

1. The weight of a piece of wire is directly proportional to its length.

A piece of wire is 25 cm long and has a weight of 6 grams.

Another piece of the same wire is 30 cm long.

Calculate the weight of the 30 cm piece of wire.

$$\begin{aligned} W &= kl \\ 6 &= k(25) \\ k &= \frac{6}{25} \end{aligned} \qquad \begin{aligned} w &= \frac{6}{25}(1) \\ &= \frac{6}{25}(30) \\ &= \frac{180}{25} \\ &= 7.2 \end{aligned}$$

..... grams
(Total 2 marks)

2. A ball falls vertically after being dropped.
The ball falls a distance d metres in a time of t seconds.
 d is directly proportional to the square of t .

The ball falls 20 metres in a time of 2 seconds.

- (a) Find a formula for d in terms of t .

$$\begin{aligned} d &= kt^2 \\ 20 &= k(2)^2 \\ 20 &= 4k \\ k &= 5 \end{aligned}$$

$$d = 5t^2$$

..... (3)

- (b) Calculate the distance the ball falls in 3 seconds.

$$d = 5(3)^2$$

$$45$$

..... m (1)

- (c) Calculate the time the ball takes to fall 605 m.

$$605 = 5t^2$$

$$121 = t^2$$

$$11$$

..... seconds (3)
(Total 7 marks)

3. The time, T seconds, it takes a water heater to boil some water is directly proportional to the mass of water, m kg, in the water heater.

When $m = 250$, $T = 600$

- (a) Find T when $m = 400$

$$T = km$$

$$600 = k(250)$$

$$k = \frac{600}{250} = \frac{12}{5} = 2.4$$

$$T = 2.4(m)$$

$$= 2.4(400)$$

$$= 960$$

$$T = \underline{\underline{960}}$$

(3)

The time, T seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power, P watts, of the water heater.

When $P = 1400$, $T = 360$

- (b) Find the value of T when $P = 900$

$$T = \frac{k}{P}$$

$$360 = \frac{k}{1400}$$

$$k = 504000$$

$$T = \frac{504000}{900}$$

$$= 560$$

$$T = \underline{\underline{560}}$$

(3)

(Total 6 marks)

4. D is proportional to S^2 .

$D = 900$ when $S = 20$

Calculate the value of D when $S = 25$

$$D = kS^2$$

$$900 = k(20)^2$$

$$\frac{900}{400} = k$$

$$k = 2.25$$

$$D = 2.25S^2$$

$$D = 2.25(25)^2$$

$$= 1406.25$$

$$D = \underline{\underline{1406.25}}$$

(Total 4 marks)

5. In a spring, the tension (T newtons) is directly proportional to its extension (x cm).

When the tension is 150 newtons, the extension is 6 cm.

- (a) Find a formula for T in terms of x .

$$\begin{aligned}T &= kx \\150 &= k(6) \\k &= 25 \\T &= 25x\end{aligned}$$

$$T = \underline{25x} \quad (3)$$

- (b) Calculate the tension, in newtons, when the extension is 15 cm.

$$\begin{aligned}T &= 25x \\&= 25(15) \\&= 375\end{aligned}$$

$$\underline{375} \text{ newtons} \quad (1)$$

- (c) Calculate the extension, in cm, when the tension is 600 newtons.

$$\begin{aligned}T &= 25x \\600 &= 25x \\\frac{600}{25} &= x \\x &= 24\end{aligned}$$

$$\underline{24} \text{ cm} \quad (1)$$

(Total 5 marks)

6. d is directly proportional to the square of t .

$d = 80$ when $t = 4$

(a) Express d in terms of t .

$$\begin{aligned}d &= k t^2 \\80 &= k (4)^2 \\80 &= 16k \\k &= 5\end{aligned}$$

$$d = 5t^2$$

$$\dots d = 5t^2 \dots$$

(3)

(b) Work out the value of d when $t = 7$

$$\begin{aligned}d &= 5t^2 \\&= 5(7)^2 \\&= 245\end{aligned}$$

$$d = \dots 245 \dots$$

(1)

(c) Work out the positive value of t when $d = 45$

$$\begin{aligned}d &= 5t^2 \\45 &= 5t^2 \\9 &= t^2 \\t &= 3\end{aligned}$$

$$t = \dots 3 \dots$$

(2)

(Total 6 marks)

7. The distance, D , travelled by a particle is directly proportional to the square of the time, t , taken.

When $t = 40$, $D = 30$

- (a) Find a formula for D in terms of t .

$$D = kt^2$$

$$30 = k(40)^2$$

$$30 = k(1600)$$

$$k = \frac{3}{160}$$

$$D = \frac{3}{160}t^2$$

- (b) Calculate the value of D when $t = 64$

$$D = \frac{3}{160}t^2$$

$$= \frac{3}{160}(64)^2$$

$$= 76.8$$

$$\underline{\underline{76.8}}$$

(1)

- (c) Calculate the value of t when $D = 12$
Give your answer correct to 3 significant figures.

