Mark schemes

Q1.

(a) $\overrightarrow{BE} = \frac{2}{3}$ **a or** $\overrightarrow{AE} = \frac{5}{3}$ **a**

oe

B1

− **a** − $\overrightarrow{BE}$ + **b**

or − their $\overrightarrow{AE}$ + **b**

M1

$-\frac{5}{3}$ **a** + **b** or **b** − $\frac{5}{3}$ **a**

A1

(b) $\overrightarrow{EF} = \frac{2}{5}\ \overrightarrow{ED}$

M1

$-\frac{2}{3}$ **a** + $\frac{2}{5}$ **b**

oe

ft their $\overrightarrow{ED}$

A1ft

[5]

Q2.

(a) 2**b** – 2**a** or –2**a** + 2**b**
or 2(**b** – **a**) or 2(–**a** + **b**)

B1

(b) **Alternative method 1**

MA + *AN*

or $\frac{1}{2}$ *OA* + $\frac{1}{2}$ *AB*

or **a** + $\frac{1}{2}$ their (2**b** – 2**a**)

oe

M1

a + **b** – **a**

A1

Alternative method 2

(M is midpoint of OA and N is midpoint of AB)

(hence) $MN = \frac{1}{2} OB$

M1

$MN = \frac{1}{2} \times 2\mathbf{b}$

By midpoint theorem, triangle AOB is an enlargement sf 2 of triangle AMN is M1, A1

M1

(c) **Alternative method 1**

Common angle *MAN*
or (Angle) *AMN* = (Angle) *AOB* because corresponding
or (Angle) *ANM* = (Angle) *ABO* because corresponding

Must be a specific angle shown to be common and if not MAN then reason ie corresponding must be stated

Check diagram if reference to say, 'x is a common angle'

B1

Sides in ratio 1 : 2

oe eg scale factor 2

B1

Alternative method 2

$\overrightarrow{OB} = 2\overrightarrow{MN}$ and $\overrightarrow{OA} = 2\overrightarrow{OM}$

Any two sides shown to be parallel vectors

oe eg $\overrightarrow{OB} = 2\mathbf{b}$, $\overrightarrow{MN} = \mathbf{b}$ and $\overrightarrow{AB} = 2\mathbf{b} - 2\mathbf{a}$,

$\overrightarrow{AN} = \mathbf{b} - \mathbf{a}$

B2

[5]

Q3.

Alternative method 1 Shows that *CB* (or *BC*) is equal and parallel to *DE* (or *ED*)

$(\overrightarrow{CB} =) -(\mathbf{b} - 2\mathbf{a}) - 2\mathbf{b} - \mathbf{a}$
or $(\overrightarrow{BC} =)\ \mathbf{b} - 2\mathbf{a} + 2\mathbf{b} + \mathbf{a}$

oe method

M1

$(\overrightarrow{CB} =)\ \mathbf{a} - 3\mathbf{b}$
or $(\overrightarrow{BC} =)\ 3\mathbf{b} - \mathbf{a}$

Must see correct method for $\overrightarrow{CB}$ or $\overrightarrow{BC}$

A1

CB is equal and parallel to *DE*

Must see a correct vector for first A1 and have a statement
oe e.g. CB is equal and parallel to ED

A1

Alternative method 2 Shows that *BE* (or *EB*) is equal and parallel to *CD* (or *DC*)

($\overrightarrow{BE}$ =) $\mathbf{a} + 2\mathbf{b}$
or ($\overrightarrow{CD}$ =) $-(\mathbf{b} - 2\mathbf{a}) - (\mathbf{a} - 3\mathbf{b})$
or ($\overrightarrow{EB}$ =) $-\mathbf{a} - 2\mathbf{b}$
or ($\overrightarrow{DC}$ =) $(\mathbf{a} - 3\mathbf{b}) + (\mathbf{b} - 2\mathbf{a})$

oe method

M1

($\overrightarrow{BE}$ =) $\mathbf{a} + 2\mathbf{b}$
and ($\overrightarrow{CD}$ =) $\mathbf{a} + 2\mathbf{b}$
or
($\overrightarrow{EB}$ =) $-\mathbf{a} - 2\mathbf{b}$
and ($\overrightarrow{DC}$ =) $-\mathbf{a} - 2\mathbf{b}$

