OCR
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H

Monday 09 November 2020 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

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J560006

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

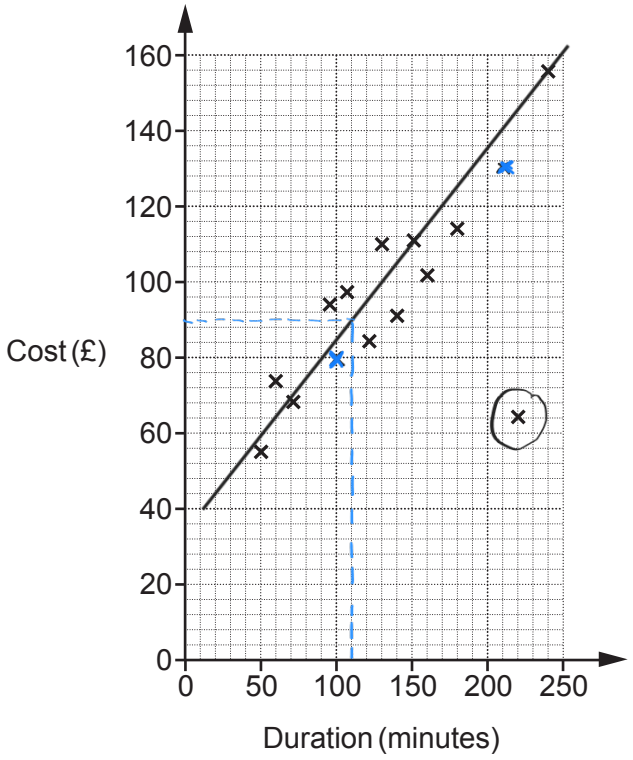
INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 A travel agent records the duration and cost of the 15 flights he sold on one day.
The data for the first 13 flights are plotted on the scatter diagram.



(a) The data for the final two flights is:

Duration	210 minutes	1 hour 40 minutes
Cost	£130	£80

(100 minutes)

Plot these flights on the scatter diagram. **[2]**

(b) The cost of one of the 15 flights had been discounted in a sale.

Circle the most likely flight on the scatter diagram. **[1]**

(c) **(i)** Draw a line of best fit on the scatter diagram. **[1]**

(ii) Use your line of best fit to estimate the duration of a flight costing £90.

(c)(ii) 110 minutes **[1]**

(d) Explain why the travel agent should not use his records to estimate the cost of a 7 hour flight.

7 hours is beyond the given data

[1]

2 Solve $3x + 4 < 19$.
Show your solution on the number line.

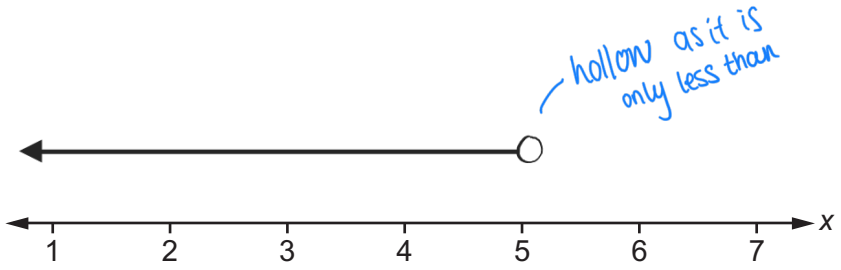
$3x + 4 < 19$

$3x < 19 - 4$

$3x < 15$

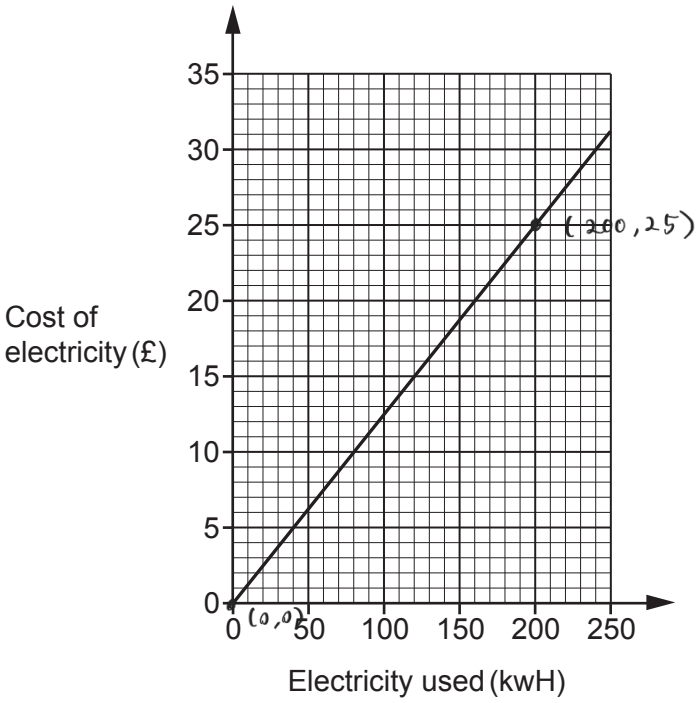
$x < 15 \div 3$

$x < 5$



[4]