$$D = \frac{3}{160}t^2$$

$$12 = \frac{3}{160}t^2$$

$$640 = t^2$$

$$t = 25.3 \quad (3 \text{ s.f.})$$

$$\underline{\underline{25.3}}$$

(2)

(Total 6 marks)

8. M is directly proportional to L^3 .

When $L = 2$, $M = 160$

Find the value of M when $L = 3$

$$M = kL^3$$

$$160 = k(2)^3$$

$$160 = 8k$$

$$k = 20$$

$$\begin{aligned} M &= 20L^3 \\ &= 20(3)^3 \\ &= 540 \end{aligned}$$

.....
540

(Total 4 marks)

9. p is inversely proportional to m .
 $p = 48$ when $m = 9$

Calculate the value of p when $m = 12$

$$p = \frac{k}{m}$$

$$48 = \frac{k}{9}$$

$$k = 432$$

$$p = \frac{432}{m}$$

$$= \frac{432}{12}$$

.....
36

(Total 2 marks)

10. r is inversely proportional to t .
 $r = 12$ when $t = 0.2$

Calculate the value of r when $t = 4$.

$$r = \frac{k}{t}$$

$$12 = \frac{k}{0.2}$$

$$k = 2.4$$

$$r = \frac{2.4}{t}$$

$$r = \frac{2.4}{4} = 0.6$$

.....
0.6

(Total 3 marks)

11. f is inversely proportional to d .

When $d = 50, f = 256$

Find the value of f when $d = 80$

$$f = \frac{k}{d}$$

$$256 = \frac{k}{50}$$

$$k = 12800$$

$$f = \frac{12800}{d}$$

$$f = \frac{12800}{80}$$

$f =$
160

(Total 3 marks)

12. y is inversely proportional to x^2 .

Given that $y = 2.5$ when $x = 24$,

(i) find an expression for y in terms of x

$$y = \frac{k}{x^2}$$

$$2.5 = \frac{k}{(24)^2}$$

$$k = 1440$$

$$y = \frac{1440}{x^2}$$

(ii) find the value of y when $x = 20$

$$y = \frac{1440}{x^2}$$

$$= \frac{1440}{(20)^2}$$

$$y = 3.6$$

(iii) find a value of x when $y = 1.6$

$$y = \frac{1440}{x^2}$$

$$1.6 = \frac{1440}{x^2}$$

$$x^2 = \frac{1440}{1.6} = 900$$

$$x = 30$$

(Total 6 marks)

13. P is inversely proportional to d^2 .

$P = 10\,000$ when $d = 0.4$

Find the value of P when $d = 0.8$

$$P = \frac{k}{d^2}$$

$$10000 = \frac{k}{(0.4)^2}$$

$$k = 1600$$

$$P = \frac{1600}{d^2}$$

$$= \frac{1600}{0.8^2}$$

$$P = 2500$$

(Total 3 marks)

14. The shutter speed, S , of a camera varies inversely as the square of the aperture setting, f .

When $f = 8$, $S = 125$

- (a) Find a formula for S in terms of f .

$$S = \frac{k}{f^2}$$
$$125 = \frac{k}{8^2}$$
$$k = 8000$$

$$S = \frac{8000}{f^2} \quad (3)$$

- (b) Hence, or otherwise, calculate the value of S when $f = 4$

$$S = \frac{8000}{4^2}$$

$$S = 500$$

$$S = 500 \quad (1)$$

(Total 4 marks)

15. q is inversely proportional to the square of t .

When $t = 4$, $q = 8.5$

(a) Find a formula for q in terms of t .

$$q = \frac{k}{t^2}$$

$$8.5 = \frac{k}{(4)^2}$$

$$k = 136$$

$$q = \frac{136}{t^2}$$

(3)

(b) Calculate the value of q when $t = 5$

$$q = \frac{136}{5^2}$$

$$= 5.44$$

$$5.44$$

(1)

(Total 4 marks)

16. P is inversely proportional to V .

When $V = 8$, $P = 5$

(a) Find a formula for P in terms of V .

$$P = \frac{k}{V}$$

$$5 = \frac{k}{8}$$

$$k = 40$$

$$P = \frac{40}{V} \dots\dots\dots (3)$$

(b) Calculate the value of P when $V = 2$

$$P = \frac{40}{2}$$

$$\dots\dots\dots 20 \dots\dots\dots (1)$$

(Total 4 marks)

17. The force, F , between two magnets is inversely proportional to the square of the distance, x , between them.

When $x = 3$, $F = 4$.

(a) Calculate F when $x = 2$.

$$F = \frac{k}{x^2}$$

$$4 = \frac{k}{3^2}$$

$$k = 36$$

$$F = \frac{36}{x^2} \\ = \frac{36}{2^2}$$

$$\dots\dots\dots 9 \dots\dots\dots (4)$$

(b) Calculate x when $F = 64$.

$$F = \frac{36}{x^2}$$

$$x = \sqrt{\frac{36}{64}}$$

$$64 = \frac{36}{x^2}$$

$$x^2 = \frac{36}{64}$$

$$\dots\dots\dots x = \frac{3}{4} \dots\dots\dots (2)$$

(Total 6 marks)