Edexcel GCSE

Mathematics

Foundation/Higher Tier

Number: Trial and improvement

Information for students

The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this selection.

Advice for students

Show all stages in any calculations.
Work steadily through the paper. Do not spend too long on one question.
If you cannot answer a question, leave it and attempt the next one.
Return at the end to those you have left out.

Information for teachers

The questions in this document are taken from the 2009 GCSE Exam Wizard and include questions from examinations set between January 2003 and June 2009 from specifications 1387, 1388, 2540, 2544, 1380 and 2381.

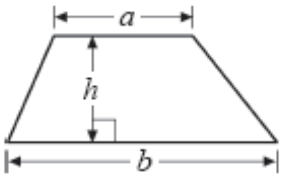
Questions are those tagged as assessing "Trial and improvement" though they might assess other areas of the specification as well. Questions are those tagged as "Foundation/Higher" so could have (though not necessarily) appeared on either a Foundation, Intermediate or Higher tier paper.

GCSE Mathematics

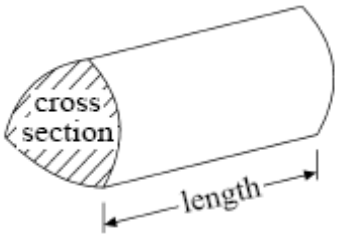
Formulae: Foundation Tier

You must not write on this formulae page.
Anything you write on this formulae page will gain NO credit.

Area of trapezium = $(a + b)h$



Volume of prism = area of cross section × length



1. The equation

$$x^3 - 2x = 67$$

has a solution between 4 and 5
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **ALL** your working.

x = ...

(Total 4 marks)

2. The equation $x^3 + 4x = 100$

has one solution which is a positive number.

Use the method of trial and improvement to find this solution.
Give your answer correct to 1 decimal place.
You must show **ALL** working.

x = ...

(Total 4 marks)

3. The equation

$$x^3 - 4x = 24$$

has a solution between 3 and 4.

Use a trial and improvement method to find this solution.

Give your answer correct to 1 decimal place.

You must show **all** your working.

$x =$ ……..

(Total 4 marks)

4. The equation $x^3 + 10x = 21$
has a solution between 1 and 2

Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **ALL** your working.

x = ...

(Total 4 marks)

5. The equation

$$x^3 - x = 20$$

has a solution between 2 and 3

Use a trial and improvement method to find this solution.
Give your solution correct to 1 decimal place.
You must show **ALL** your working.

x = …….

(Total 4 marks)

6. The equation $x^3 - 15x = 31$

has a solution between 4 and 5.

Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **ALL** your working.

$x =$ …….

(Total 4 marks)

7. The equation

$$x^2 + 2x = 410$$

has a solution between 7 and 8.

Use a trial and improvement to find this solution.
Give your solution correct to 1 decimal place.
You must show **ALL** your working.

$x =$ .....

(Total 4 marks)

8. The equation

$$x^3 + 2x = 65$$

has a solution between 3 and 4

Use a trial and improvement method to find this solution.
Give your solution to 1 decimal place
You must show **ALL** your working.

$x =$.....

(Total 4 marks)

9. Tariq and Yousef have been asked to find the solution, correct to one decimal place, of the equation

$$x^3 + 2x = 56$$

(a) Work out the value of $x^3 + 2x$ when $x = 3.65$

.....

(1)

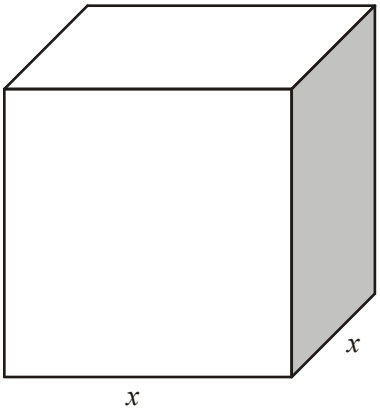
Tariq says 3.6 is the solution.
Yousef says 3.7 is the solution.

(b) Use your answer to part (a) to decide whether Tariq or Yousef is correct.
You must give a reason.

.....

(1)
(Total 2 marks)

10.



A cuboid has a square base of side x cm.
The height of the cuboid is 1 cm more than the length x cm.
The volume of the cuboid is 230 cm^3.

(a) Show that $x^3 + x^2 = 230$

(2)

The equation $x^3 + x^2 = 230$

has a solution between $x = 5$ and $x = 6$.

