Q1.

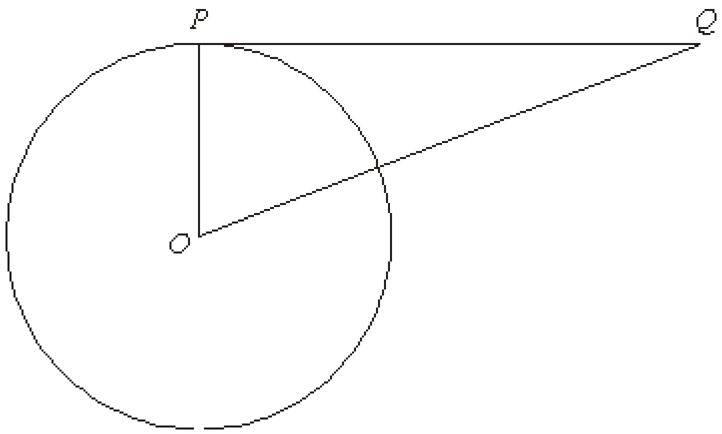


Diagram **NOT** accurately drawn

P is a point on the circumference of the circle, centre O.
PQ is a tangent to the circle.

(i) Write down the size of angle OPQ.

.... °

(ii) Give a reason for your answer.

....

....

(Total 2 marks)

Q2.

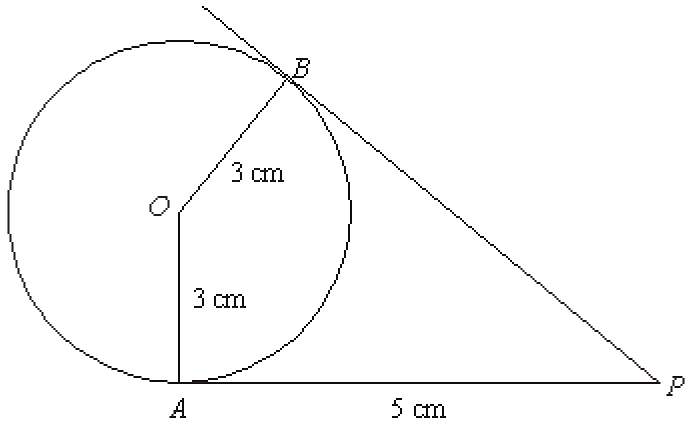


Diagram **NOT** accurately drawn

A and B are points on a circle, centre O, radius 3 cm.

PA and PB are tangents to the circle.

PA = 5 cm.

(a) Write down the size of the angle OBP.

.... °

(1)

(b) (i) Write down the length of PB.

.... cm

(ii) Give a reason for your answer.

....

....

(2)

(Total 3 marks)

Q3.

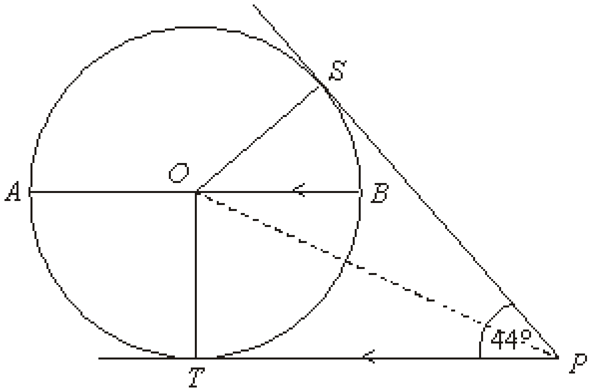


Diagram **NOT** accurately drawn

The diagram shows a circle, centre O.
A, S, B and T are points on the circumference of the circle.

PT and PS are tangents to the circle.
AB is parallel to TP.

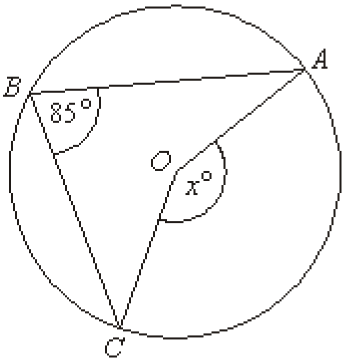
Angle SPT = 44°.

