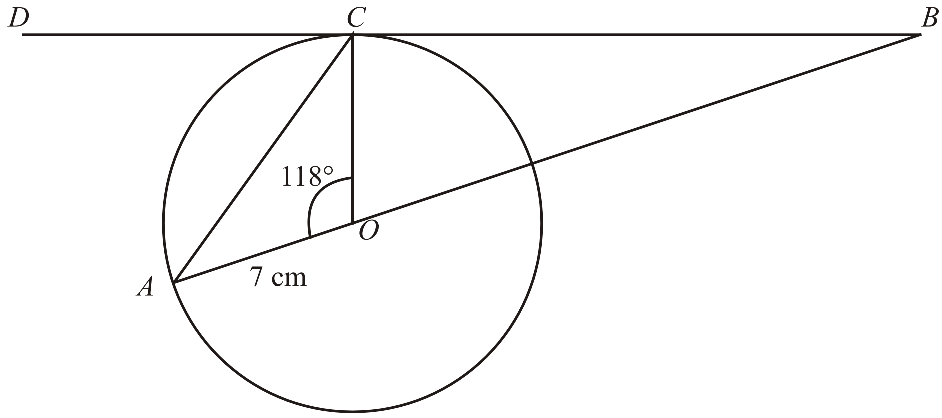
………………………………………………………………………………

………………………………………………………………………………

(Total 2 marks)

37.

Diagram **NOT** accurately drawn



A and *C* are points on a circle, centre *O*.
DCB is the tangent to the circle at *C*.
AOB is a straight line.
OA = 7 cm.
Angle *AOC* = 118°.

Work out the length of *OB*.
Give your answer correct to 3 significant figures.

OB = ... cm
(Total 4 marks)

38.

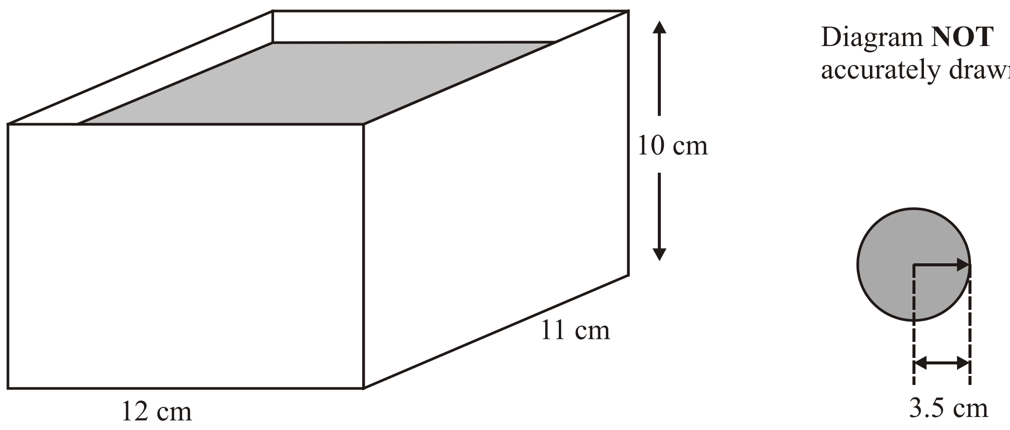


Diagram **NOT** accurately drawn

A rectangular container is 12 cm long, 11 cm wide and 10 cm high.
The container is filled with water to a depth of 8 cm.

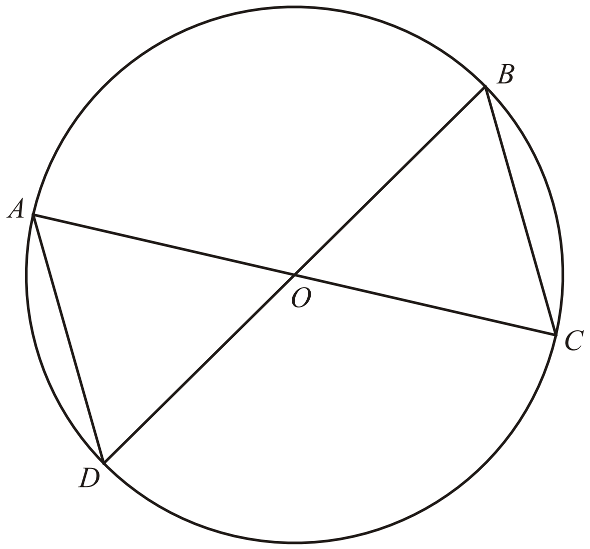
A metal sphere of radius 3.5 cm is placed in the water.
It sinks to the bottom.

Calculate the rise in the water level.
Give your answer correct to 3 significant figures.

..............................cm
(Total 4 marks)

39.

Diagram **NOT** accurately drawn



A, B, C and *D* are four points on a circle, centre *O*.
AOC and *BOD* are straight lines.

Prove that triangle *AOD* and triangle *BOC* are congruent.
You must give reasons for each stage of your proof.

(Total 3 marks)

40.

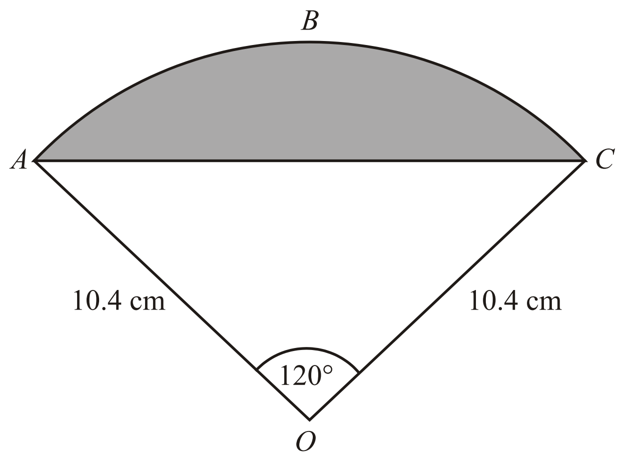


Diagram **NOT** accurately drawn

The diagram shows a sector *OABC* of a circle with centre *O*.
OA = *OC* = 10.4 cm.
Angle *AOC* = 120°.

Calculate the area of the shaded segment *ABC*.
Give your answer correct to 3 significant figures.

…………………….cm²
(Total 4 marks)

41.

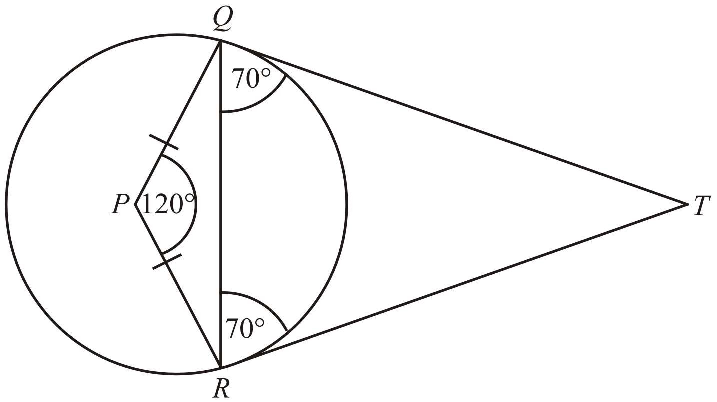


Diagram **NOT** accurately drawn

In the diagram *Q* and *R* are points on the circumference of a circle.
TQ and *TR* are tangents to the circle.
PQ = *PR*.
Angle *RQT* = angle *QRT* = 70°.
Angle *RPQ* = 120°.

Explain why *P* is not the centre of the circle.

....

....