(b) Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You must show **all** your working.

x = ...

(4)
(Total 6 marks)

1. 4.2 4

x	$x^3 - 2x$
4.1	60.7(21)
4.2	65.6(88)
4.3	70.9(07)
4.22	66.7(114)
4.23	67.2(269)
4.24	67.7(45)
4.25	68.2(656)

B2 for trial $4.1 \leq x \leq 4.3$ evaluated
(B1 for trial $4 < x < 5$ evaluated)
B1 for different trial $4.225 \leq x \leq 4.25$ evaluated
B1 (dep on at least one previous B1) for 4.2 cao

[4]

2. 4.4 4

4.0	80
4.1	85.3(2)
4.2	90.8(9)
4.3	96.7(1)
4.4	102.7(8)
4.5	109.1(3)
4.6	115.7(4)
4.7	122.6(2)
4.8	129 7(9)
4.9	137.2(5)
5.0	145
4.35	99 7(1)

B2 for trial strictly between 4 and 5 evaluated
(B1 for trial at either end)
B1 for different trial between 4.3 and 4.4 exclusive
B1 cao (dep) on at least one previous B1
NB need to see trials evaluated to at least 1dp (truncated or rounded)

[4]

3. 3.3 4

3→15
4→48
3.1→17.3(91)
3.2→19.9(68)
3.3→22.7(37)
3.4→25.7(04)
3.5→28.8(75)
3.4→25.7(04)
3.3→22.7(37)
3.35→24.1(95375)

B2 for trial between 3.3 and 3.4 inclusive
(B1 for trial between 3 and 4 inclusive)
B1 for different trial between 3.3 and 3.4 exclusive
B1 (dep on at least one previous B1) for 3.3
NB trials should be evaluated to at least 1 dp truncated or rounded

[4]

4. $1 \rightarrow 11$
$2 \rightarrow 28$
$1.1 \rightarrow 12.3(31)$
$1.2 \rightarrow 13.7(28)$
$1.3 \rightarrow 15.1(97)$
$1.4 \rightarrow 16.7(44)$
$1.5 \rightarrow 18.3(75)$
$1.6 \rightarrow 20.0(96)$
$1.7 \rightarrow 21.9(13)$
$1.8 \rightarrow 23.8(32)$
$1.9 \rightarrow 25.8(59)$
$1.65 \rightarrow 20.9(92125)$
1.7 4

B2 for trial between 1.6 and 1.7 inclusive
(B1 for a trial between 1 and 2 inclusive)
B1 for a different trial between 1.6 and 1.7 exclusive
B1 (dep on at least one previous B1) for 1.7
NB: trials should be evaluated to at least 1dp truncated or rounded.

[4]

5. 2.8 4

$2 \rightarrow 6, 3 \rightarrow 24$
$2.8 \rightarrow 19.152\ldots$
$2.9 \rightarrow 21.489\ldots$
$2.85 \rightarrow 20.299125$

B2 for trial strictly between 2 and 3 evaluated
(B1 for trial at 2 or 3 evaluated)
B1 for different trial between 2.8 and 2.9 evaluated
B1 cao (dep on at least one previous B1)
$2.5 \rightarrow 13.125$
$2.6 \rightarrow 14.976$
$2.7 \rightarrow 16.983$
$2.8 \rightarrow 19.152$
$2.9 \rightarrow 21.489$
$2.85 \rightarrow 20.299$

[4]

6. 4.7 4

4… 4	4.5… 23.6/7	4.65… 30.7/8
5… 50	4.6… 28.3	4.64… 30.2/3
	4.7… 33.3	4.66… 31.2/3

B2 for a trial strictly between 4 and 5
(B1 for a trial at 4 or 5)
B1 for a different trial strictly between 4.6 and 4.7
(all trials to be evaluated correct to 1 dp)
B1 for 4.7 (dep on at least B1 awarded)

[4]

7. 7.3 4

7.2… 387(.6) 7.5… 436 (.8)
7.3… 403(.671)
7.4… 420(.0)
7.35… 411(.765)

B2 for trial between 7.2 and 7.5 inclusive
(B1 for different trial between 7 and 8 inclusive)
B1 for different trial between 7.34 and 7.36 inclusive
B1(dep on at least 1 previous B1) for 7.3

[4]

8.

