1.

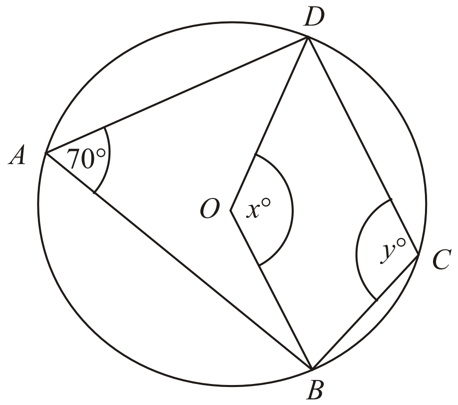


Diagram **NOT** accurately drawn

In the diagram, *A*, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
Angle $BAD = 70°$.
Angle $BOD = x°$.
Angle $BCD = y°$.

(a) (i) Work out the value of x.

x = ...

(ii) Give a reason for your answer.

....

....

(2)

(b) (i) Work out the value of y.

y = ...

(ii) Give a reason for your answer.

....

....

(2)
(Total 4 marks)

2.

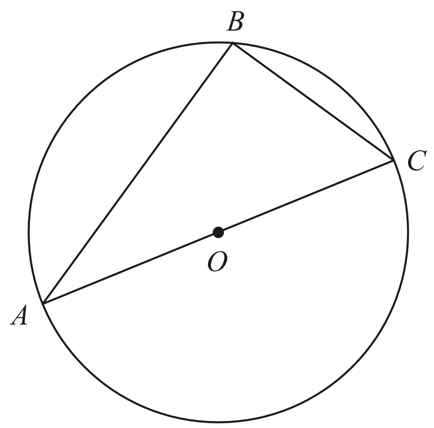


Diagram **NOT** accurately drawn

A, *B* and *C* are points on the circumference of a circle, centre *O*.
AC is a diameter of the circle.

(a) (i) Write down the size of angle *ABC*.

..........................°

(ii) Give a reason for your answer.

....

....

(2)

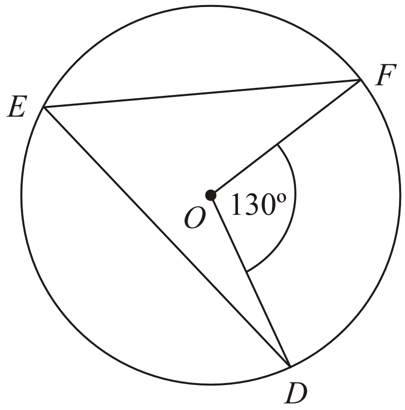


Diagram **NOT** accurately drawn

D, E and F are points on the circumference of a circle, centre O.
Angle $DOF = 130°$.

(b) (i) Work out the size of angle DEF.

..........................°

(ii) Give a reason for your answer.

....

....

(2)
(Total 4 marks)

3.

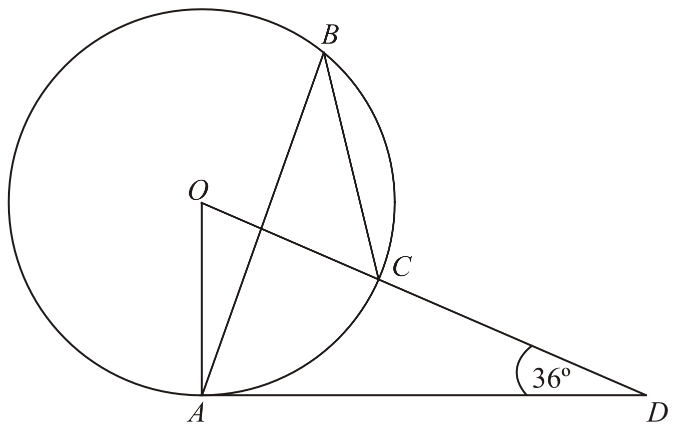


Diagram **NOT** accurately drawn

The diagram shows a circle centre *O*.
A, *B* and *C* are points on the circumference.

DCO is a straight line.
DA is a tangent to the circle.

Angle *ADO* = 36°

(a) Work out the size of angle *AOD*.

.... °

(2)

(b) (i) Work out the size of angle *ABC*.

.... °

(ii) Give a reason for your answer.

....

(3)
(Total 5 marks)

4.