Must see correct method for $\overrightarrow{CD}$ or $\overrightarrow{DC}$
oe eg ($\overrightarrow{BE}$ =) $\mathbf{a} + 2\mathbf{b}$ and ($\overrightarrow{DC}$ =) $-\mathbf{a} - 2\mathbf{b}$

A1

BE is equal and parallel to *CD*

Must see two correct vectors for first A1 and have a statement
oe e.g. BE is equal and parallel to DC

A1

Alternative method 3 Shows that two pairs of opposite sides are parallel

($\overrightarrow{CB}$ =) $-(\mathbf{b} - 2\mathbf{a}) - 2\mathbf{b} - \mathbf{a}$
or ($\overrightarrow{BC}$ =) $\mathbf{b} - 2\mathbf{a} + 2\mathbf{b} + \mathbf{a}$
or ($\overrightarrow{BE}$ =) $\mathbf{a} + 2\mathbf{b}$
or ($\overrightarrow{CD}$ =) $-(\mathbf{b} - 2\mathbf{a}) - (\mathbf{a} - 3\mathbf{b})$
or ($\overrightarrow{EB}$ =) $-\mathbf{a} - 2\mathbf{b}$
or ($\overrightarrow{DC}$ =) $(\mathbf{a} - 3\mathbf{b}) + (\mathbf{b} - 2\mathbf{a})$

oe method

M1

($\overrightarrow{CB}$ =) $\mathbf{a} - 3\mathbf{b}$
or
($\overrightarrow{BC}$ =) $3\mathbf{b} - \mathbf{a}$
or
($\overrightarrow{BE}$ =) $\mathbf{a} + 2\mathbf{b}$
and ($\overrightarrow{CD}$ =) $\mathbf{a} + 2\mathbf{b}$
or

$(\overrightarrow{EB} =)$ $-\mathbf{a} - 2\mathbf{b}$

and $(\overrightarrow{DC} =)$ $-\mathbf{a} - 2\mathbf{b}$

Must see correct method for $\overrightarrow{CB}$ or $\overrightarrow{BC}$
or $\overrightarrow{CD}$ or $\overrightarrow{DC}$
oe eg $(\overrightarrow{BE} =)$ $\mathbf{a} + 2\mathbf{b}$ and $(\overrightarrow{DC} =)$ $-\mathbf{a} - 2\mathbf{b}$

A1

$(\overrightarrow{CB} =)$ $\mathbf{a} - 3\mathbf{b}$
and $(\overrightarrow{BE} =)$ $\mathbf{a} + 2\mathbf{b}$
and $(\overrightarrow{CD} =)$ $\mathbf{a} + 2\mathbf{b}$
and *CB* is parallel to *DE*
and *BE* is parallel to *CD*

Must see three correct vectors and have two statements
oe eg $(\overrightarrow{BC} =)$ $3\mathbf{b} - \mathbf{a}$
and $(\overrightarrow{BE} =)$ $\mathbf{a} + 2\mathbf{b}$
and $(\overrightarrow{DC} =)$ $-\mathbf{a} - 2\mathbf{b}$
and BC is parallel to DE
and BE is parallel to DC

A1

Alternative method 4 Shows that two pairs of opposite sides are equal

$(\overrightarrow{CB} =)$ $-(\mathbf{b} - 2\mathbf{a}) - 2\mathbf{b} - \mathbf{a}$
or $(\overrightarrow{BC} =)$ $\mathbf{b} - 2\mathbf{a} + 2\mathbf{b} + \mathbf{a}$
or $(\overrightarrow{BE} =)$ $\mathbf{a} + 2\mathbf{b}$
or $(\overrightarrow{CD} =)$ $-(\mathbf{b} - 2\mathbf{a}) - (\mathbf{a} - 3\mathbf{b})$
or $(\overrightarrow{EB} =)$ $-\mathbf{a} - 2\mathbf{b}$
or $(\overrightarrow{DC} =)$ $(\mathbf{a} - 3\mathbf{b}) + (\mathbf{b} - 2\mathbf{a})$