$3 \rightarrow 33$	$4 \rightarrow 72$
$3.5 \rightarrow 49.8/9$	$3.6 \rightarrow 53.8/9$
$3.7 \rightarrow 58.0/1$	$3.8 \rightarrow 62.4/5$
$3.9 \rightarrow 67.1$	$3.85 \rightarrow 64.7/8$

3.9 4

(all trials to be correctly evaluated to at least 1 d.p.)
B2 for a trial strictly between 3 and 4 evaluated
(B1 for a trial at 3 or 4 evaluated)
B1 for a different trial between 3.8 and 3.9 evaluated
B1 cao (dep. on at least one previous B1)

[4]

9. (a) 55.927125 1

B1 for 55.9……. or better

(b) Yousef is correct since 55.9….. is less than 56 1

B1 for Yousef and reason

[2]

10. (a) AG 2

$x^2(x + 1) = 230$

M1 for $x \times x \times (x + 1)$ or $x \times x \times x + 1$ oe, $x^2 (x + 1)$, $x^2 \times x + 1$
A1 cao from $x \times x \times (x + 1)$, no need to see 230

(b) 5.8 4

5 – 150 6 – 252
5.1 – 158.7
5.2 – 167.6
5.3 – 177.0
5.4 – 186.6
5.5 – 196.6
5.6 – 207.0
5.7 – 217.7
5.8 – 228.8
5.9 –240.2
5.85 – 234.4

B 2 for trial between 5.8 and 5.9 inclusive evaluated
(B1 for different trial between 5 and 6 inclusive evaluated)
B1 for different trial between 5.8 and 5.85 (not including 5.8)
B1 (dep on at least one previous B1) cao for 5.8, 5.81, 5.811

[6]

1. Mathematics A

Paper 4

Centres must continue to emphasise that marks in this question as dependent on clear working being shown. Specifically the choice of a trial, and the result of each evaluation. The majority of candidates gained 3 marks, but few achieved the full marks, since they were happy to retire to a stated solution after only a few trials, or without attempting to get sufficiently near to the solution. A significant number of candidates failed to follow the instruction to round their solution to one decimal place.

Paper 6

A standard trial and improvement question which was tackled with confidence by nearly all candidates. It was nice to see many candidates testing at 4.25 and going on to give the answer of 4.2. A minority did look at the differences of the value of the function at 4.2 and 4.3 and took the nearest to 67.

Mathematics B Paper 19

This question was well answered by the majority of candidates with full working shown, as required. Although a significant number of successful candidates failed to gain all available marks by omitting to do an appropriate trial to two decimal places.

2. Paper 4

This question is written to assess candidates trial and improvement approach to the solution of equations. This can only be done if candidates show sufficient working for assessment. Evaluations of substitutions should be carried out to at least the accuracy requested in the question (in this case one decimal place). Candidates who use a different formula to the one given usually trivialise the solution, and cannot therefore be awarded marks. Half the candidates failed to show any understanding of an organised trial and improvement method, gaining no marks. Most of the remainder did show sound method in their approach, and were credited for doing so. Candidates should be reminded to work to two decimal places if the answer is to be rounded to one decimal place; a mark was reserved for this, and a further mark for rounding the answer correctly to 4.4 (ie to one decimal place). Few candidates performed both of these operations to gain full marks.

Paper 6

This was a standard trial and improvement question, in which candidates were required to find one of the solutions to a cubic equation. The demand was a little steeper than many of the previous questions in that an initial closed interval was not given to the candidates. However, this did not appear to adversely affect the facility with which candidates were able to gain at least 3 marks for the question. As usual at this level the most significant loss in marks came from failing to test at the intermediate value of 3.5 between 3 and 4. It is not sufficient to look at the value of the function at $x = 4.3$ and $x = 4.4$.

3. Higher Tier

This was a standard trial and improvement question and so was tackled competently by almost all of the entry. As always, many candidates did not use the correct procedure for determining the root correct to one decimal place. The most common incorrect method of comparing the value of $x^3 - 4x$ with 24 for values of x at 3.3 and 3.4 is unsound and results in the loss of a mark.
A small number of candidates still ignore the instructions and do not write down the values of the left hand side for their values of x. They get no marks even if they find 3.3.

