M1.

$6c(c^2 + 5)$ or $3(c^2 + 5)$

M1

$$\frac{6c\left(c^2 + 5\right)}{3\left(c^2 + 5\right)}$$

This mark implies first M1

M1

$2c$ and multiple of 2 so even

oe statement
Must see method

A1

[3]

M2.

(a) $(n - 6)^2$ could be zero (so she is wrong)
or
The sixth term is 1

oe

B1

(b) 1

B1

[2]

M3.

Alternative method 1

$4x^2 + 6xy + 6xy + 9y^2$

or $4x^2 - 6xy - 6xy + 9y^2$

Four terms, three correct with a term in x^2 and a term in y^2

or $4x^2 \pm 12xy + ay^2$ with $a \neq 0$

or $bx^2 \pm 12xy + 9y^2$ with $b \neq 0$

M1

$4x^2 + 12xy + 9y^2 - (4x^2 - 12xy + 9y^2)$

or $4x^2 + 12xy + 9y^2 - 4x^2 + 12xy - 9y^2$

oe

allow one error, which may be missing brackets

M1dep

$24xy = 360$

oe

M1

$xy = 15$ (and 15 is a multiple of 5)

A1

Alternative method 2

$(2x + 3y + 2x - 3y)(2x + 3y - (2x - 3y))$

or

$(2x + 3y + 2x - 3y)(2x + 3y - 2x + 3y)$

allow one error, which may be missing brackets

M1

their $4x$ × their $6y$

Correct simplification of both of their brackets and intention to multiply

M1dep

$24xy = 360$

oe

M1

$xy = 15$ (and 15 is a multiple of 5)

A1

Additional Guidance

Missing brackets in Alt 1 for second method mark may be recovered for M3 or M3A1

[4]

M4. $5x^2 + 30x$

or $9x^2 + 15x + 15x + 25$

or $9x^2 + 30x + 25$

M1

$5x^2 + 30x - (9x^2 + 15x + 15x + 25)$

or $5x^2 + 30x - (9x^2 + 30x + 25)$

or $5x^2 + 30x - 9x^2 - 15x - 15x - 25$

or $5x^2 + 30x - 9x^2 - 30x - 25$

allow one error

M1

$-4x^2 - 25$ or $-(4x^2 + 25)$

from fully correct algebra

A1

Argues that both terms have to be negative so expression is negative

Strand (ii) correct mathematical argument and M2 scored

argument may be that their expression is not negative for all values of x

Q1ft

[4]

M5.

(a) 29 and 23 identified

B1 (n + 9)(n + 3) or 667 or 29 or 23

B2

(b) **Alternative method 1**

$(n - 3)^2$

Allow $(n - 3)(n - 3)$ for $(n - 3)^2$

M1

$(n - 3)^2 - 9 + 14$
or
$(n - 3)^2 + 5$

Allow $(n - 3)(n - 3)$ for $(n - 3)^2$

A1

$(n - 3)^2 \geq 0$ then adding 5 so always positive
or
States minimum value is 5
or
States (3, 5) is minimum point

oe Allow $(n - 3)(n - 3)$ for $(n - 3)^2$

ft M1 A0

Must see M1 and attempt $(n - 3)^2 + k$

ft $(n - 3)^2 + k$ where $k > 0$

SC2 States minimum value is 5

or

States (3, 5) is minimum point

A1ft

Alternative method 2

Quadratic curve sketched in first quadrant with minimum point above the x-axis

Labelling on axes not required

M1

(discriminant =) −20

A1

States no (real) roots

oe Allow roots → solutions

ft M1 A0

Must see M1 and attempt a discriminant ft discriminant < 0

SC2 States minimum value is 5

or

States (3, 5) is minimum point

A1ft

Alternative method 3

$2n - 6 = 0$

oe equation

e.g. $2n = 6$ or $n = 3$

M1

(second derivative =) 2

A1

States minimum value is 5
or
States (3, 5) is minimum point

oe

ft M1 A0

Must see M1 and attempt a second derivative

ft (second derivative) > 0

SC2 States minimum value is 5

or

States (3, 5) is minimum point

A1ft

[5]

M6.