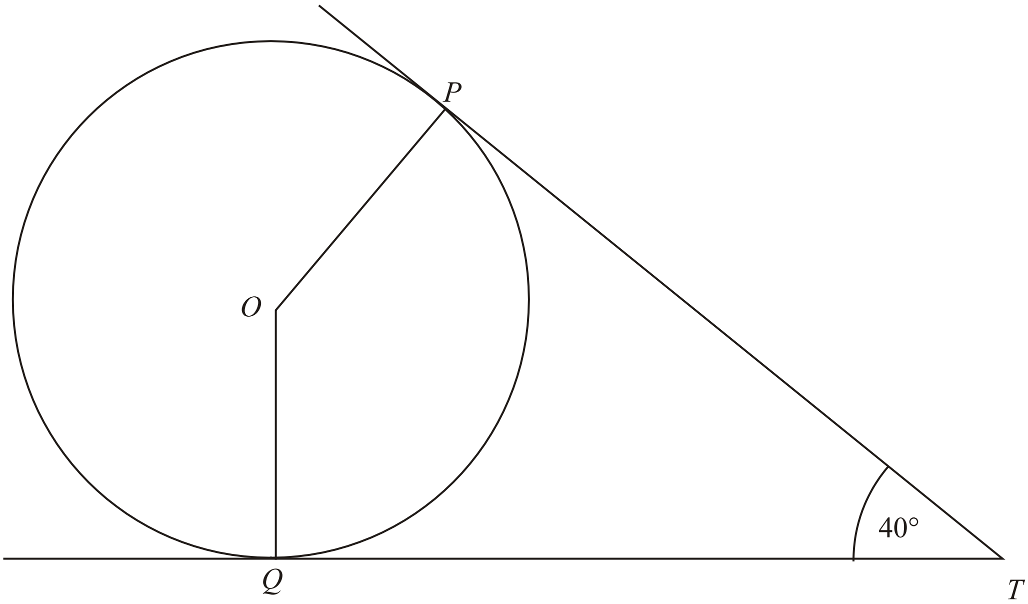
....

....

(Total 2 marks)

42.

Diagram **NOT** accurately drawn



P and *Q* are two points on a circle centre *O*.

The tangents to the circle at *P* and *Q* intersect at the point *T*.

(a) Write down the size of angle *OQT*.

..............................°

(1)

(b) Calculate the size of the obtuse angle *POQ*.

......................°
(2)

(c) Give reasons why angle *PQT* is 70°

....

....
(2)
(Total 5 marks)

43.

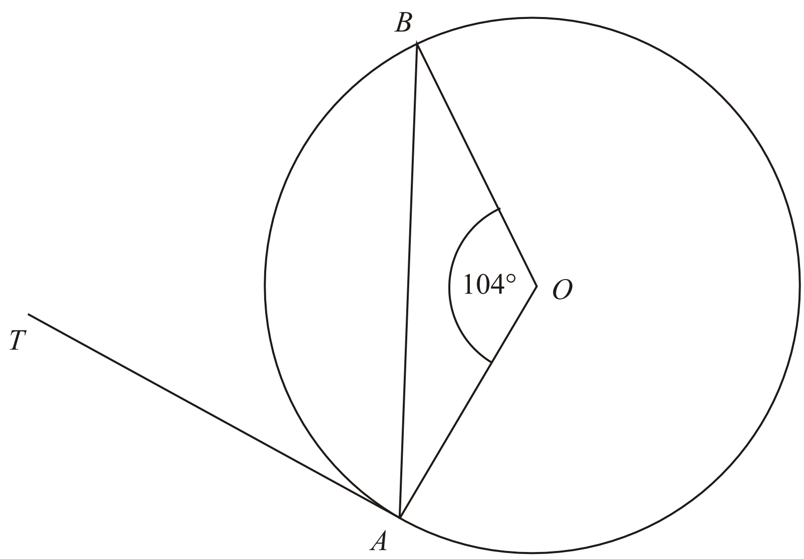


Diagram **NOT** accurately drawn

A and *B* are two points on the circumference of a circle, centre *O*.
TA is a tangent to the circle.
Angle *BOA* = 104°.

Work out the size of angle *BAT*.

.......................°

(Total 3 marks)

44.

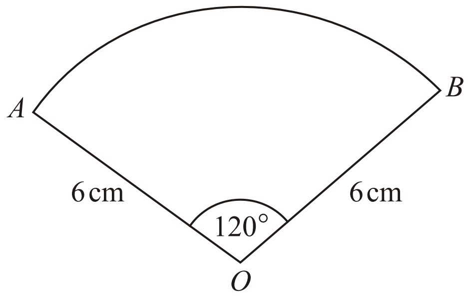


Diagram **NOT** accurately drawn

The diagram shows a sector of a circle, centre *O*.
The radius of the circle is 6 cm.
Angle *AOB* = 120°.

Work out the **perimeter** of the sector.
Give your answer in terms of π in its simplest form.

.... cm
(Total 3 marks)

1. (i) 27° 4

Tangent 90° to diameter /radius/ line from (through) centre

B1 for 27° cao
B1 for reason

(ii) 63°

180 −(90+ "27")
angle in semicircle (is 90°)/Alternate segments
/angle at centre twice at circumference

B1 ft for 90−"27" if not 63°
B1 for reason

[4]

2. (a) 90 2

Angle in a semi circle (is a right angle)

B1 cao
B1 for a valid reason

(b) 56 2

Angles in the same segment (are equal)

B1 cao
B1 for reason

(c) 112 2

Angle at the centre is twice the angle at the circumference

B1 cao
B1 for reason

[6]

3. (a) 1.75
1.65
31.05
30.95 2

B2 all correct
Or B1 for 2 or 3 correct

(b) 29.2 1

"30.95" – 1.75

B1 ft on values in (a)

(c) 5531 4

Minimum volume of bigger sphere = $\frac{4}{3} \times \pi \times$ "30.95^3" = …..

Maximum volume of smaller sphere = $\frac{4}{3} \times \pi \times$ "1.75^3" =

M1 for correct substitution of his/her "30.95" or "1.75" into $\frac{4\pi}{3}r^3$
A1 for either 124122 – 124201 or 22.4379 – 22.4542
M1 (dep) for his/her min big vol ÷ his/her max little vol
A1 cao

[7]

4. 3

OY = OX (radii)
OM = OM or OM is common
OMX = OMY = 90°
B1 for any one line
B1 for remaining two lines
B1 (dep on 2 previous Bs) for
Δ OMY ≡ Δ OMX RHS and conclusion

[3]

5. (a) 60 1

B1 for 60
B1 for BDC = 25

(b) 35 2

eg Angle *BDC* = 25
ADB = 60 – 25

[Award the mark for equivalent approaches]
B1 ft for (a) –25

(c) Ben is correct; angle *DAB* = 65 + 25 = 90 and since angle in a semi-circle is 90°, *BD* must be a diameter 1

B1 for full valid justification

[4]

6. (a) (I) *AB* = *AC* (triangle *ABC* is isosceles)
(II) *PB* = *PC* tangents (from a point to a circle are) equal
(III) *AP* = *AP* (common side)
so the 2 triangles are congruent, SSS. 3

B3 for I, II, III with congruency reason
(B2 for any two of I, II or III)
(B1 for any one of the I, II or III)

(b) 50° 4

BPC = 20°
PBC (or *PCB*) = 90–1/2 "20" (= 80°)
BAC = *PBC* = "80"

B4 for 50°
(B3 for BAC = 80°)
(B2 for PBC = 80° or PCB = 80°)
(B1 for APC = 10° or BPC = 20° or a middle angle = 90°)
SC if clear numerical slip seen eg "PBC = 180 – 90 – 10 = 70" then goes on to get correct ft angle ABC = 55 deduct 1 from total so this cand would get B4 – 1 = B3

[7]

7. (a) $(x + 8)^2 = x^2 + (x + 5)^2$

$x^2 + 16x + 64 = 2x^2 + 10x + 25$ — 4

B1 for angle OTA = 90º (implied by use of Pythagoras with OA as hypotenuse)

M1 $(x + 8)^2 = x^2 + (x + 5)^2$ oe

M1 for correct squaring of x + 8 or x + 5

A1 for completion following correct working

(b) 9.93 — 3

$(+ 6 + \sqrt{(6^2 - - 4 \times 39)}) /2$

$= (6 \pm \sqrt{(36 + 156)}) /2$

$= (6 \pm \sqrt{192}) /2 = (6 + 13.856) /2$

M1 for substitution into quadratic formula, allow sign errors in b and c

M1 for $x = (6 \pm \sqrt{192}) /2$ [+ alone will do]

A1 for 9.92 – 9.93

M1 for $(x - 3)^2 - 9 - 39$

M1 for $x = 3 \pm \sqrt{48}$ [+ alone will do it]

A1 for 9.92 – 9.93

T&I= 9.93 gets M1,M0,A0

[7]