3 The graph shows the cost of electricity with Company A.



(a) Use the information in the graph to estimate the cost of electricity for a customer who uses 450 kwH of electricity.

gradient $= \frac{25-0}{200-0} = \frac{25}{200} = \frac{1}{8}$

y-intercept $= 0$

$y = mx + c$

$y = \frac{1}{8}x$

$y = \frac{1}{8}x$

$y = \frac{1}{8}(450)$

$y = 56.25$

(a) £ 56.25 **[3]**

(b) Company B charges 14.3 pence per kwH of electricity used.

If Company B's cost of electricity was plotted on the same axes as Company A's cost of electricity, which line would be steeper?
Explain how you know.

Company A has a gradient of $\frac{1}{8}$ which is equal to £0.125

Company A = 12.5 pence per kwH

Company B = 14.3 pence per kwH

Company B would have the steeper line because it has a greater gradient due to the higher cost of electricity which is 14.3 pence per kwH compared to 12.5 pence per kwH for Company A. **[3]**

4 Nina estimates the value of $\sqrt{\left(\frac{3.93 \times 393}{0.546 \times 220}\right)^3}$ by rounding each number to 1 significant figure.

(a) Show that Nina's answer is 64. **[3]**

$3.93 \approx 4$ (1sf)
$393 \approx 400$ (1sf)
$0.546 \approx 0.5$ (1sf)
$220 \approx 200$ (1sf)

$= \sqrt{\left(\frac{4 \times 400}{0.5 \times 200}\right)^3}$

$= \sqrt{\left(\frac{1600}{100}\right)^3}$

$= \sqrt{(16)^3}$

$= \sqrt{4096}$

$= 64$

(b) Calculate the error in her estimated answer as a percentage of the exact answer.

The exact answer :

$= \sqrt{\left(\frac{3.93 \times 393}{0.546 \times 220}\right)^3}$

$= \sqrt{(12.8579)^3}$

$= 46.1057$

≈ 46.11

Error : $64 - 46.11$

$= 17.89$

Percentage error $= \frac{17.89}{46.11} \times 100$

$= 38.798$

$\approx 38.8\%$

(b) 38.8 % **[4]**

5 **(a)** Work out the size of the exterior angle of a regular 12-sided polygon.

$$\text{Size of exterior angle} = \frac{360^\circ}{\text{number of sides}}$$

$$= \frac{360^\circ}{12}$$

$$= 30^\circ$$

(a) 30 ° **[2]**

(b) Use your answer to part **(a)** to write down the size of the interior angle of a regular 12-sided polygon.

$$\text{Interior angle} = 180^\circ - 30^\circ$$

$$= 150^\circ$$

(b) 150 ° **[1]**

6 A truck is used to transport some wood panels.
Each wood panel is a cuboid measuring 2.4 m by 1.2 m by 1.8 **cm**.
The density of each wood panel is 750 kg/m^3.

The truck can carry 15 tonnes of these wood panels.

Calculate the maximum number of wood panels that the truck can carry.
Show how you decide.

