

1. Given that $f(x) = x - 4$ find:

a) $f(5)$

$$f(5) = 5 - 4$$

..... 1 (1)

b) $f(3)$

$$f(3) = 3 - 4$$

..... -1 (1)

2. Given that $g(x) = 2x^2 - 10$ find:

a) $g(2)$

$$\begin{aligned} g(2) &= 2(2)^2 - 10 \\ &= 8 - 10 \end{aligned}$$

..... -2 (1)

b) $g(-2)$

$$\begin{aligned} g(-2) &= 2(-2)^2 - 10 \\ &= 8 - 10 \end{aligned}$$

..... -2 (1)

c) Solve: $g(x) = 8$

$$2x^2 - 10 = 8$$

$$2x^2 = 18$$

$$x^2 = 9$$

..... $x = \pm 3$ (3)

3. Given that $f(x) = 3x - 5$ find:

$$\begin{aligned} \text{a) } f(3) &= 3(3) - 5 \\ &= 9 - 5 \end{aligned}$$

$$\dots\dots\dots 4 \dots\dots (1)$$

$$\begin{aligned} \text{b) } f(-2) &= 3(-2) - 5 \\ &= -6 - 5 \end{aligned}$$

$$\dots\dots\dots -11 \dots\dots (1)$$

c) Solve: $f(x) = 1$

$$\begin{aligned} 3x - 5 &= 1 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

$$\dots\dots\dots (2)$$

4. Given that $f(x) = x^2 - 3$ find:

$$\begin{aligned} \text{a) } f(10) &= (10)^2 - 3 \\ &= 100 - 3 \end{aligned}$$

$$\dots\dots\dots 97 \dots\dots (1)$$

$$\begin{aligned} \text{b) } f(-1) &= (-1)^2 - 3 \\ &= 1 - 3 \end{aligned}$$

$$\dots\dots\dots -2 \dots\dots (1)$$

c) Find: $f^{-1}(x)$

$$\begin{aligned} y &= x^2 - 3 \\ y + 3 &= x^2 \\ \sqrt{y+3} &= x \end{aligned}$$

$$f^{-1}(x) = \sqrt{x+3}$$

$$\dots\dots\dots f^{-1}(x) = \sqrt{x+3} \dots\dots (2)$$

5. Given that $f(x) = 2x - 4$ and $g(x) = 3x + 5$

a) Find: $gf(3)$

$$\begin{aligned} f(3) &= 2(3) - 4 \\ &= 6 - 4 \\ &= 2 \end{aligned}$$

$$\begin{aligned} g(2) &= 3(2) + 5 \\ &= 6 + 5 \end{aligned}$$

$$\dots\dots\dots 11 \dots\dots\dots (2)$$

b) Work out an expression for: $f^{-1}(x)$

$$y = 2x - 4$$

$$y + 4 = 2x$$

$$\frac{1}{2}(y + 4) = x$$

$$f^{-1}(x) = \frac{1}{2}(x + 4)$$

$$f^{-1}(x) = \frac{1}{2}(x + 4) (2)$$

c) Solve: $f(x) = g(x)$

$$2x - 4 = 3x + 5$$

$$-4 = x + 5$$

$$x = -9$$

$$\dots\dots\dots x = -9 \dots\dots\dots (2)$$

6. Given that $f(x) = 3x + 1$ and $g(x) = x^2$

a) Write down an expression for: $fg(x)$

$$3x^2 + 1 \dots \dots \dots (2)$$

b) Work out an expression for: $gf(x)$

$$\dots \dots \dots (3x + 1)^2 \dots \dots \dots (2)$$

c) Solve: $fg(x) = gf(x)$

$$3x^2 + 1 = (3x + 1)^2$$

$$3x^2 + 1 = 9x^2 + 3x + 3x + 1$$

$$0 = 6x^2 + 6x$$

$$0 = x^2 + x$$

$$0 = x(x + 1)$$

$$x = 0 \quad x = -1$$

$$\dots \dots \dots x = 0 \quad x = -1 \dots \dots \dots (3)$$

7. Given that $f(x) = x^2 - 17$ and $g(x) = x + 3$

a) Work out an expression for: $g^{-1}(x)$

$$y = x + 3$$

$$y - 3 = x$$

$$g^{-1}(x) = x - 3 \quad (2)$$

b) Work out an expression for: $f^{-1}(x)$

$$y = x^2 - 17$$

$$y + 17 = x^2$$

$$\sqrt{y + 17} = x$$

$$f^{-1}(x) = \sqrt{x + 17} \quad (2)$$

c) Solve: $f^{-1}(x) = g^{-1}(x)$

$$\sqrt{x + 17} = x - 3$$

$$x + 17 = (x - 3)^2$$

$$x + 17 = x^2 - 6x + 9$$

$$0 = x^2 - 7x - 8$$

$$0 = (x - 8)(x + 1)$$

$$x = 8 \quad x = -1$$

..... (4)

8. A function f is defined such that

$$f(x) = x^2 - 1$$

a) Find an expression for: $f(x-2)$

$$\begin{aligned} f(x-2) &= (x-2)^2 - 1 \\ &= x^2 - 2x - 2x + 4 - 1 \\ &= x^2 - 4x + 3 \end{aligned}$$

$$\underline{\underline{x^2 - 4x + 3}} \quad (2)$$

b) Hence solve: $f(x-2) = 0$

$$\begin{aligned} x^2 - 4x + 3 &= 0 \\ (x-3)(x-1) &= 0 \\ x=3 \quad x=1 \end{aligned}$$

$$\underline{\underline{x=3 \quad x=1}} \quad (2)$$

9. A function f is defined such that

$$f(x) = 4x - 1$$

a) Find: $f^{-1}(x)$

$$\begin{aligned}y &= 4x - 1 \\y + 1 &= 4x \\ \frac{y+1}{4} &= x \\ f^{-1}(y) &= \frac{y+1}{4}\end{aligned}$$

..... (2)

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant}$$

Given that $fg(2) = 12$

b) Work out the value of k

$$\begin{aligned}g(2) &= k(2)^2 \\ &= 4k \\ f(4k) &= 4(4k) - 1 \\ &= 16k - 1\end{aligned}$$

$$16k - 1 = 12$$

$$16k = 13$$

$$k = \frac{13}{16}$$

..... (2)