8. (a) 55º — 2

90 –35 = 55º

Angle in a semi-circle = 90º

B1 for 55º

*B1 for (angle in) a **semi-circle** = 90º*

(b) 35º — 1

90 – "55"

B1 for 35º ft

(c) 110º — 2

180 – 2 × 35 = 110º

M1 for complete method or for twice "(a)"

A1 cao

Candidates may choose to use Isosceles triangles or Angle subtended at centre is twice angle subtended at circumference

[5]

9. (a) Circle centre *O* Line 3

B1 correct circle, within overlay
B2 correct line tol ± 1mm at (4, 4) and (0, –2)
(B1 for any straight line with the correct intercept on the y axis)

(b) $x = 6.4$,
$y = 7.7$
$x = -4.6$,
$y = -8.9$ 2

B2 Two paired solutions, ft from a line and a curve with at least B1 scored in (a)
B1 Any two correct values, ft from a line and a curve with at least B1 scored in (a)
Tol ± 0.2

(c) $q = 9$ 2

$(x + 3)^2 - 9$

B1 for $x^2 + 6x + 9$ seen
B1 for $q = 9$

(d) 3, 10 3

$(x + 3)^2 - 9 + (y - 2)^2 - 4 - 87 = 0$
$(x + 3)^2 + (y - 2)^2 = 100$

M1 for completing the square
A1 for $(y - 2)^2 - 4$ seen
A1 any correct answer

[10]

10. (a) (i) 150 2

B1 accept 150 or 210

(ii)

B1 for angle at the centre is twice the angle at the circumference

(b) 30 — 3

360 – 90 – 90 – "150" or 180 – "150"

B1 identifies angle between radius and tangent as 90° (may be in working or on diagram)
M1 360 – 90 – 90 – "150"
A1 ft from (a)(i) excluding a negative answer
Or
B1 for 90
M1 for $2 \times (180 - 90 - \text{“}\frac{150}{2}\text{“})$
A1 ft from (a)(i) excluding a negative answer
Or
B3 for 180 – (a)
SC: 180 – "210" can get B1 for 90°and/or B1 for "cyclic quadrilateral"

[5]

11. (a) Angle in a semicircle — 1

B1 oe

(b) 20 — 3

$12^2 + 16^2 = 400$
$\sqrt{400} = 20$

M1 for $12^2 + 16^2$
M1 for $\sqrt{144 + 256}$
A1 cao

(c) 314 — 3

$\pi \times 10^2$

M1 for $\pi \times \left(\frac{"20"}{2}\right)^2$
M1 (indep) for correct order of evaluation of $\pi \times r^2$ *for any r*
A1 for 314 – 315 inclusive

[7]

12. $QRP = 57°$
$x = 2 \times 57 = 114°$ 3

M1 for QOP = 2 × "57" or QRP = 57° in working or on the diagram.
A1 114° cao
B1 angle at circumference is half angle at centre

[3]

13. (a) Reason 1

B1 for 'radius is perpendicular to tangent' oe

(b) $\tan x = \frac{5.8}{12.5} (= 0.464)$

$P = \tan^{-1}(0.464) = 24.89\ldots$
24.9 3

M1 for $\tan\ (x =) \frac{5.8}{12.5}$ *or* $\sin\ (x =) \frac{5.8}{'OP'}$ *or* $\cos\ (x =) \frac{12.5}{'OP'}$

M1 for $\tan^{-1} \frac{5.8}{12.5}$ *oe correct use of inverse*

A1 for 24.9 (or better)
*(**SC** M1M1A0 for either 0.434(4..) or 27.6(5..) seen)*

(c) $OP^2 = 12.5^2 + 5.8^2$

$OP = \sqrt{12.5^2 + 5.8^2} \left(= \sqrt{189.89}\right) = 13.78...$

$PC = OP - 5.8$
7.98 4

M1 for $12.5^2 + 5.8^2$

M1 for $\sqrt{12.5^2 + 5.8^2}$

OR

M1 for $\cos$ *'24.9'* $\frac{12.5}{OP}$ *or* $\sin$ *'24.9'* $= \frac{5.8}{OP}$

M1for OP = 12.5÷ cos '24.9' or 5.8 ÷ sin '24.9'
A1 for 13.7 to 13.8
B1ft (dep on OP > 12.5) for adding or subtracting 5.8

[8]

14. (a) 'minimum possible diameter is twice minimum possible radius' oe
minimum possible diameter = 2 × 14.15 = 28.3 cm 2

M1 for 'minimum possible diameter is twice minimum possible radius' or 2 × 14.15 seen
A1 for 28.3 cao

(b) **upper bound**, in cm, for radius of outer circle is 15.85
lower bound, in cm, for radius of inner circle is 14.15
area, in cm^2, of shaded region
$=\pi R^2 - \pi r^2$
$= \pi(15.85)^2 - \pi(14.15)^2$
$= 51\pi$
$k = 51$ 4

B1 for 15.85 or 789.2(3...) seen
B1 for 14.15 or 629.0(1...) seen
M1 for using $\pi R^2 - \pi r^2$
A1 cao (accept final answer left as 51π)

[6]

15. (a) $\dfrac{29}{\sin 45^\circ} = \dfrac{18.4}{\sin \angle PEA}$

$\sin \angle PEA = \dfrac{18.4 \times \sin 45^\circ}{29} (= 0.4486...)$

$\angle PEA = 26.6569...$
26.7 3

M1 for correct substitution in sine rule
M1 (dep) for rearrangement to get $\dfrac{18.4 \times \sin 45^\circ}{29}$ oe
(award if 0.448(6...) or 0.538(8...) or 0.412(0...) seen)
A1 cao for answers rounding to 26.7

(b) $\angle EPF = 2 \times [45 + \text{'26.7'}] (= 143.4)$

$\text{arc } EF = \dfrac{143.4}{360} \times \pi \times 58$

72.6 4

M1 for valid method to find $\angle EPF$ (award if 143(.4)° seen)
M2 (dep) for $\dfrac{\text{'143.4'}}{360} \times \pi \times 58$
(M1 for either $\dfrac{\text{'143.4'}}{360} \times k_1$ or for $k_2 \times 58\pi$, where $k_2 < 1$)
A1 for 72.5 to 72.6 inclusive
***SC** award B2 for 61.1*

[7]

16. (a) 90
90° 2

B1 90°
B1 angle in semi circle (= 90°)

(b) $70 \div 2$

$35°$ — 2

B1 35° or 325°
B1 angle at centre = twice angle at circumference
OR
B1 angle on a straight line with isosceles triangle

[4]

17. (a) $\frac{120}{360}$ or $\frac{1}{3}$

$\frac{120}{360} \times 2\pi \times 10.4$

$21.7 – 21.8$ — 3

B1 for $\frac{120}{360}$ *or* $\frac{1}{3}$ *seen*
M1 for $\frac{120}{360} \times 2\pi \times 10.4$
A1 for 21.7 – 21.8

(b) Area Sector = $\pi(10.4)^2 \div 3 = 113.26488$

Area Triangle = $\frac{1}{2}(10.4)(10.4)\sin 120°$

$= 46.8346$

Area segment = 66.43 ....