Work out the size of angle SOB.

..................................°

(Total 4 marks)

Q4.



In the diagram, A, B and C are points on the circumference of a circle, centre O.

Angle ABC = 85°.

(i) Work out the size of the angle marked $x°$.

.... °

(ii) Give a reason for your answer.

....

....

(Total 2 marks)

Q5.

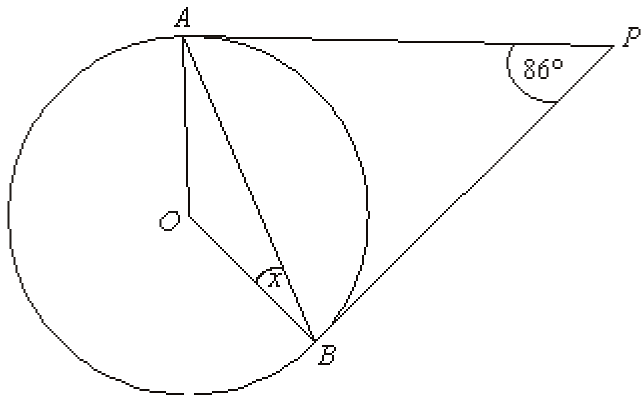


Diagram **NOT** accurately drawn

A and B are points on the circumference of a circle, centre O.
PA and PB are tangents to the circle.
Angle APB is 86°.

Work out the size of the angle marked x.

..........................°

(Total 2 marks)

Q6.

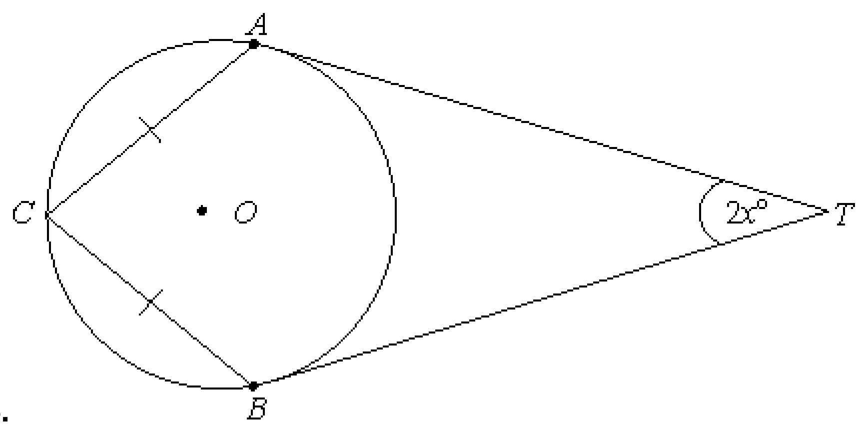


Diagram **NOT** accurately drawn

A, B and C are points on the circumference of the circle, centre O.
TA and TB are tangents to the circle.
$CA = CB$.
Angle $ATB = 2x°$.

Prove that angle $ACB = (90 - x)°$.

(Total 5 marks)

Q7.

