1. (a) The equation

$$x^3 + 4x^2 = 100$$

has a solution between 3 and 4
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **ALL** your working.

x = ...

(4)

The diagram shows a cuboid.

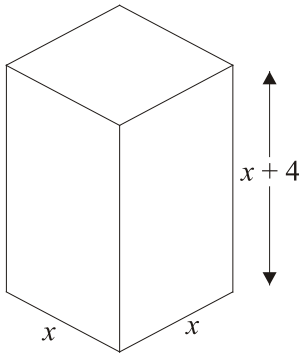

Diagram **NOT** accurately drawn

The base of the cuboid is a square of side x cm.
The height of the cuboid is $(x + 4)$ cm.
The volume of the cuboid is 100 cm^3.

(b) (i) Show that $x^3 + 4x^2 = 100$

(ii) Use your answer to part (a) to write down the height of the cuboid, correct to 1 decimal place.

..... cm

(2)

(Total 6 marks)

2. The equation

$x^3 - x = 30$

has a solution between 3 and 4
Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You must show **all** your working.

x = ...

(Total 4 marks)

3. The equation

$$x^3 - 5x = 60$$

has a solution between 4 and 5.
Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You must show **all** your working.

.....

(Total 4 marks)

4. The equation

$$x^3 + 2x = 26$$

has a solution between 2 and 3

Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **all** your working.

x = ...

(Total 4 marks)

5. The equation

$$x^3 + 20x = 71$$

has a solution between 2 and 3

Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **ALL** your working.

x = ...

(Total 4 marks)

6. The equation

$$x^3 + 4x = 26$$

has a solution between 2 and 3
Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You **must** show all your working.

x = ...

(Total 4 marks)

1. (a) 4

3.1	68.2(31)
3.2	73.7(28)
3.3	79.4(97)
3.4	85.5(44)
3.5	91.8(75)
3.6	98.4(96)
3.7	105.4(13)
3.65	101.9(1725)

= 3.6

B2 for trial $3.1 \leq x \leq 3.7$ evaluated
(B1 for trial $3 < x < 4$ evaluated)
B1 for different trial $3.615 \leq x \leq 3.65$ evaluated
B1 for 3.6, (dep on at least one of 2 previous Bs) or anything that rounds to 3.6
Values evaluated can be rounded or truncated, but to at least 1 d.p.

(b) (i) $x^2(x + 4) = 100$ 2

B1 for $x^2(x + 4)$ seen or $x \times x \times x + 4$
OR
"3.6"^{3} + 4 × "3.6"^{2} ≈ 100 (dep on 3.6 ≤ (a) ≤ 3.7);
(46.656 + 4 × 51.84)
B1 ft from "3.6" ie "3.6" + 4

[6]

2.

3	24	3.7	46.9(53)
4	60	3.8	51.0(72)
3.1	26.6(91)	3.9	55.4(19)
3.2	29.5(68)	3.21	29.8(66…)
3.3	32.6(37)	3.22	30.1(66…)
3.4	35.9(04)	3.23	30.4(68..)
3.5	39.3(75)	3.24	30.7(72...)
3.6	43.0(56)	3.25	31.0(78...) or 31

= 3.2 4

B2 for trial between 3.2 and 3.3 inclusive
(B1 for trial between 3 and 4 inclusive)
B1 for different trial between 3.21 and 3.25 inclusive
B1 (dep on at least one previous B1) cao for 3.2.
NB: embedded answers: –B1; award Bs for evaluations rounded or truncated to at least 1 dp or for 31

[4]

3.

x	$x^3 - 5x$
4	44
4.1	48.4(2)
4.2	53.0(8)
4.3	58.0(0)
4.4	63.1(8)
4.5	68.6(2)
4.6	74.3(3)
5	100
4.35	60.5(6)

= 4.3 4

B2 for trial between 4.3 and 4.4 inclusive
(B1 for trial between 4 and 5 inclusive)
B1 for different trial between 4.3 and 4.4 exclusive
B1 (dep on at least one previous B1) for 4.3 cao
NB Trials should be evaluated to at least 1 dp truncated or rounded, apart from those which when done so would give .0 which can be rounded to the nearest integer

[4]

4.