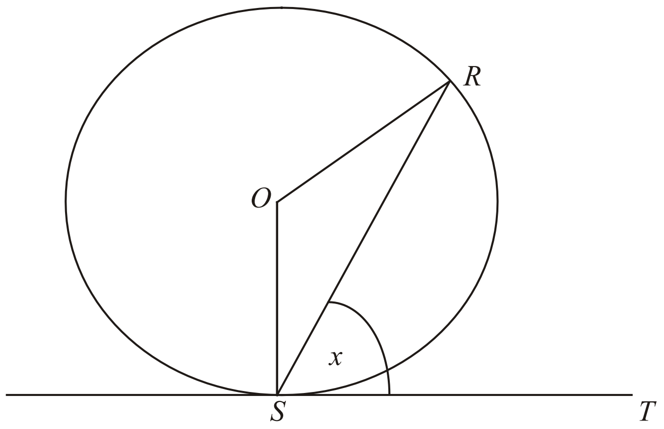


Diagram **NOT** accurately drawn

R and *S* are two points on a circle, centre *O*.
TS is a tangent to the circle.
Angle $RST = x$.

Prove that angle $ROS = 2x$.
You must give reasons for each stage of your working.

(Total 4 marks)

5.

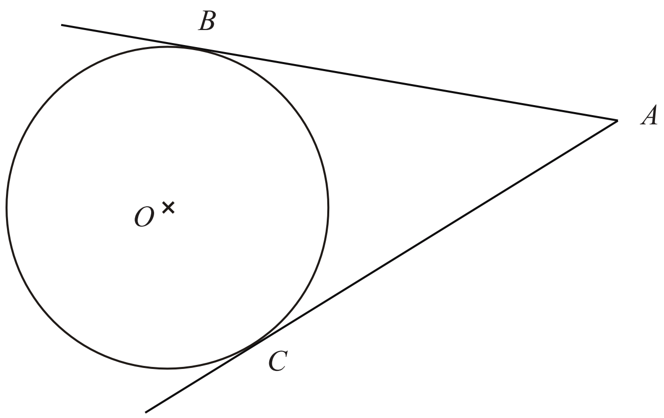


Diagram **NOT** accurately drawn

B and *C* are points on a circle, centre *O*.
AB and *AC* are tangents to the circle.
Angle *BOC* = 130°.

Work out the size of angle *BAO*.

..............................°
(Total 3 marks)

6.

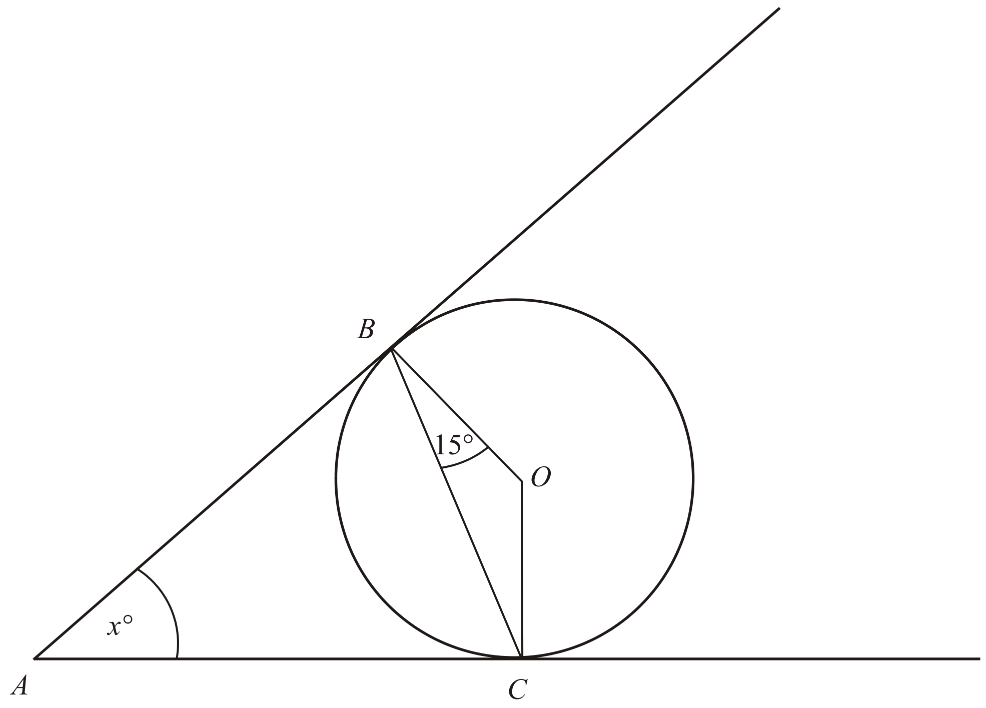


Diagram **NOT** accurately drawn

B and *C* are two points on a circle, centre *O*.

