

1. The n th even number is $2n$.

The next even number after $2n$ is $2n + 2$

(a) Explain why.

..... Even numbers go up in 2s. The 2 times table:.....
..... $\begin{array}{ccccccc} & +2 & +2 & +2 & +2 & & \\ \nearrow & & & & & & \\ 2 & 4 & 6 & 8 & 10 & & \end{array}$

(1)

(b) Write down an expression, in terms of n , for the next even number after $2n + 2$

..... $2n + 4$

(1)

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

$$2n + (2n+2) + (2n+4)$$

$$6n + 6$$

$$\nearrow 6(n+1)$$

Because we can factorise out 6, $6n+6$ is always a multiple of 6.

(3)

(5 marks)

2. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n .

$$\begin{aligned} & (3n+1)(3n+1) - (3n-1)(3n-1) \\ & (9n^2+3n+3n+1) - (9n^2-3n-3n+1) \\ & (9n^2+6n+1) - (9n^2-6n+1) \\ & 9n^2+6n+1 - 9n^2+6n-1 \\ & 12n \\ & \underline{\underline{4(3n)}} \end{aligned}$$

(3 marks)

3. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

$$n + n+1$$

$$2n+1$$

$2n$ is an even number. An even number plus one is always odd.

(3 marks)

4. Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n .

$$\begin{aligned} & (2n+3)(2n+3) - (2n-3)(2n-3) \\ & (4n^2 + 6n + 6n + 9) - (4n^2 - 6n - 6n + 9) \\ & (4n^2 + 12n + 9) - (4n^2 - 12n + 9) \\ & \quad 24n \\ & \quad \underline{\underline{8(3n)}} \end{aligned}$$

(3 marks)

- *5. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

n and $n+1$ (two consecutive integers)

$$(n+1)^2 - n^2 \quad (\text{diff. between squares})$$

$$(n+1)(n+1) - n^2$$

$$n^2 + n + n + 1 - n^2$$

$$\underline{2n+1}$$

$$\text{Sum of } n \text{ and } n+1 = \underline{2n+1}$$

(4 marks)

6. Prove that $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 5, for all positive integer values of n .

$$\begin{aligned} & (5n+1)(5n+1) - (5n-1)(5n-1) \\ & (25n^2+5n+5n+1) - (25n^2-5n-5n+1) \\ & (25n^2+10n+1) - (25n^2-10n+1) \\ & \quad 20n \\ & \quad \underline{\underline{5(4n)}} \end{aligned}$$

(3 marks)

7. If $2n$ is always even for all positive integer values of n , prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

$$2n \quad \text{and} \quad 2n+2$$

$$(2n)^2 + (2n+2)^2$$

$$4n^2 + (2n+2)(2n+2)$$

$$4n^2 + 4n^2 + 4n + 4n + 4$$

$$8n^2 + 8n + 4$$

$$\underline{\underline{4(2n^2 + 2n + 1)}}$$

(3 marks)

8. Prove that

$(n+1)^2 - (n-1)^2 + 1$ is always odd for all positive integer values of n .

$$(n+1)(n+1) - (n-1)(n-1) + 1$$

$$(n^2 + n + n + 1) - (n^2 - n - n + 1) + 1$$

$$(n^2 + 2n + 1) - (n^2 - 2n + 1) + 1$$

$$n^2 + 2n + 1 - n^2 + 2n - 1 + 1$$

$$4n + 1$$

The four times table is always even. An even plus one is odd.

(3 marks)

9. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

n and $n+1$

$$n^2 + (n+1)^2$$

$$n^2 + (n+1)(n+1)$$

$$n^2 + n^2 + n + n + 1$$

$$2n^2 + 2n + 1$$

$$2n(n+1) + 1$$

$$2(n)(n+1) + 1$$

2 x an even is always a multiple of 4, either n or $n+1$ is even. $\therefore 2(n)(n+1)$ is divisible by 4. There is a remainder of 1.

(4 marks)