66.4 — 4

M1 for $\pi(10.4)^2 \div 3$ *or* $\pi(10.4)^2 \times \frac{120}{360}$ *oe*

M1 for $\frac{1}{2}(10.4)(10.4)\sin 120°$ *or any other valid method for area triangle OAC*
M1 (dep on at least 1 of the previous Ms) for area of sector – area of triangle OAC, providing the answer is positive.
A1 66.35 – 66.5

[7]

18. (a) 70 — 2

for 180 – (20 + 90) or angle CDA = 90° seen
A1 cao

(b) 20 — 2

B1 cao
B1 for angles in the same segment (are equal) or angles subtended by same arc at circumference

[4]

19. $\frac{60}{360} \times 2 \times \pi \times 12 = 4\pi$ — 3

M2 for $\frac{60}{360} \times 2 \times \pi \times 12$, *accept numerical* π

(M1 for $\frac{60}{360} \times k$, *where k in terms of* π, *or* $n \times 2 \times \pi \times 12$, $n < 1$*)*

A1 for 4π *or* $\frac{a\pi}{b}$ *cao, where a and b are correct integers*

[3]

20. (a) (i) $180 - 2 \times 25 = 130$

M1 for $180 - 2 \times 25$
A1 cao

(ii) Reason — 3

B1 for mentioning isosceles and equal (or base) angles or equal sides and equal (or base) angles

(b) $180 - 95 = 85$ — 1

B1 cao

[4]

21. (a) $\frac{1}{3} \times \pi \times 5^2 \times 8 = \pi \times 25 \times 8 \div 3 = 209.4395$

209 – 210 — 2

M1 for $\frac{1}{3} \times \pi \times 5^2 \times 8$

A1 for 209 – 210

(b) Base radius = $\frac{216}{360} \times 15 = 9$

Height = $\sqrt{(15^2 - 9^2)} = 12$ 4

M1 for 216 ÷ 360
A1 for 9
M1 for $\sqrt{(15^2 - \text{“}9\text{”}^2)}$, where “9” < 15
A1 cao

[6]

22. (a) (i) 130 ÷ 2 2

= 65

B1 cao

(ii) Reason

B1 ‘angle at centre is twice the angle at the circumference’
Allow “origin & O & middle” and “edge & perimeter”

(b) *RQP* = 55°
RSP = 180° – *RQP* = 125 2

M1 full method for RSP
A1 cao
(SC B1 for methods that depend on QRS = 90° and PQO = 27.5° leading to 125°)

[4]

23. $\frac{150}{360} \times \pi \times 13^2 = 0.416 \times 530.9291585 = 221.22\ldots221$ 2

M1 for $\frac{150}{360} \times \pi\ 13^2$ *or* $\pi \times 13^2 \div 2.4$ *oe*
A1 220 – 222

[2]

24. $\frac{1}{2} \times 6 \times 6 \times \sin 60 - \frac{60}{360} \times \pi \times 3^2$

$= 15.588 - 4.712$

10.8 – 10.9 4

M1 for $\frac{1}{2} \times 6 \times 6 \times \sin 60$ *or for* $0.5 \times 6 \times \sqrt{6^2 - 3^2}$ *or 15.5 – 15.6 or 14.5 – 14.6 or ±5.48(65...)*

M1 for $\frac{60}{360} \times \pi \times 3^2$ *(= 4.712…)*

M1 (dep on 1 previous M1) for 'area of triangle' – 'area of sector'

A1 for 10.8 – 10.9

SC: B3 for 10.1 – 10.2 or 9.84 – 9.85

[4]

25. (a) 30 3

angle *TPO* = angle *OPN* = angle *PNT*

$2x + x + 90 = 180$

M1 for angle TPO = angle OPN = angle PNT

M1 (dep) for 2x + x + 90 = 180 (oe)

A1 cao

(b) *PS* = *SN* 3

M1 for identifying triangle NOP as isosceles

M1 for identifying angle NSO or angle PSO as 90°

A1 for use of symmetry together with result

OR M2 for 3 relevant statements

[M1 for 2 relevant statements]

A1 for use of congruent triangles together with result

OR M1 for $\tan ONS = \frac{OS}{SN}$ *and* $\tan OPS = \frac{OS}{SP}$ *oe*

[use of sine or cos accepted if isosceles triangle identified]

M1 for $SN = \frac{OS}{\tan ONS}$ *and* $SP = \frac{OS}{\tan OPS}$

A1 for result

[6]

26. 34 — 3

180 – (90 + 56)

M1 for 180 − (90 + 56)
A1 cao
B1 for angle between tangent and radius is 90°
[SC B1 if angle ATO is taken as 56° and angle AOT calculated giving 34°]

[3]

27. (a) (i) 43 — 2

B1 cao

(ii) Angle at centre

B1 for angle at centre = 2 × angle at circumference

(b) 137 — 1

180 – 43

B1 ft for 180 − "43"

[3]

28. (a) 58° — 1

B1 cao

(b) 29° — 2

Angle BCD = 122°

Angle DBC = $\frac{1}{2}(180 - 122)$

M1 for angle BCD = 180 − "58" or $\frac{360-116}{2}$

A1 cao

[3]

29. reasons — 2

B1 for numerical reason involving comparison of angle
[eg ∠ATC = 100° should be 90°, ∠ATB = 60° should be 50°, ∠ATB = 50° gives 170° in triangle should be 180°, ∠BTC ≠ ∠BAT]
B1 for alternate segment theorem <u>OR</u> angle between tangent and radius is 90°

[2]

30. 382 — 4

$12.6^2 - 5.7^2 = 126.27$
height = 11.2 (369…)
V = $\pi(5.7)^2$ ("$\sqrt{126.27}$") ÷ 3

M1 $12.6^2 = h^2 + 5.7^2$
M1 $\sqrt{12.6^2 - 5.7^2}$ $(= \sqrt{126.27})$
M1 (dep on 1st M1) for $\pi(5.7)^2$ ("$\sqrt{126.27}$") $\div 3$
A1 for $380.8 \le ans < 382.5$

[4]

31. (a) 96° — 2

B1 for 96°
B1 for Angle at centre equals twice the angle at the circumference (Accept reflex angle with reasons)
SC: if answer 264°, B1 B1 possible

(b) — 3

M1 for using angle at centre is twice…
M1 for using angles at point 0 add up to 360°
A1 for conclusion using evidence presented
Alternative
M1 for using radii <u>and</u> isosceles triangles
M1(dep) for using interior angles of a quadrilateral
A1 for conclusion using evidence presented

[5]

32. 36° — 3

108 ÷ 2 (= 54)
180 – 90 – "54"

M1 for $180 - 90 - 108 \div 2$ ***or*** $\frac{1}{2}(360 - 2 \times 90 - 108)$
A1 for 36° as a final answer
B1 (indep) for angle between tangent and radius is 90°
OR *tangents from an external point are equal*

[3]

33. (i) 76 2

$180 - 2 \times 52$

B1 cao

(ii) tgts to a circle are equal in length

B1 for mention of equal tgts to a circle from a point

[2]

34. (a) Angle P is common
Angle PSR = angle PTQ = 90°
(angle between rad and tgt)
PS = PT (tgts from a point)
Triangles are congruent ASA 3

(b) Triangle STQ and triangle STR
are congruent since triangle
PST is common to triangles
PQT and PRS 2

B2 for 2 of angle P common ***or*** *angle $P\hat{S}R$ = angle $P\hat{T}Q$ = 90°*
or *PS = ST*
[B1 for any 1 of above]
B1 for complete proof with ASA including two circle theorem reasons
M1 for valid method to find area or use congruence
A1 for fully correct method with reasons

[5]

35. 75 cm^2 4

$2\pi \times 10 \times \frac{x}{360} = 15$

$x = \frac{270}{\pi}$

$A = \pi \times 10^2 \times \frac{x}{360}$

M1 for $\pi \times 10^2$ (= 314…) or $2 \times \pi \times 10$ (= 62.8…)
M1 for $\frac{2 \times \pi \times r}{15}$ (= 4.18)
M1 for $\frac{\pi \times r^2}{"4.18"}$
A1 for $74.9 \le ans \le 75.1$
<u>Alternative method</u>
M1 for $\pi \times 10^2$ (= 314.1…) or $2 \times \pi \times 10$ (= 62.8…)
M1 for $\frac{15}{2 \times \pi \times r}$ (= 0.238…)
M1 for $\pi \times r^2 \times$ "0.238"
A1 for $74.9 \le ans \le 75.1$
<u>Alternative method</u>
M1 for $\pi \times 10^2$ (= 314.1…) or $2 \times \pi \times 10$ (= 62.8…)
M1 for $\frac{15 \times 360}{2 \times \pi \times r}$ (= 85.9)
M1 for $\frac{"x"}{360} \times \square \times r^2$
A1 for $74.9 \le ans \le 75.1$

[4]

36. (i) 150 2

B1 accept 150 or 210

(ii)

B1 for angle at the centre is twice the angle at the circumference

[2]

37. BOC is a right angle triangle

Angle BOC = 62°

Cos 62 = $\frac{7}{OB}$

OB = $\frac{7}{\cos 62}$

14.91... 4

B1 for <OCB = 90° and one other correct angle in triangle OBC

M1 for cos62 $\frac{7}{OB}$ or sin 28 = $\frac{7}{OB}$

M1 for OB $\frac{7}{cos62}$ or OB = $\frac{7}{sin28}$

A1 for 14.89 – 14.93

[4]

38. Vol sphere = $\frac{4}{3} \times \pi \times 3.5^3$

= 179.59...