Angle *OBC* = 15°.

AB and *AC* are tangents to the circle.

(a) Calculate the size of the angle marked *x*°.

....................°

(2)

(b) Give reasons for your answer.

....

....

....

(2)
(Total 4 marks)

7.

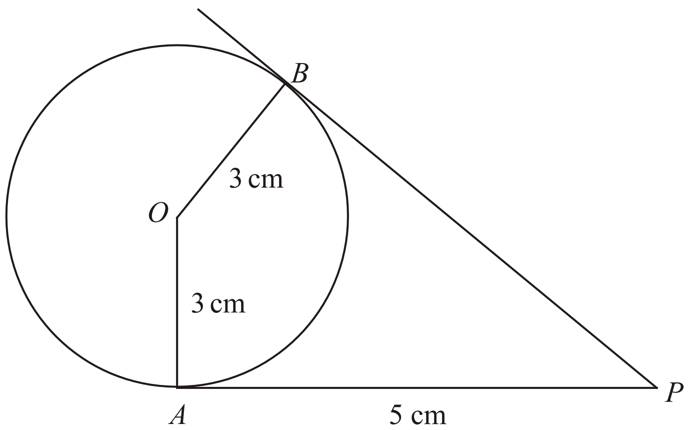


Diagram **NOT** accurately drawn

A and *B* are points on a circle, centre *O*, radius 3 cm.

PA and *PB* are tangents to the circle.

PA = 5 cm.

(a) Write down the size of the angle *OBP*.

.... °

(1)

(b) (i) Write down the length of *PB*.

.... cm

(ii) Give a reason for your answer.

....

....

(2)
(Total 3 marks)

8.

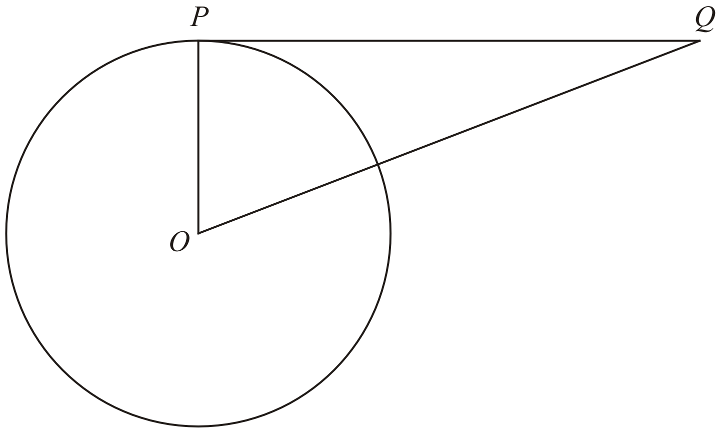


Diagram **NOT** accurately drawn

P is a point on the circumference of the circle, centre *O*.
PQ is a tangent to the circle.

(i) Write down the size of angle *OPQ*.

.... °

(ii) Give a reason for your answer.

....

....

(Total 2 marks)

9.

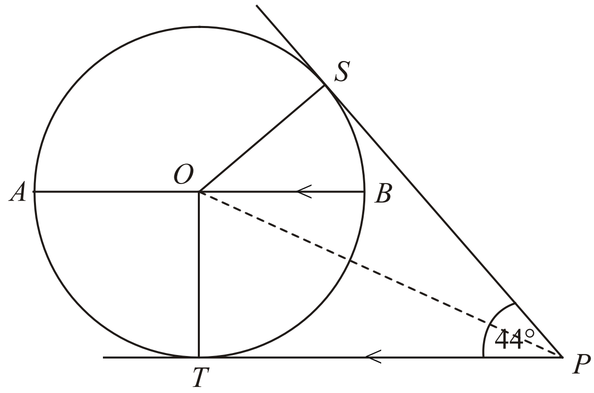


Diagram **NOT** accurately drawn

The diagram shows a circle, centre *O*.
A, S, B and *T* are points on the circumference of the circle.

PT and *PS* are tangents to the circle.
AB is parallel to *TP*.

Angle *SPT* = 44°.

Work out the size of angle *SOB*.

..................................°
(Total 4 marks)

10.