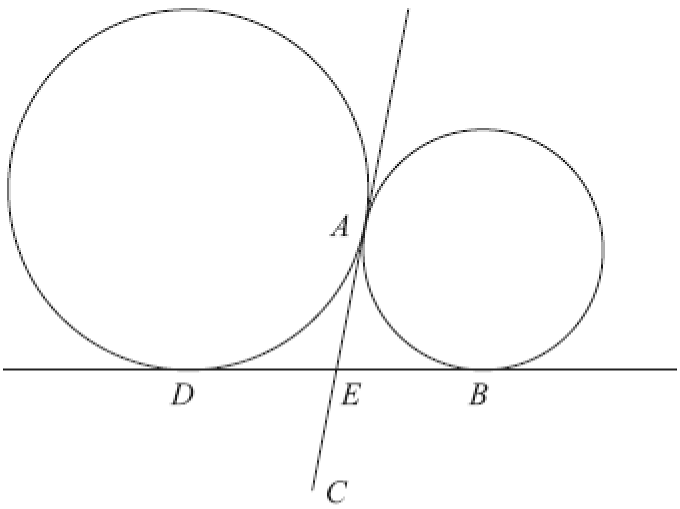


Diagram **NOT** accurately drawn

A and D are two points on the circumference of a circle.
A and B are two points on the circumference of a smaller circle.
DB and AC are tangents to both circles.

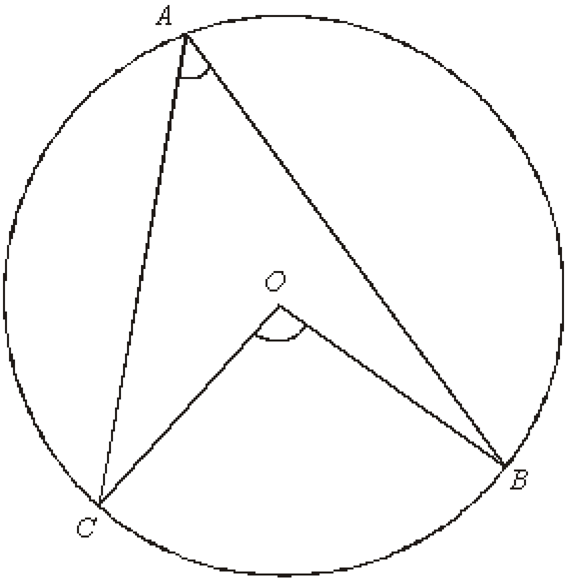
E is the intersection of DB and AC.
E is the midpoint of AC.

Prove that $ABCD$ is a rectangle.

(Total 4 marks)

Q8.

Diagram **NOT** accurately drawn



A, *B* and *C* are points on the circle with centre *O*.

Prove that the angle subtended by arc *BC* at the centre of the circle is twice the angle subtended by arc *BC* at point *A*.

(Total 4 marks)

Q9.

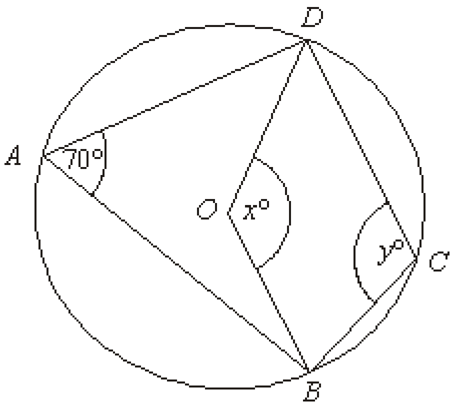


Diagram **NOT** accurately drawn

In the diagram, *A*, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
Angle *BAD* = 70°.
Angle *BOD* = x°.
Angle *BCD* = y°

(a) (i) Work out the value of x.

x = ...

(ii) Give a reason for your answer.

....

....

(2)

(b) (i) Work out the value of y.

y = ...

(ii) Give a reason for your answer.

....

....

(2)
(Total 4 marks)

Q10.

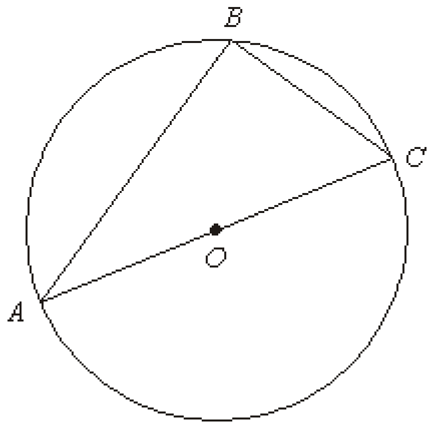


Diagram **NOT** accurately drawn

A, B and C are points on the circumference of a circle, centre O.
AC is a diameter of the circle.