Intermediate Tier

Solving an equation by trial and improvement is a technique that should have been well practiced by candidates and consequently this question was well attempted. About half of the candidates were able to gain 3 or 4 marks. Many carried out trials at 3.3 and 3.4 but a significant number of candidates then chose the value that gave an answer closest to 24 and did not carry out a trial between 3.3 and 3.4. However, at least one further trial between 3.3 and 3.4 (most often at 3.35) was frequently seen but a very common error was for 3.4 or 3.35 to be selected as the answer rather than 3.3. Some candidates carried out far too many trials and gave answers correct to 3 or more decimal places.

4. **Intermediate Tier**

As in previous examinations, many candidates knew how to do trial and improvement and gained at least 2 marks for a trial at 1.6 or 1.7 but fewer than 10% of candidates went on to give a fully correct solution. The trial at 1.65 was often omitted and even when this trial was carried out the final answer was often given as 1.6 or 1.65. Some candidates evaluated x^3 as $3x$ and a few calculated x^3 but then only added 10 each time. Most candidates gave the results of their trials to at least 1 decimal place, but some carried out far too many trials.

Higher Tier

This question was done reasonably well by many candidates; the majority using appropriate trials to evaluate $x^3 + 10x$ to an appropriate degree of accuracy. Many gained the first two marks for a trial of 1.6 or 1.7, fewer gained the mark for a trial between these values. A common approach was to look at the differences between 21 and their evaluations for 1.6 and 1.7, and erroneously conclude an answer 1.6.

5. Many candidates and centres are well practised in trial and improvement methods and the majority usually gained 3 or 4 marks. One mark was usually lost for either an answer of 2.9 or failure to trial a value between 2.8 and 2.9 or in many cases giving an answer to more than one decimal place.

6. Over half (52%) gained at least one mark, and usually two by correctly evaluating a trial between 4 and 5; however very few (17%) managed a complete solution, more often scoring 3 out of the 4 marks. Many candidates having accurately trialled at 4.65, by not fully reading the question concluded that this was the desired answer.
Failure to score at all was either through making no attempt or through an attempt at solving the cubic algebraically.

7. The majority of candidates answered this question well. 40% of candidates gained full marks and a further 40% of candidates gained 3 out of 4 marks. Some candidates never actually evaluated the answers to their trials and just put 'too big' or 'too small', such an approach will gain no marks as the question clearly states that all working should be shown. A number of candidates misunderstood the reference to giving the solution to one decimal place and searched for an answer giving 410 to one decimal place and thus lost the final mark by giving the final answer to too many decimal places. The most common error was from those candidates who found 7.3 but never used a value for x to more than 1 decimal place. These candidates generally compared their results of trials using 7.3 and 7.4 with 410 to see which was closest. This is an incorrect method as it assumes linearity.

8. The majority of candidates gained at least 2 marks for a correctly evaluated trial, to at least one decimal place, of a value of x between 3 and 4. This was often followed by correct trials of 3.8 and 3.9, with 3.9 being chosen since the evaluated answer was "closer to that of 3.9" rather than confirming the response by trialling a value in the interval between 3.8 and 3.9. This led to the loss of one mark. Often when these values were trialled they were then quoted as the final answer, also losing one mark.

9. In part (a) many candidates correctly substituted 3.65 and evaluated the cubic expression to gain the mark, only 3% gave acceptable explanation as to why 3.7 was the correct solution to one decimal place. "3.7 is correct because you round 3.65 up" or equivalent, was the most common reason given for Yousef being correct.

10. **Paper 4**

Few candidates did any work in part (a) that related to the problem and many attempted to substitute numbers at this stage. Those who realised that the height of the cuboid was $x + 1$ usually gained at least one mark. Trial and improvement was generally popular and most candidates had some success in part (b). A surprising number did not evaluate a trial in the range $5.8 < x \leq 5.85$, losing the final mark. Some did not fully evaluate their trials and gained no marks. A few candidates incorrectly evaluated expressions such as $5^3 + 6^2$, i. e. they used x and $x + 1$.

Paper 6

Part (a) was the first question on the paper where an equation was to be derived, which was then to be subsequently solved. Many candidates had the correct idea of multiplying length by width by height, but lost marks through poor algebra, such as $x \times x \times x + 1$. Some candidates tried to identify the cube term as a volume and the square term as an area and then argued that adding them together gave the total volume.
The second part was well done, with no evidence that candidates on this paper were put off by the squared term.