Q1.

c is a positive integer.

Prove that $\frac{6c^3 + 30c}{3c^2 + 15}$ is an even number.

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(Total 3 marks)

Q2.

In the formula $T = (n - 6)^2 + 1$ n is a positive integer.

(a) Kim says,

"The value of T is always greater than 1
because $(n - 6)^2$ is always greater than 0"

Comment on her statement.

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(1)

(b) What is the only value of T that is a square number?

Answer ....

(1)
(Total 2 marks)

Q3.

$(2x + 3y)^2 - (2x - 3y)^2 = 360$

Show that xy is a multiple of 5

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(Total 4 marks)

Q4. Prove that $5x(x + 6) - (3x + 5)^2$ is negative for all values of x.

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(Total 4 marks)

Q5.

(a) The nth term of a sequence is $n^2 + 12n + 27$

By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers.

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(2)

(b) The nth term of a different sequence is $n^2 - 6n + 14$

By completing the square, or otherwise, show that every term is positive.

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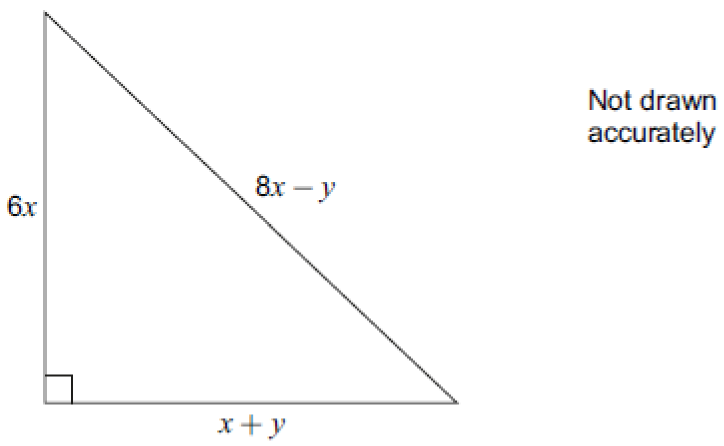
(3)
(Total 5 marks)

Q6.

The diagram shows a right-angled triangle.



Prove algebraically that $x : y = 2 : 3$

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(Total 6 marks)

Q7. Prove that $\dfrac{3n+5}{3n} - \dfrac{n}{n-1} \equiv \dfrac{2n-5}{3n(n-1)}$

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(Total 3 marks)

Q8.

(a) n is a positive integer.

Write down the **next** odd number after $2n - 1$

Answer..

(1)

(b) Prove that the product of two consecutive odd numbers is **always** one less than a multiple of 4.

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(3)
(Total 4 marks)

Q9. n is an integer.

Show that $\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number.

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(Total 3 marks)

Q10.

Prove that $(5n + 3)(n - 1) + n(n + 2)$
is a multiple of 3 for all integer values of n.

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(Total 4 marks)

Q11. n is an integer.

$$S = \frac{1}{2}n(n+1)$$

Prove that $8S + 1$ is an odd square number.

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(Total 5 marks)

Q12. Prove that $\dfrac{3n-1}{n} - \dfrac{3n+1}{n-2} \equiv \dfrac{2-8n}{n(n-2)}$

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(Total 4 marks)

Q13.

The n^{th} term of the linear sequence

2 7 12 17 ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that **all** the terms in the new sequence are multiples of 5 .

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(Total 4 marks)