(a) (i) Write down the size of angle ABC.

..........................°

(ii) Give a reason for your answer.

....

....

(2)

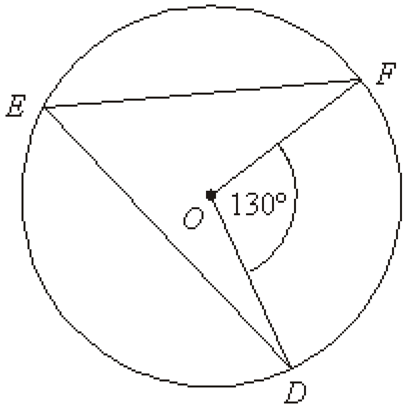


Diagram **NOT** accurately drawn

D, E and F are points on the circumference of a circle, centre O.
Angle DOF = 130°.

(b) (i) Work out the size of angle DEF.

..........................°

(ii) Give a reason for your answer.

....

....

(2)
(Total 4 marks)

Q11.

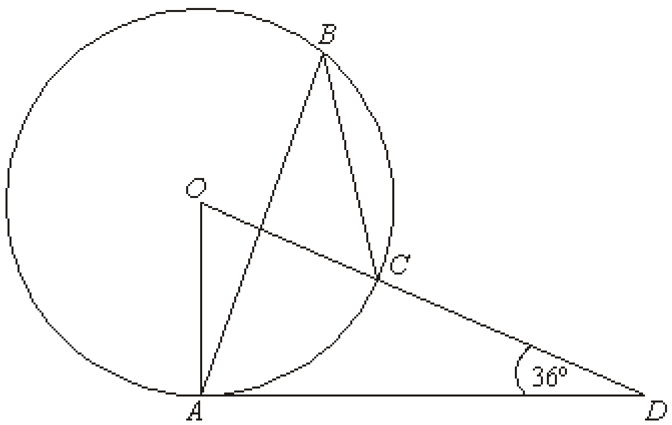


Diagram **NOT** accurately drawn

The diagram shows a circle centre O.

A, B and C are points on the circumference.

DCO is a straight line.
DA is a tangent to the circle.

Angle ADO = 36°

(a) Work out the size of angle AOD.

.... °

(2)

(b) (i) Work out the size of angle ABC.

.... °

(ii) Give a reason for your answer.

....

(3)
(Total 5 marks)

M1.

	Working	Answer	Mark	Additional Guidance
(i)		90° and reason	2	**B1** for 90°
(ii)				**B1** for angle between tangent and radius (is 90°)
				Total for Question: 2 marks

M2.

	Answer	Mark	Additional Guidance
(a)	90	1	**B1** for 90 cao Watch for angle marked on diagram
(b)(i)	5	2	**B1** for 5 cao
(ii)	Reason		**B1** for tangents from an external point are equal in length
			Total for Question: 3 marks

M3.

Working	Answer	Mark	Additional Guidance

SOT = 360 – (90 + 90 + 44) = 136°	46°	4	**Using triangle *SOP*** **B1** recognition of tangent/radius property (can be awarded for a right angle marked on the diagram) **M1** 180 – 90 – 22 or sight of 68º **M1** SOP – 22 **A1** cao
SOT = 360 – (90 + 90 + 44) = 136° 136 – 90			**Using quadrilateral SPTO** **B1** recognition of tangent/radius property (can be awarded for a right angle marked on the diagram) **M1** 360 – (2 × 90 + 44) or sight of 136º **M1** *SOT* − 90 **A1** cao
SOT = 360 – (90 + 90 + 44) = 136° 136 ÷ 2 – 22			**Alternative method for quadrilateral *SPTO*** **B1** recognition of tangent/radius property (can be awarded for a right angle marked on the diagram) **M1** 360 – (2 × 90 + 44) or sight of 136º **M1** for 136 ÷ 2 – 22 **A1** cao
			Total for Question: 4 marks

M4.