oe

M1

$(\overrightarrow{CB} =)$ $\mathbf{a} - 3\mathbf{b}$
or
$(\overrightarrow{BC} =)$ $3\mathbf{b} - \mathbf{a}$
or
$(\overrightarrow{BE} =)$ $\mathbf{a} + 2\mathbf{b}$
and $(\overrightarrow{CD} =)$ $\mathbf{a} + 2\mathbf{b}$
or
$(\overrightarrow{EB} =)$ $-\mathbf{a} - 2\mathbf{b}$
and $(\overrightarrow{DC} =)$ $-\mathbf{a} - 2\mathbf{b}$

Must see correct method for $\overrightarrow{CB}$ or $\overrightarrow{BC}$
or $\overrightarrow{CD}$ or $\overrightarrow{DC}$
oe eg $(\overrightarrow{BE} =)$ $\mathbf{a} + 2\mathbf{b}$ and $(\overrightarrow{DC} =)$ $-\mathbf{a} - 2\mathbf{b}$

A1

($\overrightarrow{CB}$ =) **a** – 3**b**
and ($\overrightarrow{BE}$ =) **a** + 2**b**
and ($\overrightarrow{CD}$ =) **a** + 2**b**
and *CB* is equal to *DE*
and *BE* is equal to *CD*

Must see three correct vectors and have two statements

oe eg ($\overrightarrow{BC}$ =) 3**b** – **a**
and ($\overrightarrow{BE}$ =) **a** + 2**b**
and ($\overrightarrow{DC}$ =) –**a** – 2**b**
and BC is equal to DE
and BE is equal to DC

A1

Additional Guidance

Choose the method that gives most marks

Ignore incorrect vectors if not contradictory

For parallel allow in the same direction or in the opposite direction

For equal to allow = or the same as

Condone incorrect notation if unambiguous
eg *CB* = –(b – 2a) – 2b – a

M1

[3]

Q4.

(a) 4**b**

B1

(b) ($\overrightarrow{ED}$ =) $\frac{1}{3}$ (**a** + 3**b**) or ($\overrightarrow{ED}$ =) $\frac{1}{3}$ **a** + **b**

B1

$\overrightarrow{EC}$ = their ($\frac{1}{3}$ **a** + **b**) − $\frac{1}{3}$ **a**
or $\overrightarrow{EC}$ = **b**

M1

Valid justification

eg $\overrightarrow{ED}$ = $\frac{1}{3}$ ***a*** + ***b*** *and* $\overrightarrow{EC}$ = ***b***
and $\overrightarrow{AB}$ = 4 $\overrightarrow{EC}$ *(so* $\overrightarrow{AB}$ *is a multiple of* $\overrightarrow{EC}$*)*

A1

[4]

Q5.

(a) Opposite sides parallel (same direction) and equal (same length)
or opposite sides are equal vectors

*Strand (i). Must mention that opposite sides are parallel and equal **or** equal vectors*

Q1

(b) **b − c** or **−c + b**

B1

(c) $LP = \frac{1}{2}\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$

LP = must be stated or LP = LA + AP

B1 for $\frac{1}{2}\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$

B2

Alternative 1

$\frac{1}{2}\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a}) = \mathbf{a} + \frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{a}$

B1 for $\frac{1}{2}\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$

B2

Alternative 2

$(LP) = -\frac{1}{2}\mathbf{a} + \mathbf{b} + (\mathbf{c} - \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{c})$

This is LP = LO + OB + BC + CP

M1

$-\frac{1}{2}\mathbf{a} + \mathbf{b} + \mathbf{c} - \mathbf{b} + \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{c}$

A1

Alternative 3

$(LP) = -\frac{1}{2}\mathbf{a} + \mathbf{c} + \frac{1}{2}(\mathbf{a} - \mathbf{c})$

This is LP = LO + OC + CP

M1

$-\frac{1}{2}\mathbf{a} + \mathbf{c} + \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{c}$

A1

Alternative 4

OC = **c** and *L* and *P* are midpoints

Using midpoint theorem. This may be expressed differently but if evidence that mid-point theorem used then award M1

M1

$LP = \frac{1}{2}OC$

This is for accurately describing the results using the *mid-point theorem.*

A1

Alternative 5

Written explanation such as
(Journey of) *L* to *A* to *P* is half (the journey of) *O* to *A* to *C* so *LP* is half *OC*.