Height = $\frac{\text{"179.59"}}{12 \times 11}$

1.36 4

M1 for $\frac{4}{3} \times \pi \times 3.5^3$ =(179.59...)

M1 for 12 × 11 × x = "179.59..." or 1 cm rise is 132cm^3 of water.~

M1 for x = $\frac{\text{"179.59"}}{12 \times 11}$

A1 for 1.36 – 1.364

Alternative method

M1 for $\frac{4}{3} \times \pi \times 3.5^3$(= 179.59...)

M1 for 12 × 11 × 8 + "179.59"(= 1235.59...)

M1 for $\frac{\text{"1235.59.."}}{12 \times 11}$ OR $\frac{12 \times 11 \times 10 - \text{"1235.59"}}{12 \times 11}$

A1 for 1.36 – 1.364

[4]

39.	OA = OC radii OB = OD radii Angle AOD = angle COB vert opp		3	
		M1 for one valid statement *M1 for two further valid statements* *A1 for complete proof including SAS (oe)*		
				[3]

40.	Area Sector = $\pi(10.4)^2 \div 3 = 113.26488$ Area Triangle = $\frac{1}{2}(10.4)(10.4)\sin 120°$ = 46.8346 Area segment = 66.43 .... 66.4		4	
		M1 for $\pi(10.4)^2 \div 3$ *or* $\pi(10.4)^2 \times \frac{120}{360}$ *oe* *M1 for* $\frac{1}{2}(10.4)(10.4)\sin 120°$ *or any other valid method for area triangle OAC* *M1 (dep on at least 1 of the previous Ms) for area of sector - area of triangle OAC* *A1 66.35 – 66.5*		
				[4]

41.	½(180 – 120) = 30° Angle *PQR* = 30° so *PQ* is not a radius		2	
		M1 for angle between tangent and radius is 90° ***or*** *sight of right angle marked on diagram* *A1 for angle PQR = 30° not 20°* ***or*** *angle PQT = 100° not 90°* ***or*** *QTR = 40° not 60°*		
				[2]

42.	(a) 90		1	
		B1 cao		
	(b) 140		2	
		M1 for sight of 20° *or 360 – 90 – 90 – 40* *A1 for 140°* *SC: Award B1 for an answer or 220*		

(c) 2

B1 for Angle between <u>tangent</u> and <u>radius</u> = <u>90°</u>
or <u>Tangents</u> from a <u>point</u> to a circle are <u>equal</u>

B1 for <u>Isosceles</u> triangle POQ so angle OQP = 20°
or Angles in a <u>triangle</u> add up to <u>180°</u>

[5]

43. 180 – 104 (= 76) 3
"76" ÷ 2 (= 38)
90 – "38"

104 ÷ 2
= 52

M1 for (180 – 104) ÷ 2 or 180 – 90 – 104 ÷ 2 or 38 seen on the diagram for angle BAO
M1 for 90 – "angle BAO" (This could be implied by the values of angle BAO and angle BAT shown on the diagram)
A1 for 52° (shown to be their final answer, other than just shown on the diagram)

***Alternative method** (using angle at centre and alternate segment theorems)*
M2 for 104 ÷ 2 seen leading directly to their final answer.
A1 cao

Angle BAO (A) = 38 with no working gets M1 (unless contradicted on the diagram with 38, say at angle BAT.
*104 ÷ 2 = 52° **only** seen is a correct solution, since the candidate could be using the 'angle at the centre' and 'alternate segment' theorems.*
This gets M2A1
***Warning**: sight of 104 ÷ 2 = 52° gets no marks if it is related to a different angle (say BTA or AOM, where M is the midpoint of AB) and is then followed by incorrect methods to find the required angle.*

Another method is to draw a tangent through B. This is usually then joined to T which is not strictly correct. (however it is pretty close) <ATB is then found to be 76° (360 – 104 – 90 – 90) and <BAT is found using triangle ATB which is assumed is isosceles. This method is acceptable for full marks.

[3]

44. $\frac{120}{360} \times \pi \times 2 \times 6$

$4\pi + 12$ 3

M1 for $\frac{120}{360} \times \pi \times 2 \times 6$ *oe allow 3.14, 3.142,* $\frac{22}{7}$ *for* π

A1 for 4π *or anything in the closed interval [12.56, 12.57],*

or $12\frac{4}{7}$ *oe or* $\frac{a\pi}{b}$ *where a and b are integers with* $a = 4b$

A1 $4\pi + 12$ *or* $\pi 4 + 12$ *oe*

SC (B2 for a fully correct, but unsimplified expression for the perimeter, including $\left(\frac{2\pi r}{3}\right) + 12$ *or* $\left(\frac{2\pi r}{3}\right) + 2r$

Or for any value in the closed interval [24.56, 24.57])

[3]

1. Paper 3

Many weaker candidates failed to understand the three-letter notation for describing angles. Many candidates were able to give the numerical values of the two angles requested. Very few candidates gave appropriate reasons for their deductions. As with question 10, merely stating the calculations undertaken is *not* a reason for the properties used. As candidates use the circle theorems in both parts to obtain the angles, their reasons should make some reference to those theorems, if not by name, then by description.

Paper 5

Although most candidates obtained the correct numerical values for the two angles only a minority of these candidates could give valid reasons for their answers. Some explanations were vague and were not awarded the mark, for example, ‘the angle between the chord *AC* and the tangent *EC* is 90°. ’ A minority of the weaker candidates indicated that they did not fully understand the three-letter angle notation.

2. **Paper 3**

This question was not answered well. In parts (a) and (b) the angles were correct in about a quarter of cases but fewer candidates wrote '112°' in part (c). Candidates clearly found it difficult to give reasons for their answers and circle theorems were rarely used. Many candidates explained about angles in a triangle or merely wrote down how they had calculated the angle. Numerical calculations are not sufficient and candidates would be advised to familiarise themselves with the appropriate wording

Paper 5

Although most candidates obtained the correct numerical values for some of the angles, only a minority of these candidates could give valid reasons for their answers. As stated in the general comments, the examiners required reasons which were geometric and based on the circle properties. Some of the better candidates gave three acceptable reasons as (i) 'Angle in a semi-circle is 90°' (ii) 'Angles in the same segment are equal' (iii) 'Angle subtended at the centre is twice the angle subtended at the circumference'. In part (iii) some candidates used the fact that triangle *OPQ* is isosceles to show that the size of angle *POQ* is 112°. In this case the examiners were looking for a reason based on radii of the circle are equal (in length). Some of the weaker candidates indicated that they did not fully understand the three-letter angle notation or assumed that *SQ* crossed *PR* at right angles.

3. Candidates benefited somewhat from the structure of the question. However, many candidates got the upper and lower bounds of *R* wrong, most often giving 30.5 and 31.5.
Many got part (b) correct on follow-through.
In part (c), there was a great deal to unpack. The candidates had to choose the correct formula for the volume using their lower bound and then dividing by the volume using their upper bound and then rounding down. Most found it difficult to earn full marks.

4. This question was designed to test the ability of candidates to demonstrate a formal proof. There were two methods of approach:

A formal proof that triangles *OMX* and *OMY* are congruent (RHS).

A formal use, with justification, of Pythagoras in each of the triangles *OMX* and *OMY*.

In general, candidates omitted essential details and so lost marks.

5. **Mathematics A Paper 3**

This was a demanding question for candidates. Part (a) was answered quite well and in part (b) some candidates realised that angle *BDC* = 25° but it was often assumed that the required angle was half the answer to part (a), i.e. angle *ADB* = angle *BDC*. Few candidates thought that Ben was correct in part (c). Despite the "Diagram NOT accurately drawn" warning, many candidates said that the line *DB* could not be a diameter as it did not pass through the centre of the circle. Many of those answering "yes" gave no geometrical reason.