	Answer	Mark	Additional Guidance
(i)	170°	1	**B1** cao
(ii)	Reason	1	**B1** for Angle at centre is twice angle at circumference (accept edge, middle, *O* origin) oe
			Total for Question: 2 marks

M5.

Working	Answer	Mark	Additional Guidance
$\frac{1}{2}$ (180-86)= 47 90 – 47 =	43	2	**M1** for $\frac{1}{2}$ (180 – 86) or 47 or for 90 – '47' or $\frac{1}{2}$ (180 – "94") **A1** for 43 cao
Total for Question: 2 marks			

M6.

Working	Answer	Mark	Additional Guidance
AOT = 90 – x (Angle between tangent and radius is 90°) AOC = 90 + x (Tangents from an external point are equal) ACB = 2(180 – (90 + x)) ÷ 2 = 90 – x **Or** Obtuse angle BOA = 180 – 2x (Angle between tangent and radius is 90°) Reflex angle BOA = 180 + 2x (Tangents from an external point are equal) ACB = (360 – (180 + 2x)) ÷ 2 – 90 – x		5	**B1** for AOT = 90 – x or OAT = 90° or OBT= 90° (may be shown on diagram) **B1** for AOC = 90 + x **B1** for completing the proof **C2** for 2 reasons: Angle between tangent and radius is 90° **and** Tangents from an external point are equal. **QWC: proof should be clearly laid out with technical language correct** [C1 for 1 of: Angle between tangent and radius is 90° **or** Tangents from an external point are equal, **QWC: proof should be clearly laid out with technical language correct**] **OR** **B1** for obtuse angle BOA = 180 – 2x or OAT = 90° or OBT= 90° (may be shown on diagram) **B1** for reflex angle BOA = 180 + 2x **B1** for completing the proof

			C2 for 2 reasons: Angle between tangent and radius is 90° **and** Tangents from an external point are equal. **QWC: proof should be clearly laid out with technical language correct** [C1 for 1 of: Angle between tangent and radius is 90° **or** Tangents from an external point are equal, **QWC: proof should be clearly laid out with technical language correct**]
			Total for Question: 5 marks

Working	Answer	Mark	Additional Guidance
Alternative method AOB = 360 – $2x$ – 90 – 90 = 180 – $2x$ (Angle between tangent and radius is 90°) ACB = (180 – $2x$)/2 (Angle at the circumference is half angle at the centre)		5	**Alternative method** **B1** for AOB = 360 – $2x$ – 90 – 90 **B1** for ACB = (180 – $2x$)/2 **B1** for completing the proof **C2** for 2 reasons: Angle between tangent and radius is 90° **and** Angle at the circumference is half angle at the centre **QWC: proof should be clearly laid out with technical language correct** [C1 for 1 of: Angle between tangent and radius is 90° **or** Angle at the circumference is half angle at the centre **QWC: proof should be clearly laid out with technical language correct**] B3 maybe awarded for a fully correct alternative method.
			Total for Question: 5 marks

M7.

Working	Answer	Mark	Additional Guidance
$DE = AE$, and $AE = EB$ (tangents from an external point are equal in length) so $DE = EB$ $AE = EC$ (given) Therefore $AE = DE = EB = EC$ So $DB = AC$ If the diagonals are equal and bisect each other then the quadrilateral is a rectangle. OR If $AE = DE = EB = EC$ then there are four isosceles triangles ADE, AEB, BEC, DEC in which the angles DAB, ABC, BCD, CDA are all the same. Since $ABCD$ is a quadrilateral this makes all four angles 90°, and $ABCD$ must therefore be a rectangle.	Proof	4	**B1** for $DE = AE$ or $AE = EB$ (can be implied by triangle AED is isosceles or triangle AEB is isosceles or indication on the diagram) OR tangents from an external point are equal in length **B1** for $AE = DE = EB = EC$ **B1** for $DB = AC$, (dep on **B2**) OR consideration of 4 isosceles triangles in $ABCD$ **C1** fully correct proof. Proof should be clearly laid out with technical language correct and fully correct reasons
			Total for Question: 4 marks

M8.