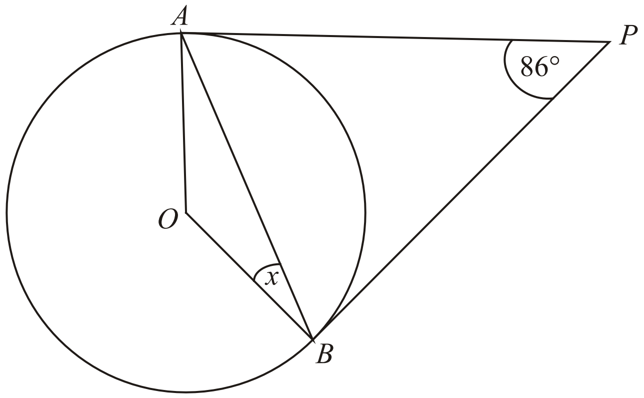


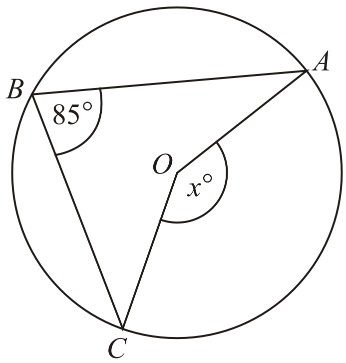
Diagram **NOT** accurately drawn

A and *B* are points on the circumference of a circle, centre *O*.
PA and *PB* are tangents to the circle.
Angle *APB* is 86°.

Work out the size of the angle marked *x*.

..........................°
(Total 2 marks)

11.



In the diagram, A, B and C are points on the circumference of a circle, centre O.

Angle $ABC = 85°$.

(i) Work out the size of the angle marked $x°$.

.... °

(ii) Give a reason for your answer.

....

....

(Total 2 marks)

1. (a) (i) 2×70

140 — 2

B1 for 140 cao

(ii) Reason

B1 for 'angle at centre is twice angle at circumference'

(b) (i) $180 - 70$ or $\frac{1}{2} \times 220$

110 — 2

B1 for 110 cao

(ii) Reason

B1 for 'opposites angles in a cyclic quadrilateral sum to 180 degrees'
or 'angle at centre is twice angle at circumference'

[4]

2. (a) (i) 90
angle in a semi-circle = 90° 2

B1 cao
B1 for angle in a semi-circle (= 90°) or angle at the centre is twice the angle at the circumference or angle subtended by a diameter = 90°.

(b) (i) 130 ÷ 2

65

B1 cao

(ii) angle at centre is twice the angle at the circumference 2

B1 for angle at the centre is twice the angle at the circumference.

[4]

3. (a) *AOD* = 90 – 36 or 180 – (90 + 36)
54 2

M1 AOD = 90 – 36 or 180 – (90 + 36)
A1 cao

(b) (i) *ABC* = *AOD* ÷ 2
27 2

M1 ABC = AOD ÷ 2
A1 ft from '54'

(ii) Reason 1

B1 Angle at centre = twice angle at circumference

[5]

4. angle $RSO = 90° - x$
(angle between tangent and radius = 90°)
angle RSO = angle $SRO = 90° - x$
(base angles of an isosceles triangle)
angle $ROS = 180° - (90° - x) - (90° - x) = 2x$

Alternative:
Angle TSR = angle $SPR = x$ where P is a point on the circumference on the major segment (angles in alternate segment)
Angle $ROS = 2x$
(angle at centre = twice angle at circumference) 4

B1 for stating (this may be just indicated on the diagram) or using the fact that angle OST = 90°
M1 for angle ROS = 180° – (90° – x) – (90° – x) or equivalent
A1 for completing the proof to show that angle ROS = 2x
B1 for reasons "angle between tangent and radius = 90°" ***and*** *"base angles of an isosceles triangle"*

Alternative:
B1 for identifying angle SPR = x
B1 for reason "angles in alternate segment theorem"
B1 for "angle at centre = twice angle at circumference"
B1 for completion of proof

[4]

5. $180 - 90 - \frac{130}{2}$
25 3

M2 for a complete correct method
eg $180 - 90 - \frac{130}{2}$ *oe*
or ½(360 – 90 – 90 – 130)
or angle BAO marked as 25° on the diagram or angle BAO worked out as 25°

(M1 for angle OBA or angle OCA = 90°
or angle BOA or angle COA = 65°
or both angles ABC and ACB = 65°
(these could be marked on the diagram or implied by calculation))

A1 cao

S C Award M2 A0 for angle A = 50° indicated on the diagram or implied by the working or 360 – (90 + 90 + 130 oe) = 50