B1 if intention seen but explanation not complete or slight error

B2

(d) $MN = \frac{1}{2}\mathbf{b} + \frac{1}{2}(\mathbf{c} - \mathbf{b})$

M1

$LP = MN = \frac{1}{2}\mathbf{c}$ …… *LMNP* is a

parallelogram (as opposite sides are the same vector)

By choosing MN it is opposite LP so no need to say opposite sides but a 'conclusion' must be stated or implied

A1

Alternative 1

$LM = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

M1

$LM = PN = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ …… *LMNP* is a parallelogram

(as opposite sides are the same vector).

By choosing LM and PN no need to say opposite sides but a 'conclusion' must be stated or implied

A1

Alternative 2

LP parallel to *OC* and $\frac{1}{2}OC$ (midpoint theorem)

M1

MN parallel to *OC* and $\frac{1}{2}OC$ (midpoint theorem)

so *LMNP* is a parallelogram as opposite sides parallel and the same length

A1

[6]

Q6.

(a) $\overrightarrow{BC} = 2\mathbf{a} - 3\mathbf{b}$ or

$\overrightarrow{CB} = -2\mathbf{a} + 3\mathbf{b}$ or

$\overline{AM} = \mathbf{a}$ or $\overline{MA} = -\mathbf{a}$ or

$\overline{BN} = \frac{2}{5}\overline{BC}$ or $\overline{CN} = -\frac{3}{5}\overline{BC}$

oe

M1

$\mathbf{a} + \frac{3}{5}(-2\mathbf{a} + 3\mathbf{b})$

$-\mathbf{a} + 3\mathbf{b} + \frac{2}{5}(2\mathbf{a} - 3\mathbf{b})$

oe

M1

$-\frac{1}{5}\mathbf{a} + \frac{9}{5}\mathbf{b}$

oe eg $-0.2\mathbf{a} + 1.8\mathbf{b}$ or $\frac{1}{5}(9\mathbf{b} - \mathbf{a})$

Must collect terms

A1

(b) $\overrightarrow{MN}$ is not a multiple of $\overrightarrow{AB}$

oe

B1ft

[4]

Q7.

(a) *MN* = ½***x*** + ½***y***

oe
MN = ½*BC* + ½*CD*
MN = *MC* **+** *CN*

B1

BD = ***x* + *y***

oe
BD = *BC* + *CD*

B1

BD is a multiple of *MN*

oe

Q1

(b) 2 : 1

B1

[4]

Q8.

(a) $\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

oe

B1

$\overline{QS} = -\boldsymbol{a} + \boldsymbol{b}$

or $\overline{SQ} = \boldsymbol{a} - \boldsymbol{b}$

oe

M1

$\overline{QN} = -\frac{1}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

or $\overline{SN} = \frac{2}{3}\boldsymbol{a} - \frac{2}{3}\boldsymbol{b}$

oe

M1dep

(b) $\overline{PN} = \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

or $\overline{NM} = \frac{1}{3}\boldsymbol{a} + \frac{1}{6}\boldsymbol{b}$

oe

A1

Valid reason

Strand (ii)
e.g. PN is a multiple of PM
PN is a multiple of NM

$\overline{PN} = \frac{1}{3}(2\boldsymbol{a} + \boldsymbol{b})$ *and* $\overline{PM} = \frac{1}{2}(2\boldsymbol{a} + \boldsymbol{b})$

$\overline{PN} = \frac{2}{3}(\boldsymbol{a} + \frac{1}{2}\boldsymbol{b})$ *and* $\frac{2}{3}\,\overline{PM}$

Q1

[5]

Q9.

(a) 5**a** + 3**b** + 6**a** − **7**b

M1

11**a** − 4**b**

A1

(b) 22

ft their 11 × 8 ÷ their 4
*Accept 22**a** (− 8**b**)*

B1 ft

[3]