2 → 12	
3 → 33	2.5 → 20.(625)
2.1 → 13.(461)	2.6 → 22.(776)
2.2 → 15.(048)	2.7 → 25.(083)
2.3 → 16.(767)	2.8 → 27.(5(52)
2.4 → 18.(624)	2.9 → 30.(189)
2.73 → 25.8(06)	**2.74 → 26.0(508) or 26 ←**
2.75 → 26.2(96)	2.76 → 26.5(45)

2.7 4

B2 for trial between 2.7 and 2.8 inclusive
(B1 for trial between 2 and 3 inclusive)
B1 for different trial between 2.73 and 2.75 inclusive
B1 (dep on at least one previous B1) for 2.7 only
NB trials where x has 1 d.p should be rounded or truncated to at least 2SF; trials where x has 2 d.p. or more should be rounded or truncated to at least 3SF

[4]

5.

2	48
3	87
2.5	65.(625)
2.6	69.(576)
2.7	73.(683)
2.65	71.6(09)
2.61	69.9(79)
2.62	70.3(84)
2.63	70.7(91)
2.64	71.1(99)
2.66	72.(021)
2.67	72.4(34)
2.68	72.8(48)
2.69	73.2(65)

2.6 4

B2 for trial $2.6 \leq x \leq 2.7$ evaluated
(B1 for trial $2 \leq x \leq 3$ evaluated)

B1 for different trial $2.6 < x \leq 2.65$

B1(dep on at least one previous B1) for 2.6

Values evaluated can be rounded or truncated, but to at least 2sf when x has 1dp and 3sf when x has 2dp

***NB** Allow 72 for evaluation using $x = 2.66$*

***NB** No working scores no marks even if answer is correct*

[4]

6. $2 \rightarrow 16$ 4
2.5

B2 for trial between 2.5 and 2.6 inclusive
(B1 for a trial between 2 and 3 inclusive)

$3 \rightarrow 39$

$2.5 \rightarrow 25.(625)$

$2.1 \rightarrow 17.(661)$ $2.6 \rightarrow 27.(976)$

$2.2 \rightarrow 19.(448)$ $2.7 \rightarrow 30.(483)$

$2.3 \rightarrow 21.(367)$ $2.8 \rightarrow 33.(152)$

$2.4 \rightarrow 23.(424)$ $2.9 \rightarrow 35.(989)$

$2.51 \rightarrow 25.8(53)$ $2.54 \rightarrow 26.5(47)$

$2.52 \rightarrow 26.0(83)$ $2.55 \rightarrow 26.7(813)$

$2.53 \rightarrow 26.3(14)$

B1 for a different trial between 2.51 and 2.55 inclusive
B1 (dep on at least one previous B1) for 2.5 only

NB trials where x has 1 d.p. should be rounded or truncated to at least 2SF;
trials where x has 2 d.p. should be rounded or truncated to at least 3SF

[4]

1. Paper 5524

Most candidates were familiar with the methods needed, but the common error was not to pursue their trials to the second decimal place. In part (b) most attempted numerical, rather than algebraic justification, and in the last part few understood that x+4 meant merely adding 4 to their answer to part (a).

Paper 5526

Trial and improvement continues to be a mark earner for most students at this tier. The usual error was to omit the trial at 3.65 in order to establish where the solution lay in the interval 3.6 to 3.7. In addition, the other 2 main errors were to find $(4x)^2$ instead of $4x^2$ or not to give the result of their trials to at least 1dp. There were some interesting answers to the subsequent parts. Many candidates saw that they had to derive the given expression from a consideration of the algebraic dimensions of the cuboid. Even with this insight, there were candidates who wrote $x \times x \times x + 4$. Weaker candidates did not see this approach at all and interpreted this part as a demand to check their answer.