[3]

6.	(a)	Angle $ABO = 90°$ Angle $BAC = 180° - 2 \times 75°$ *M1 90 – 15 (= 75), ie for* ***using*** *angle between tangent and radius is 90°* *A1 cao* <u>Alternative</u> Angle $BOC = 180 - 2 \times 15 = 150$ Angle $BAC = 360 - 150 - 90 \times 2$ *<u>Alternative</u>* *M1 for 360 – "angle BOC" – 90 × 2 ie for* ***using*** *angle between tangent and radius is 90°* *A1 cao* 30	2	
	(b)	*B1 for angle between <u>tangent</u> and <u>radius</u> is 90°* *B1 for isosceles triangle / length of tangents from point to circumference are equal.* ***OR*** *B1 for angle between tangent and radius is 90°* *B1 for isosceles triangle + angles in a quadrilateral*	2	
				[4]
7.	(a)	90 *B1 for 90 cao* *Watch for angle marked on diagram*	1	
	(b)	(i) 5 *B1 for 5 cao* (ii) Reason *B1 for tangents from an external point are equal in length*	2	
				[3]
8.	(i)	90° and reason *B1 for 90°*	1	
	(ii)	*B1 for angle between tangent and radius (is 90°)*	1	
				[2]

9. $SOT = 360 - (90 + 90 + 44)$
$= 136°$

$SOT = 360 - (90 + 90 + 44)$
$= 136°$
$136 - 90$

$SOT = 360 - (90 + 90 + 44)$
$= 136°$
$136 \div 2 - 22$

46° 4

Using triangle SOP
B1 recognition of tangent/radius property (can be awarded for a right angle marked on the diagram)
M1 180 – 90 – 22 or sight of 68º
M1 SOP – 22
A1 cao

Using quadrilateral SPTO
B1 recognition of tangent/radius property (can be awarded for a right angle marked on the diagram)
M1 360 – (2 × 90 + 44) or sight of 136º
M1 SOT – 90
A1 cao

Alternative method for quadrilateral SPTO
B1 recognition of tangent/radius property (can be awarded for a right angle marked on the diagram)
M1 360 – (2 × 90 + 44) or sight of 136º
M1for 136 ÷ 2 – 22
A1 cao

[4]

10. $\frac{1}{2}(180 - 86) = 47$
$90 - 47 = 43$ 2

M1 for $\frac{1}{2}(180 - 86)$ *or 47 or for 90 – '47' or* $\frac{1}{2}(180 - \text{"94"})$
A1 for 43 cao

[2]

11. (i) 170° 1

B1 cao

(ii) Reason 1

B1 for Angle at centre is twice angle at circumference (accept edge, middle, O origin)oe

[2]

1. Part (a)(i) was generally done well. Most candidates realised that they needed to double the angle at the circumference to get the angle at the centre, but in part (a)(ii), only the best candidates were able to quote the circle theorem accurately. A typical answer here was 'the angle in the middle is double the angle at the edge'. A common unacceptable answer was $BOD = 2 \times BAD$. In part (b)(i), only about a quarter of the candidates were able to work out the correct value for y. many thought that x and y were equal and said as much in part (b)(ii), e.g. 'opposite angles in cyclic quadrilateral are equal'. Again, only the best candidates were able to quote the circle theorem accurately. A common unacceptable answer was 'circle in a quadrilateral, opposite angle add to 180°'. A significant number of candidates thought that *BODC* was the cyclic quadrilateral and gave the angle as 40°. Candidates should be advised to learn the circle theorems accurately.

2. A correct answer of 90° was the most common response to (a) part (i), however very few candidates were able to offer an acceptable reason in (ii). Reasons such as 'triangles with hypotenuse as a diameter always give a right angle' or 'lines drawn from a diameter always make a right angle' were the best of the unacceptable offerings. 'Angles in a semicircle' or 'angles subtended by a diameter' were accepted for the award of the mark.
In part (b), the correct angle of 65° was usually seen but often supported by an incorrect reason. It should be noted that 'Arrow head theory', or similar is not an acceptable reason.

3. Many candidates answered part (a) correctly, recognising the right angle between radius and tangent and using the angle sum of a triangle to work out the size of angle *AOD*. There was, though, some evidence of poor arithmetic with some candidates unable to subtract 126 from 180 correctly. Correct answers to (b)(i) were much rarer.