$(8x - y)^2 = (6x)^2 + (x + y)^2$

oe

Allow $(8x - y)(8x - y)$ and $(x + y)(x + y)$

Condone $6x^2$

M1

Expands $(8x - y)^2$ to 4 terms with 3 correct from
$64x^2 - 8xy - 8xy + y^2$

oe

If going straight to 3 terms must be

$64x^2 - 16xy + ky^2$ ($k \neq 0$) or

$ax^2 - 16xy + y^2$ ($a \neq 0$)

M1

Expands $(x + y)^2$ to 4 terms with 3 correct from
$x^2 + xy + xy + y^2$

oe

If going straight to 3 terms must be
$x^2 + 2xy + ay^2$ ($a \neq 0$) or
$bx^2 + 2xy + y^2$ ($b \neq 0$)

M1

$27x^2 - 18xy$ (= 0) or $27x^2 = 18xy$
or better
e.g.1 $9x^2 - 6xy$ (= 0)
e.g.2 $3x^2 = 2xy$

$64x - 16y = 36x + x + 2y$
or equivalent linear equation
e.g.1 $64x - 16y - 36x = x + 2y$
e.g.2 $64x - 16y - x - 2y = 36x$

A1

Any correct factorisation of
their $px^2 + qxy$ or correct division of
their $px^2 = qxy$ by a multiple of x
(p and q non zero)
e.g.1 $9x(3x - 2y)$ (= 0)
e.g.2 $3x(9x - 6y)$ (= 0)
e.g.3 $27x = 18y$
e.g.4 $9x = 6y$

Correct collection and correct simplification of terms for their linear equation in x and y
e.g. $27x = 18y$
To gain this mark there must have been some expansion of brackets seen

M1

$3x = 2y$ **or** $\frac{x}{y} = \frac{2}{3}$ **or** $\frac{y}{x} = \frac{2}{3}$

or $x = \frac{2}{3}y$ or $y = \frac{3}{2}x$ **or**

$\frac{x}{2} = \frac{y}{3}$ **or** $\frac{2}{x} = \frac{3}{y}$

Must see M1 M1 M1 A1
Do not allow if a contradictory statement is also seen

M1

[6]

M7. $(3n + 5)(n - 1)$ or $3n \times n$

M1

$(3n + 5)(n - 1) - 3n \times n$

M1dep

$$\frac{3n^2 - 3n + 5n - 5 - 3n^2}{3n(n-1)}$$

or $$\frac{3n^2 + 2n - 5 - 3n^2}{3n(n-1)}$$

Denominator used

A1

Additional Guidance

Ignore repetition of right hand side – see script S3

[3]

M8.

(a) $2n + 1$ **or** $1 + 2n$

B1

(b) $(2n - 1)(2n + 1)$

oe e.g. $(2n + 1)(2n + 3)$ ***or*** $(2n - 3)(2n - 1)$

M1

$4n^2 - 2n + 2n - 1$ **or** $4n^2 - 1$

oe e.g. $4n^2 + 2n + 6n + 3$

or $4n^2 - 6n - 2n + 3$

A1

$4n^2$ is a multiple of 4

so $4n^2 - 1$ is one less

oe clear explanation from their (correct) expression

A1

[4]

M9. $\frac{n(n-1)+n(n+1)}{2}$

This mark is for combining fractions ***or*** *if fractions dealt with separately, for combining n^2 terms correctly*

$\frac{n^2-n+n^2+n}{4}$ *is B0 as incorrect combining of fractions*

B1

$\frac{n^2-n+n^2+n}{2} = \frac{2n^2}{2}$

This mark is for eliminating −n and n either by showing by crossing or writing on same line and writing next line without them

$\frac{n^2}{2} - \frac{n}{2} + \frac{n^2}{2} + \frac{n}{2}$

B1

$\frac{2n^2}{2} = n^2$

This mark is for cancelling 2 top and bottom

$\frac{n^2}{2} + \frac{n^2}{2} = n^2$

B1

Alternative Method

$\frac{n}{2}((n-1)+(n+1))$

This mark is for factorising out a common factor.