	Working	Answer	Mark	Additional Guidance
QWC (i, ii, iii)	Join *AO* and produce to *P* Mark equal angles in		4	**M1** for Joining *AO* and producing to "*P*" **M1** for marking equal angles in isosceles

	isosceles triangle *AOC* or *AOB* Mark angle *COP* as twice angle *CAO* or mark angle *BOP* as twice angle *BAO* Identify angle *A* as half angle *BOC*			triangle *AOC* or *AOB* giving reason that triangles are isosceles because radii are equal **M1** for marking angle *COP* as twice angle *CAO* or marking Angle *BOP* as twice angle *BAO* giving reason that exterior angle of a triangle is equal to the interior and opposite angles o.e. **QWC: Working should be logical and sequential in structure; following on from labelling the extended line** **A1** for Identifying angle *A* as half angle *BOC* if M3 awarded **QWC: All labelling and angle notation should be consistent**
				Total for Question: 4 marks

M9.

	Working	Answer	Mark	Additional Guidance
(a)(i)	2×70	140	2	**B1** for 140 cao
(ii)		Reason		**B1** for 'angle at centre is twice angle at circumference'
(b)(i)	$180 - 70$ or $\frac{1}{2} \times 220$	110	2	**B1** for 110 cao
(ii)		Reason		**B1** for 'opposites angles in a cyclic quadrilateral sum to 180 degrees' or 'angle at centre is twice angle at circumference'
				Total for Question: 4 marks

M10.

	Working	Answer	Mark	Additional Guidance
(a)(i)		90	2	**B1** cao **B1** for angle in a semi-circle (= 90°) or angle at the centre is twice the angle at the circumference or angle subtended by a diameter = 90°.
(ii)		angle in a semi-circle = 90°		
(b)(i)	130 ÷ 2	65	2	**B1** cao
(ii)		angle at centre is twice the angle at the circumference		**B1** for angle at the centre is twice the angle at the circumference.
				Total for Question: 4 marks

M11.

	Working	Answer	Mark	Additional Guidance
(a)	*AOD* = 90 – 36 or 180 – (90 + 36)	54	2	**M1** *AOD* = 90 – 36 or 180 – (90 + 36) **A1** cao
(b)(i)	*ABC* = *AOD* ÷ 2	27	2	**M1** *ABC* = *AOD* ÷ 2 **A1** ft from ‘54’
(ii)		Reason	1	**B1** Angle at centre = twice angle at circumference
				Total for Question: 5 marks

E1. Over 70% of candidates gained the mark for writing down the correct size of the angle in part (i) of this question. However, many candidates did not understand the notation used to identify the angle OPQ and gave 180º as their answer. Many then went on to explain in part (ii) that the angles in a triangle sum to 180º. Disappointingly, only about one third of the candidates who answered part (i) correctly could give a clear and succinct reason in part (ii). All that was required was a clear reference to the angle between the tangent and the radius.

E2. In this question candidates were able to give the answer of 90º for (a) and 5 cm for (b)(i) but very few candidates were able to give a complete reason as to why PB had a length of 5 cm.

E3. This was a very successful question for the 28% of candidates that gained all four marks. In fact though 25% of candidates scored no marks 12% scored the mark for recognising that there were 90° between the tangent and radius of a circle and a further 35% gained two marks for correctly calculating the value of either angle *TOP* or *SOT* or *SOP*. There were a number of valid methods for solving this question and all were awarded marks if the solution was correct. A surprising number of candidates had no understanding of which angle SOB referred to .Often they mentioned angle O which was of course meaningless as there were many angles with this point as a vertex.