2. Higher Tier

Most candidates were able to demonstrate the method of trial and improvement. The vast majority showed enough working out to satisfy the rubric for this question. Generally speaking, calculations were carried out accurately. Many candidates lost a mark because typically they did not carry out a calculation at $x = 3.25$, rather making a judgement on the proximity of $x^3 - x$ to 30 at $x = 3.2$ compared to at $x = 3.3$.

Some candidates ignored the rubric to give an answer correct to 1 decimal place and did further calculations. They lost a mark.

Intermediate Tier

Almost all candidates attempted this question with varying degrees of success. Many obtained three marks, losing a mark by failing to do a 2 decimal place trial, which is necessary in order to justify the 3.2 as a secure answer (to 1 d.p.). Others lost the final mark by failing to round their final answer as directed. The weakest candidates managed to calculate the cubed part of the equation, but then subtracted a constant, allowing them to gain 1 mark from the initial trial of 3 or 4. Most candidates are now remembering to write the value of x on the answer line rather than the actual evaluation.

3. This was a well – answered standard trial and improvement question. The most efficient way of scoring full marks is to note that the left hand side is below 60 when

$x = 4.3$ and above 60 when $x = 4.4$. Then test the value of the left hand side when $x = 4.35$. Very few candidates lost marks by not giving sufficient accuracy (correct to at least 1 decimal place) for the value of the left-hand side. Sadly many candidates, even at this tier, compare with 60 the value of the left hand side at $x = 4.3$ and $x = 4.4$ and plump for 4.3 because the value of the left hand side is closer to 60. This, of course, is mathematically unsound.

4. This was a standard trial and improvement question. Most candidates scored marks on it. Responses tended to come in 4 groups:

- An answer of 2.7 , including a trial at 2.75 – scoring 4 marks
- An answer of 2.7, without a trial at 2.75 or equivalent – 3 marks
- An answer of 2.8 with some correct working – 1/2 marks
- A bizarre or incomplete answer

Many candidates still test the value of the function at $x = 2.7$ and at $x = 2.8$ and compare these values with 26. This is mathematically unsound and is worth a demonstration to students why.

On the positive side, nearly all candidates could work out the value of the cubic correctly for several values of x and many of these candidates worked fairly systematically recording values in a table. The most common error was to forget to change the value of the x term as x changed or to give the value of x as 2.74 or 2.73.
Some candidates still choose to ignore the instructions and do not write down the values of the cubic – they score no marks.

5. **Specification A**

While it was pleasing to see that most candidates now have a good grasp of this part of the syllabus and consequently scored well on this question there is still a lack of understanding for the need to calculate a value for $x = 2.65$ (or between 2.6 and 2.65). Candidates need to be taught that evaluating at 2.6 and 2.7 and finding out which is nearer to 71 is incorrect mathematically. Failure to round their answer to 2.6 was also common, many trying to 'do better' than 1dp.

Specification B

A standard trial and improvement question which most of the candidature were able to show some method on. The setting out of the trials was generally good, making it a lot easier for markers to award marks and also for candidates to follow their own progress towards the root. As usual, many candidates got 3 out of the 4 marks for trials at 2.6, 2.7 and then putting down 2.6 as it gives a trial closer to 71. However, many candidates knew they had to evaluate a trial at 2.65 bad picked up all 4 marks. Some candidates did further trials and wrote down (often wrong) answers correct to 2 or more decimal places. They were not awarded the final mark as they had not demonstrated they fully understood the logic of the algorithm, which should be based on the bisection method or on decimal search.

6. Solving an equation by trial and improvement is a technique that should have been well practised by candidates and this question was answered well with many candidates gaining at least 3 of the 4 marks. Most carried out trials at 2.5 and 2.6 but a significant number of candidates then chose the value that gave an answer closest to 26 and did not carry out a trial between 2.5 and 2.6. Some of those who did carry out at least one further trial between 2.5 and 2.6 (most often at 2.55) then failed to give the final answer correct to 1 decimal place as instructed.