Mathematics B Paper 16

This question was not well done and often marks earned were fortuitous rather than convincing, with candidates computing answers by merely subtracting the 'numbers' given in the question. In part (b) a candidate could gain one mark for subtracting 25° from their answer to part (a). On many occasions an answer to part (a) of 50° was followed by an answer of 25° in part (b); some candidates showed their working as 50/2, thinking that BD bisected angle D, others simply wrote down the answer.
The majority of candidates felt that Ben was incorrect in part (c) since the line DB did not pass through the centre. Very few managed to relate the correct answer to correct geometric theory.

6. Very few candidates displayed good presentation of a geometric proof. The writing of three equalities with concise reasoning was often replaced by a rambling essay usually centred around the idea of 'showing' the triangles to be similar. The idea of congruency was lost to the majority. Although very few candidates gained full marks for part (b) many did gain partial credit, normally from the work they did on the diagram as labelling of angles in the working space was often absent. Weaker candidates, gaining any credit for this question, normally showed angle *APC* as 10° or marked a relevant right angle. Better candidates went on to show angle *PBC* as 80° but it was usually only the top grade candidates who indicated that BAC was also 80°. Such candidates usually went on to obtain the correct answer for the size of angle *ABC*. A common error, was to assume that angle BAP was 10° (the same as BPA).

7. Candidates found the first part hard. Many did realise that the angle at *T* was a right angle, so the method into doing the part was to use Pythagoras. Some candidates multiplied x by $(x + 5)$ and set this equal to $(x \quad 8)$.
Part (b) was answered more successfully but there were still many poor attempts, many based on the use of -6^2, rather than $(-6)^2$ in the quadratic formula.

8. **Paper 3**

In part (a) many candidates worked out the size of angle *BCA as* 55° but very few gave a valid reason. Part (b) was often done well, even after an incorrect part (a). Part (c) was answered less well with many candidates failing to realise that they were dealing with an isosceles triangle. A significant number of candidates did not appear to be familiar with angle notation and did not appreciate which angle they were expected to find in each part of this question.

Paper 5

There was an improvement in the performance of candidates in this circle geometry question. 80% of candidates were able to calculate the missing angle in part (a) and about half of these candidates were able to correctly state a reason for their answer. Part (b) was also well answered with again 80% of candidates able to work out the correct answer. Part (c) was not so well answered as candidates had to make two steps in their solution. Only 57% of candidates were successful in this part.

9. This was a long thematic question which most candidates were able to score some marks on. Part (a) required the candidates to draw a circle and a straight line. The circle was rarely recognised and many candidates were unable to draw the straight line. A sizable minority of candidates 'simplified' the circle equation to
'$x + y = 10$'.
Part (b) required candidates to identify the point (s) of intersection of their graphs.
Part (c) was a standard completing the square and the success rate was pleasingly high. A few candidates found the value of q in the identity by substituting a value of x into the identity and then solving for q.
Part (d) was intended to follow the theme of completing the square and linking to the equation of a circle. Most candidates wisely ignored this idea and used their calculator to search out a suitable combination of values.

10. **Specification A**

Higher Tier

Most candidates were able to score at least one mark in part (a) and one mark in part (b).
In part (a), candidates were usually able to identify the angle *AOB* as 150° or 210°, and many were able to write down an appropriate reason. Only the best candidates were able to give a 'textbook' circle theorem- many described the centre as the middle or inner point, and the circumference as the edge, outside, rim, etc.
In part (b), most candidates identified angle *OBP,* or angle *OAP,* as a right angle- usually this was shown in the diagram. Relatively few solutions used the 90° to find angle *APB*. Many scored the marks for simply writing 180° – 150° = 30°- perhaps luckily in some cases as *AOBP* was seldom identified as a cyclic quadrilateral.

Intermediate Tier

This question was also poorly answered. In part (a) the correct answer of 150° was regularly seen, but rarely any explanation of quality. There was a need to mention both circumference and centre. Many candidates recognised that the angle between the tangent and radius was 90°, but then failed to do anything with it. Only the more able made any headway with this question.

Specification B

Part (a)(i) was answered correctly by the vast majority of candidates. The reasons given in (a)(ii) were not, however, always accurate enough. Candidates should be reminded of the necessity of quoting circle theorems as accurately as possible. Most candidates did attempt a reason rather than show their working as has happened in previous papers which was encouraging. In part (b) over 60% of candidates were able to score full marks. There was, however, evidence of careless arithmetic errors. The minority of candidates who failed to score in part (b) generally failed to recognise that the angle between the radius and the tangent was 90°.

11. Specification A

Higher Tier

The most economical answer of ‘angle in a semi-circle’ was rarely seen. Many candidates failed to earn the mark because their reason was so vague or they merely stated that it is 90^o because it is a right angle.
Most candidates could apply Pythagoras correctly to get 20cm. A few correctly stated that the triangle was an enlargement of the Pythagorean triple 3, 4, 5.
Part (c) was also answered well although some candidates used $\pi \times 100$ or $\pi \times 400$ as well as $\frac{12 \times 16}{2}$.

Intermediate Tier

Many of the candidates who gave a reason in part (a) struggled to use correct geometric language and “angle in a semi-circle” was mentioned by surprisingly few. “Because it is a right angle” was a very common response. Part (b) was answered quite well and candidates who used Pythagoras’ theorem usually obtained the correct answer. Many candidates, though, were unsure of what to do. Some simply added 12 and 16 and quite a few attempted to use trigonometry – usually without success. Part (c) was done less well with only a quarter of candidates gaining full marks. Marks were often lost because candidates confused the formula for the area of a circle with that for the circumference. Those that used the correct formula were usually able to evaluate $\pi \times r^2$ in the correct order.

Specification B

Higher Tier

In part (a) only about one quarter of candidates were able to quote that the ‘angle in a semi-circle is 90°’. Some candidates correctly used the fact that the angle at the circumference would be half the angle at the centre but, on the whole, part (a) was poorly done. Parts (b) was answered correctly by over 90% of candidates. In part (c), however only about 75% of candidates were able to score full marks. The wrong formula was occasionally used for the area of a circle. Some candidates misread the question and gave the area of the triangle instead of the area of the circle.

Intermediate Tier

Very few candidates offered an acceptable reason for angle C being 90°; many merely arguing that it was 90° because "it is a right angle". Many made reasonable attempts to explain the reason with reference to geometric facts but we were really looking for accuracy in awarding this mark. If candidates failed to give the preferred "angles in a semicircle = 90°" then acceptable answers needed to refer to the angle subtended by the ends of a diameter on the circumference of a circle being equal to 90°.
In part (b), when the decision to use Pythagoras was made correct working usually followed. Answers of 20 came from different calculations; eg 4 × 5 = 20, and 16 – 12 = 4 ∴ 16 + 4 = 20, the recognition of a 3, 4, 5 triangle enlarged by scale factor 4 not always convincing. A common error by weaker candidates was an answer of 28 (12 + 16).
In part (c) Many candidates used their answer from (b) to find a radius and hence the area of a circle, and gained at least one or two marks; however $(\pi r)^2$ was often evaluated by mistake. A substantial number of candidates used incorrect formulae, either finding the circumference by mistake or using $2\pi r^2$ or $2\pi d$.

12. Intermediate Tier

In this question it was essential that candidates explained what they were doing, and gave a clear indication as to which angles they were finding or using in calculations. Few did. Many subtracted 123 from 180, but gave little indication as to which angle they were trying to find. Reasons given rarely made any reference to the angle at the centre, or the angle at the circumference; giving an indication that one angle was double the other was insufficient.

Higher Tier

This was very well answered with most candidates scoring at least 2 out of the 3 marks available. The final mark was for a clear statement of the circle theorem relating the angle subtended at the centre to the angle subtended by the same arc at the circumference. This was often omitted or a comment on the specific angles in the diagram given.

13. Intermediate Tier

In part (a) less than 10% of candidates were able to give a correct reason. Some good answers were seen, from well-prepared candidates, in part (b) but only 15% gained all three marks. A significant number recognised that tangent was needed and identified the correct figures. Some used the correct ratio but were unable to find the angle; others used the ratio the wrong way up. Candidates using a correct method sometimes rounded values too soon and lost accuracy. Some did not appear to appreciate that they were finding the size of an angle and used Pythagoras' theorem. Candidates were a little more successful in part (c). Many applied Pythagoras' theorem correctly to calculate the length of *OP* although a few chose to use sine or cosine and their angle from part (b). Some then failed to add or subtract 5.8 in order to find the length of *PC,* often choosing to halve the length of *OP* instead.