E4. A straightforward circle theorem question in which most students got 170°. A few got themselves confused and thought this was about cyclic quadrilaterals and others worked out the reflex angle instead as 170°. Explanations were good but still in many cases focussing on the particular (‘angle AOC’) rather than the general (‘angle at the centre’).or using reference to the ‘arrowhead’.

E5. About two thirds of responses to this question were awarded at least one mark with just over a half of candidates achieving full marks.

Many candidates demonstrated that they knew that the tangent and radius met at 90°. However, a significant number of candidates gave “4°” as their answer – obtaining this from doubling 86 and subtracting from 180° before halving or from subtracting 86° from 90°. The candidates who gained one mark often worked out that angle ABP was 47° but could go no further.

##

Many candidates gained a mark for correctly identifying a right-angle at OAT and/or OBT even if they made no further progress. Others assumed CAT or CBT were 90° or even that ACB was. A variety of proofs were attempted but in this question where marks were awarded for Quality of Written Communication, it was essential that theorems were quoted accurately using correct mathematical language.

##

This was undoubtedly the question where candidates were least successful. 95% of candidates failed to score with 4% scoring 1 mark for one correct statement that could lead to the proof. Many incorrect arguments were based on tangents meeting radii at right angles (true but irrelevant) or the intersection of *AC* and *DB* at *E* forming four right angles (false). The weakest responses included “stories” of how you could join up *A*, *B*,*C* and *D* and it would be a rectangle, and statements like *AD* = *BC* without any justification.

Some candidates did gain one mark for stating either *AE* = *ED* or *AE* = *EB* or for the rule that tangents from an external point are equal in length. This mark was also available via annotation on the diagram or a statement of triangle *AED* or *AEB* being isosceles. However, very few candidates gained any further credit. Many candidates were unable to formulate any kind of argument/proof preferring just to write lists of circle theorems and/or rules about angles in the hope that one or more of them would be relevant. The common confusion between the meaning of equal and parallel was also seen. A minority stated such gems as Theorem 7 or Theorem 5 and one candidate in desperation wrote ‘The angle between the tangent and the radius is 90°, but there isn't a radius.

E9. Part (a)(i) was generally done well. Most candidates realised that they needed to double the angle at the circumference to get the angle at the centre, but in part (a)(ii), only the best candidates were able to quote the circle theorem accurately. A typical answer here was 'the angle in the middle is double the angle at the edge'. A common unacceptable answer was *BOD* = 2 × *BAD*. In part (b)(i), only about a quarter of the candidates were able to work out the correct value for y. many thought that x and y were equal and said as much in part (b)(ii), e.g. 'opposite angles in cyclic quadrilateral are equal'. Again, only the best candidates were able to quote the circle theorem accurately. A common unacceptable answer was 'circle in a quadrilateral, opposite angle add to 180°'. A significant number of candidates thought that *BODC* was the cyclic quadrilateral and gave the angle as 40°. Candidates should be advised to learn the circle theorems accurately.

E10. A correct answer of 90° was the most common response to (a) part (i), however very few candidates were able to offer an acceptable reason in (ii). Reasons such as 'triangles with hypotenuse as a diameter always give a right angle' or 'lines drawn from a diameter always make a right angle' were the best of the unacceptable offerings. 'Angles in a semicircle' or 'angles subtended by a diameter' were accepted for the award of the mark. In part (b), the correct angle of 65° was usually seen but often supported by an incorrect reason. It should be noted that 'Arrow head theory', or similar is not an acceptable reason.

E11. Many candidates answered part (a) correctly, recognising the right angle between radius and tangent and using the angle sum of a triangle to work out the size of angle *AOD*. There was, though, some evidence of poor arithmetic with some candidates unable to subtract 126 from 180 correctly. Correct answers to (b)(i) were much rarer.

Many candidates had remembered that angles in the same segment are equal but had forgotten that the two angles both need to be on the circumference of the circle. Hence a very common error was for angle *ABC* to be given as 54o (the same as angle *AOD*). The majority of the candidates who answered (b)(i) correctly were able to give the correct reason in (b)(ii).