Many candidates had remembered that angles in the same segment are equal but had forgotten that the two angles both need to be on the circumference of the circle. Hence a very common error was for angle *ABC* to be given as 54o (the same as angle *AOD*). The majority of the candidates who answered (b)(i) correctly were able to give the correct reason in (b)(ii).

4. The first of the four marks awarded in this question was a generous one for recognising or using the fact that the angle between a radius and a tangent is 90°. Many candidates failed to secure the award of this mark. Of those that did, the great majority attempted their proof by showing the result to be true for a particular value of x, neglecting the required general approach, and often never giving acceptable geometric reasons. Many candidates tried to use the given value of angle *ROS* and work 'backwards'. This approach usually failed.

A large number of candidates drew the line *RT* and made the assumption of taking *RT* to be another tangent, and then assuming triangle *RST* to be equilateral.

Only a few candidates succeeded in completing a proof for the general case.

5. This question divided students into two categories, the most fruitful being those who applied geometric principles to finding the angle and, unfortunately, those who didn't.

Drawing in the radii *OB* and *OC* created the quadrilateral *ABOC* and, using the fact that the radius and the tangent meet at right-angles and that angle *BOC* (given) was 130º, allowed the calculation of angle *BAC* as 360º– 90º – 90º – 130º to give 50º. This working earned 2 marks for the complete method. Had the student referred back to the question at this point they would have realised that it was angle *BAO* which was required and not *BAC.* Many candidates had the angle of 25° written on the diagram and in the correct angle but then wrote 50° on the answer line thus confusing the required angle with angle *BAO*. The second most popular method was to use one of the right-angled triangles, either *OBA* or *OCA*, and recognise that line *OA* bisected the angle at *BOC* to give 130º ÷ 2 or 65º. This could then be used in one of the triangles to obtain *BAO* directly.

The isosceles triangle ABC also featured in some solutions as a valid method but seemed to breakdown by focussing the attention on triangle *BOC* rather than the intended one. It is important in this type of question to show stages in the working or to make it clear which angle they have calculated. In this case merely writing 50º (as a result of 180–130) without identifying which angle it was scored no marks.

Some were muddled with properties of a circle using "angle at the centre is twice that of the circumference and giving 65 as their final answer.

Overall, 29% scored all 3 marks, 21% scored 2 marks generally for identifying angle *BAC* as 50°, and 11% scored one mark generally for identifying angle *OBA* or angle *OCA* as 90°.

6. In part (a) the majority of the more able candidates correctly found 30 to be the value of x, however explaining their reasons in part (b) proved far too difficult at all levels. Those that realised that angles *ABO* and *ACO* where 90°could not always correctly explain why; 'the angle from a circle (centre) to a tangent is 90°' or 'tangents meet circles at 90°', being two of the better efforts yet still gaining no credit.

7. In this question candidates were able to give the answer of 90º for (a) and 5 cm for (b)(i) but very few candidates were able to give a complete reason as to why PB had a length of 5 cm.

8. Over 70% of candidates gained the mark for writing down the correct size of the angle in part (i) of this question. However, many candidates did not understand the notation used to identify the angle OPQ and gave 180º as their answer. Many then went on to explain in part (ii) that the angles in a triangle sum to 180º. Disappointingly, only about one third of the candidates who answered part (i) correctly could give a clear and succinct reason in part (ii). All that was required was a clear reference to the angle between the tangent and the radius.

9. This was a very successful question for the 28% of candidates that gained all four marks. In fact though 25% of candidates scored no marks 12% scored the mark for recognising that there were 90º between the tangent and radius of a circle and a further 35% gained two marks for correctly calculating the value of either angle *TOP* or *SOT* or *SOP*. There were a number of valid methods for solving this question and all were awarded marks if the solution was correct. A surprising number of candidates had no understanding of which angle SOB referred to .Often they mentioned angle O which was of course meaningless as there were many angles with this point as a vertex.

10. About two thirds of responses to this question were awarded at least one mark with just over a half of candidates achieving full marks.

Many candidates demonstrated that they knew that the tangent and radius met at 90º. However, a significant number of candidates gave “4º” as their answer – obtaining this from doubling 86 and subtracting from 180º before halving or from subtracting 86º from 90º. The candidates who gained one mark often worked out that angle ABP was 47º but could go no further.

11. A straightforward circle theorem question in which most students got 170°. A few got themselves confused and thought this was about cyclic quadrilaterals and others worked out the reflex angle instead as 170°. Explanations were good but still in many cases focussing on the particular (‘angle AOC’) rather than the general (‘angle at the centre’).or using reference to the ‘arrowhead’.