Higher Tier

In part (a), just over half the candidates were able to find suitable words to describe the required circle theorem.
Part (b) was generally done much better, with most candidates successfully using the tangent ratio to find the angle. Some attempted more indirect approaches, such as the sine ratio, with mixed success.
In part (c), the majority of candidates were able to use Pythagoras' theorem to find *OP,* but many lost accuracy by inappropriate approximation or rounding. Equal credit was given to those candidates attempting to find $OP \pm 5.8$. Some thought C was at the midpoint of *OP*.

14. This question was generally done well by able candidates. Most had some appreciation of upper and lower bounds and could apply it in context. In part (a), candidates generally gave a clear method to find the minimum diameter. In part (b), many could apply $\pi R2^2 - nr^2$ to find the area, but some did not choose appropriate values for R and r. Common errors were $R = 15.8$, $r =$ 14.2= and $R = 15.85$, $r = 14.25$.

15. In part (a), about half the candidates recognised the need to use the sine formula, but a small minority attempted erroneously to use the tangent ratio. Only the best candidates were able to gain much credit in part (b). A significant number thought that the arc *EF* had centre *A,* and consequently used the cosine rule to find *AE.* Candidates should be encouraged to use all stated information and not make assumptions about diagrams. Some candidates attempted to calculate the *length EF.*

16. **Specification A**

Intermediate Tier

Some candidates realised that the angle in part (a) was 90°, fewer gave the correct answer in part (b). Most candidates relied on their working out to justify their answers, but what was actually required was reasoning related to a circle theorem.

Higher Tier

Many candidates have difficulty writing down precise circle theorems, but they continue to make good progress in this area. In part (a), whereas the majority of candidates could give the angle 90°, less than half were able to state the angle in a semi-circle theorem. Some, using the angle at the centre theorem for this part, were less successful as many did not explain that the angle at O should be 180°. In part (b), the use and explanation of the angle at the centre theorem was less popular than using the isosceles triangle method. Candidates' attempts at explaining this did not always include both the features "isosceles" and "angle on a straight line".

Specification B

Intermediate Tier

Only the higher achieving candidates scored well in this question. Many candidates clearly do not understand standard form. In part (c), many candidates felt that they were being asked to write 16×10^7 as an ordinary number and so 160 000 000 was often seen. Typical errors in parts (a) and (b) were 4.56^5, 4.56×10^3 (since there are 3 noughts in 456 000) and 3.4×10^4 and 34×10^{-5}.

Higher Tier

Intermediate Tier

Many got the correct answers of 90° in part (a) and 35° in part (b) but very few were able to give acceptable reasons why: i.e. angle in a semicircle, and angle at centre = twice the angle at the circumference. Instead ambiguous alternatives were offered but gained no credit. It was common to see an answer of 70° in both parts.

Higher Tier

In part (a) the majority of candidates were able to give the correct size of the angle but few candidates were able to give a correct reason. Those candidates who did know the correct reason were frequently unable to express their knowledge accurately. Chord, for examples, was a common word used for the diameter. The same pattern continued into part (b). In part (b) many candidates chose to refer to the isosceles triangle *BOC* rather than the appropriate circle theorem. In this case, the mark for the reason was frequently not gained. Only stating that triangle *BOC* was isosceles was not sufficient reason in itself, candidates also needed to refer to angles on a straight line. In this part, a number of candidates referred to the 'arrow head' theorem this gained no marks. Circle theorems must be properly quoted. The most successful candidates kept their reasons short and to the point.

17. Most candidates recognised that they had to find one third of something! Common errors were to use the formula for the area of a circle or to use 10.4 for the diameter of the circle in part (a). A few candidates assumed that the shaded region was a semicircle and calculated the length of the supposed diameter 18.0. Part (b) required the difference between the area of the sector and the area of the triangle *OCA*. Many candidates could do this correctly by using $\frac{1}{2}ab\sin C$ for the area of the triangle. Others made life more difficult for themselves by using the cosine rule to find the length of *AC*, followed by calculating the height of the triangle and then the area.

18. **Paper 5523**

About one quarter of candidates were successful in part (a). Most of those who identified the right angle at D went on to give an answer of 70 °, often with little working shown. Slightly fewer candidates worked out angle DBC as 20 ° in part (b). Many of the incorrect answers were based on unfounded assumptions about isosceles triangles. It was very disappointing that so few of those who worked out the angle correctly were able to mention angles in the same segment in their explanation. Most reasons were based on types of triangle, or the angle sum of a triangle, or spurious arguments about corresponding or opposite angles.

Paper 5525

Most candidates calculated the angles in parts (a) and (b) correctly, but only the best could identify and state accurately the correct circle theorem. Popular incorrect answers were "alternate segment theorem", "angles in opposite segment" and "bow theorem".

19. Virtually all the candidates attempted this question, but with varying success. Some used the circumference formula with 12 instead of 24; some calculated the perimeter of the sector, adding 24 to the arc length; Some calculated the area of the sector instead of the length.

Other common errors involved a misuse of the fraction $\frac{60}{360}$, which resulted in multiplying the arc length by 6 or by $\frac{1}{4}$.

A significant number of candidates gave their final answer unsimplified as $\frac{60}{360} \times \pi \times 24$. Very few used 3.14 in their calculations.

20. In part (a) many candidates correctly gave the required angle as 130° in (i), realising that angle EBA was 25° and that the angles in a triangle add up to 180°. Some candidates, however, had difficulty with angle notation and did not identify the angle required. The final answer was often given as 180° or 25°, in some cases with the correct angle written on the diagram. Many candidates failed to provide a sufficient explanation to gain the mark in (ii). Those who did mention an isosceles triangle often went no further and did not mention equal sides. Due to the lines on the sides AB and AE, these sides were often said to be parallel or the triangle was said to be equilateral. Many had difficulty identifying the angles and sides with letters, referring, for example, to angle AE and side A. Most candidates did attempt to give reasons of some sort, with few giving just working. Part (b) was answered very poorly. A significant number of candidates gave an answer of 95°, assuming the opposite angles to be equal. Some incorrectly assumed that some angles in the diagram were right angles.

21. Part (a) proved to be straightforward. However, part (b) proved to be challenging. In particular many candidates could not visualise how the sector could turn into the curved surface of the cone and consequently concentrated on the 144° instead of the 216°. Many candidates assumed that the base radius of the cone had to be 15cm and then worked out $15^2 + 15^2$, mistaking the position of the right angle. Of those that got the correct answer, most did it by finding the arc length of the sector and then realising that this would become the circumference of the base of the cone. They then found the radius of the base (9cm) from $\frac{\text{arc lenght}}{2\pi}$. A correct, but less common successful approach was to calculate the area of the sector and then use the formula for the curved surface area of the cone to find the radius from $\frac{\text{area of sector}}{\pi + 15}$.

22. **Higher Tier**

The angle in part (a) was usually found, but the reason given was sometimes too vague. Some candidates thought that stating 'Angle *CBA* is half of angle *COA*' was sufficient.

Part (b) was a two – step problem. Successful responses consisted of using angle at the centre is twice the angle at the circumference followed by opposite angles of a cyclic quadrilateral are supplementary, or working out the reflex angle and then using angle at the centre etc..

Weaker candidates assumed that the required angle was 110, the same as the angle at the centre or assumed that *PORS* was a cyclic quadrilateral. A few candidates assumed that *QOS* was a diameter and get the correct answer in this special case by using, for example, the angle in a semi circle property.

Intermediate Tier

This question was not well answered. Some gave the angle as 65°, but 230° was also common. Candidate attempts to give a reason was rarely linked to theorems and geometrical descriptions.

23. The most common successful approach was to multiply πR^2 by $\frac{150}{360}$, although a few candidates did the equivalent by dividing by 2.4. Common errors included assuming the sector was one third of a circle or just working out the area of a circle. Some candidates halved the given 13 and thought that the radius was 6.5 cm.