$\frac{n}{4}(n - 1 + n + 1)$ *is B0 as incorrect factorisation*

B1

$\frac{n}{2}$ (2n)

This mark is for combining terms inside bracket correctly

B1

n^2

1n^2 is OK

B1

[3]

M10.

$5n^2 - 5n + 3n - 3$

oe 4 terms with 3 correct including a term in n^2

M1

$5n^2 - 5n + 3n - 3$

Fully correct

oe e.g. $5n^2 - 2n - 3$

A1

$6n^2 - 3$

A1

$3(2n^2 - 1)$ **or** states that both terms are multiples of 3

oe

A1

[4]

M11. $8 \times \frac{1}{2} n(n + 1)$ (+1)

M1

$4n(n + 1)$ (+ 1)

or $4n^2 + 4n$ (+ 1)

)

M1dep

$(2n + 1)^2$ or $(2n + 1)(2n + 1)$

A1

$(2n + 1)^2$ is a square number

oe

or $2n + 1$ is odd

and odd × odd = odd

odd² = odd

or multiple of 4 is even

and even + 1 = odd

or

n(n + 1) is odd × even or even × odd

so n(n + 1) is even

or $4(n^2 + n)$ is even

and even + 1 = odd

***and** even × 4 = even*

***and** even + 1 = odd*

or $4n^2$ is even **and** $4n$ is even

and even + 1 = odd

A1

$(2n + 1)^2$ is a square number

and

or $2n + 1$ is odd

and odd × odd = odd

Strand (ii)

Both parts of the proof required.

or multiple of 4 is even

and even + 1 = odd

or

$n(n + 1)$ is odd × even or even × odd

so $n(n + 1)$ is even

or $4(n^2 + n)$ is even

and even + 1 = odd

***and** even × 4 = even*

***and** even + 1 = odd*

or $4n^2$ is even **and** $4n$ is even

and even + 1 = odd

SC1 for 8 × S = even

***and** even + 1 = odd*

Q1

[5]

M12. $(3n − 1)(n − 2)$ or $(3n + 1)n$

or $n(n − 2)$ as denominator on LHS

M1

$(3n − 1)(n − 2) − (3n + 1)n$

M1 dep

$3n^2 − 6n − n + 2$ or $− 3n^2 − n$

dep on first M1 only

M1 dep

$3n^2 − 6n − n + 2$ and $− 3n^2 − n$

Correct common denominators must be used for 4 marks to be awarded

A1

[4]

M13.

$(5n - 3)^2 + 1$

M1

$25n^2 - 15n - 15n + 9 + 1$

Allow one error

Must have an n^2 term

M1

$25n^2 - 30n + 10$

A1

$5(5n^2 - 6n + 2)$

oe

e.g. shows that all terms divide by 5 or explains why the expression is a multiple of 5

B1ft

Alternative method 1

Use of $an^2 + bn + c$ for terms of quadratic sequence

i.e. any one of

$a + b + c = 5$

$4a + 2b + c = 50$

$9a + 3b + c = 145$

M1

$3a + b = 45$

$5a + b = 95$

For eliminating c

M1

$25n^2 - 30n + 10$

A1

$5(5n^2 - 6n + 2)$

oe

e.g. shows that all terms divide by 5 or explains why the expression is a multiple of 5

B1ft

Alternative method 2

5 50 145 290

45 95 145

2nd difference of 50 ÷ 2 (= 25)

$25n^2$

M1

Subtracts their $25n^2$ from terms of sequence

−20 −50 −80

$-30n$

M1

$25n^2 - 30n + 10$

A1

$5(5n^2 - 6n + 2)$

oe

e.g. shows that all terms divide by 5 or explains why the expression is a multiple of 5

B1ft

[4]