24. This question was reported by many as being a good discriminator.

The most efficient way to tackle the question was to realise that the angle of the sector was 60.This enabled the candidates to use the ½*ab* sin*C* formula for the triangle. However many candidates resorted to the cosine rule to find it or decided because it was a sixth of the circle they needed to use sin 6. A number of candidates were able to calculate one of the areas correctly; more frequently the sector, and then the subtraction carried out The most common error was to use half base × height for the triangle area, using 6 as the height. Some did use Pythagoras to find the height but often made errors. Quite a few found one or other of the two areas and offered this as their answer.

25. The majority of candidates failed to score any marks on this question. Some correct solutions were seen but the vast majority of answers made unfounded assumptions such as triangle *ROS* was equilateral. In part (b) the majority of candidates were unable to present their argument convincingly. Candidates should be reminded to identify angles they are discussing rather than use vague statements such as ‘it’s a right angle’ without any reference being given.

26. This question was poorly done and full marks were rarely seen. Angle AOB was often taken as 56°. If angle AOT was labelled correctly, angle A was often taken to be 56°s also. Many candidates recognised that angle A was 90° without being able to explain why; “tangent” and “radius” needed explicit mention to gain the reasoning mark.

27. Circle theorems are clearly not well known by Intermediate level candidates. Often candidates who correctly worked out the answer of 43° for angle *BAD*, could not explain the reason why, choosing instead to describe their calculation or saying that angle *BAD* is half of angle *BOD*. In part (b) the supplement to their answer in part (a) was often given. This could have been more guesswork than a knowledge of angles in a cyclic quadrilateral.
A significant number of candidates took angles *ABC* and *ADC* to be right angles.

28. Part (a) was well answered. Part (b) was poorly answered. The majority of candidates thought that *OBCD* was a cyclic quadrilateral and proceeded accordingly. The answer of 32° was the most popular incorrect answer seen. The correct answer of 29° was very rarely given.

29. A number of candidates were able to score at least one mark in this question by comparing the size of a calculated angle with the value it should have if *TC* were a tangent. However, only a very small minority of candidates were able to score full marks by stating a circle theorem correctly. There was evidence that candidates were not able to understand the usual letter notation for angles such as angle *BTC*.

30. Fully correct answers to this question were seen from over half the candidates. The majority of candidates did obtain the correct height using Pythagoras' Theorem and generally then went on to obtain an acceptable value for the volume. Weaker candidates just used $h = 12.6$ in the formula for the volume of a cone.

31. Part (a) was well answered. The vast majority of candidates were able to evaluate the angle correctly and write down a correct reason for their answer. A minority of candidates described their working rather than quoting the appropriate circle theorem. Very few candidates were able to prove the given statement in part (b). The majority of candidates quoted the theorem relating to cyclic quadrilaterals. Some good, fully correct proofs were seen using a variety of approaches.

32. Only a very small minority of candidates were able to gain full marks on this question. A number of candidates were unable to identify angle BAO and gave the size of angle BAC instead. A final answer of 72° was a common incorrect answer. Candidates should be reminded that circle theorems must be quoted correctly. To say that 'the tangent hits the circle at 90°' as so many candidates did is not sufficient; the tangent and the radius must both be mentioned.

33. $x = 76$ was often seen in part (i). Failure tended to be a result of poor arithmetic although an answer of 52 was not uncommon.
The word tangent rarely appeared in the reason in part (ii); candidates preferring to give reasons associated with isosceles triangles or merely angles in a triangle. Very many candidates still explain how they arrived at their answer instead of using geometric theory to justify it.

34. Just under half of the candidates were able to gain credit for writing down some correct statements. Candidates should be reminded of the necessity to give reasons for statements made when proving triangles are congruent. Very few candidates spotted the overlapping common triangle which lead to an easy answer to part (b). The majority going back to try to prove that the two triangles were congruent; if this method was used then insufficient reasons were generally given.

35. Just over 40% of candidates were unable to gain any marks for this question. This was disappointing given that a mark was available for the correct expression for the area or circumference of the given circle. A number of different correct methods were seen but the most common method used was to find the fraction of the circumference using the given arc length and then apply this to the area. Some candidates found the angle of the sector and then used this successfully. There were some correct solutions seen coming from using the sector to make a cone and using the formula for the curved surface area of a cone.

36. When a correct answer of 150° was found in part (i) it was usually followed by a reason explaining its derivation (eg angle at *O* is double the angle at *C*) rather than quoting the geometric theory. Common errors were answers of 105° or 75° in part (i) with the explanation that the two angles must either add up to 180° or be equal in size.

37. Over half of all candidates gained a mark by identifying the right angle and working out one other angle in triangle *OBC*. After this, success was varied. Those candidates who recognised that *OC* was a radius of the circle and therefore of length 7cm were generally able to go and find a correct solution. A disappointingly large number of candidates failed to recognise that *OC* was of length 7cm and tried to incorrectly use trigonometry in triangle *OAC* to evaluate the length of *OC*. Fully correct solutions to this question were given by approximately 12% of candidates.

38. Too many candidates were unable to copy the formula for the volume of the sphere correctly from the formula page. $\frac{3}{4}\pi r^3$ and $\frac{4}{3}\pi r^2$ were two common incorrect formula frequently quoted. Some candidates also used the formula for the surface area of a sphere or the area of a circle. Fully correct solutions were seen from about 17% of candidates using a variety of methods.

39. Very few fully correct solutions were seen to this question. Many candidates recognised the need to provide a formal proof but few were able to provide this. Solutions were generally very poorly set out; candidates should be encouraged to set out proofs in an ordered way with appropriate reasons stated. A number of candidates assumed *AD* and *BC* were parallel or that triangles *AOD* and *BOC* were equilateral. Failure to distinguish between triangle AOD and angle AOD was also common.

40. Fully correct solutions were seen from about 30% of candidates. Those candidates that failed to gain full marks were generally able to pick up a mark for showing a correct method to calculate the area of the sector. The area of the triangle proved more problematic with many candidates giving the area of the triangle as $\frac{1}{2} \times 10.4^2$. The better candidates used $\frac{1}{2}ab\sin C$ for the area of the triangle. Some candidates took the long route of working out the length of *AC* and the height of the triangle and used these successfully to find the area of the triangle. Full marks were awarded for a solution using this method, provided that accuracy was maintained through to the final answer.

41. This question was very poorly answered with 80% of candidates scoring no marks. It was a pleasant surprise when candidates knew what they were talking about and managed to convey their ideas across…if not always eloquently. The tangent/ radius connection was often stated in about 10% of cases and most went on to discuss the 20º/30º situation, though some did get confused expecting the 70º to be 90º. Only 10% of candidates were able to produce a fully reasoned argument.

42. It was disappointing to find that only just over half the candidates realised that angle *OQT* was 90°. This, in turn, went on to affect their calculations in part (b) as they could then have gone on to use the sum of the angles of the quadrilateral in an attempt to work out the size of angle *POQ* or recognise that *OPQ* was an isosceles triangle with the acute angles being 20° each. The mean mark for this part of the question was just under 1. Candidates often struggle to provide mathematical reasons for the size of an angle and this year was no exception with 0.37 being the mean mark for this part. Those who did score one of the marks tended to score it for mention of an isosceles triangle or for stating that the sum of the angles of a triangle is 180°. The few that did mention tangent theorems tended to talk about the angle between the tangent and the circle rather than the tangent and the radius.

43. Although a correct answer of 52° was often seen, the preceding argument was not always convincing. Many candidates were unsure of the angle denoted by *BAT*, weaker ones giving three angles for their answer, *B* and *A* and *T*.

The majority of candidates used the triangle *OBA*, recognising it as an isosceles triangle, and usually finding the correct base angles of 38°, although arithmetic errors were common.

Some candidates assumed that *TB* was also a tangent and proceeded using equal tangent theory. Some credit was given here.

44. The sector is, of course, in this case one third of its circle so the fraction demand was reasonable for a higher tier paper, although some candidates assumed it was a quarter of a circle.. Many candidates used the area formula and thus scored no marks. Of those that used the correct formula many could not simplify completely the expression for the arc length. Those that did get the arc length, did, however often go on to add 12 to get an expression for the perimeter although a few spoiled things at the end by writing $12 + 4\pi = 16\pi$.