convert to m

Volume of each wood panel = $2.4 \times 1.2 \times (1.8 \div 100)$

$= 2.4 \times 1.2 \times 0.018$

$= 0.05184 \text{ m}^3$

Mass of wood panel = Density × volume

$= 750 \times 0.05184$

$= 38.88 \text{ kg}$

1 tonne = 1000 kg

15 tonne = 15 000 kg

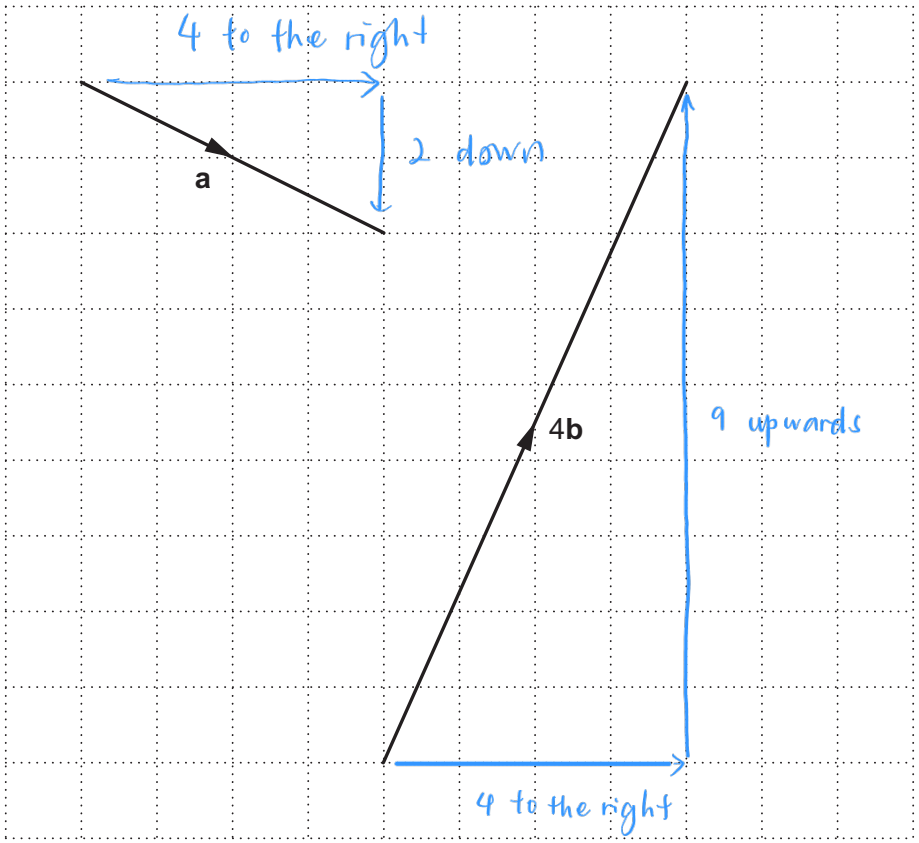
Maximum number of wood panels that can be carried $= \frac{15000}{38.88}$

$= 385.802$ (round down)

$\approx$ 385 wood panels

385 wood panels ..... **[6]**

7 Vectors **a** and 4**b** are drawn on the grid.



(a) Write vector **a** as a column vector.

(a) $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ **[2]**

(b) Find vector **b** as a column vector.

$4\mathbf{b} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$

$\mathbf{b} = \begin{pmatrix} 1 \\ 9/4 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ \frac{9}{4} \end{pmatrix}$ **[2]**

8 Li has t toy bricks.
She only has red bricks and blue bricks.

Li picks two bricks, one after the other.

If the first brick she picks is red, the probability that the second brick is red is $\frac{2}{3}$.

If the first brick she picks is blue, the probability that the second brick is red is $\frac{7}{10}$.

Calculate the value of t.

t = toy bricks

r = red bricks

b = blue bricks

$t = r + b$

① $\frac{2}{3} = \frac{r-1}{t-1}$

$2(t-1) = 3(r-1)$

$2t - 2 = 3r - 3$

$2t - 2 + 3 = 3r$

$2t + 1 = 3r$

$3r = 2t + 1$

$r = \frac{2t+1}{3}$ — ①

② $\frac{7}{10} = \frac{r}{t-1}$

$7(t-1) = 10r$

$7t - 7 = 10r$

$r = \frac{7t-7}{10}$ — ②

Compare ① with ②

$\frac{2t+1}{3} = \frac{7t-7}{10}$

$10(2t+1) = 3(7t-7)$

$20t + 10 = 21t - 21$

$10 + 21 = 21t - 20t$

$31 = t$

$t =$ 31 **[4]**

9 x is directly proportional to y.
y is directly proportional to z.

When $x = 10$, $y = 60$.
When $y = 8$, $z = 1.6$.

Find a formula for z in terms of x.

① $x = ky$

$10 = k(60)$

$k = \frac{10}{60} = \frac{1}{6}$

$x = \frac{1}{6}y$

$6x = y$ — ①

② $y = kz$

$8 = k(1.6)$

$k = \frac{8}{1.6} = 5$

$y = 5z$ — ②

Substitute ② into ①

$6x = 5z$

$5z = 6x$

$z = \frac{6}{5}x$

$z = \frac{6}{5}x$

..... **[4]**

10 Paintings are sold in an art gallery.
The cost of a painting has k% commission added to it.
Tax of 15% is then added to the total cost to give the price to pay.

Layla correctly calculates the price to pay by multiplying the cost of the painting by 1.403.

Work out the value of k.

Cost of the painting $= x$

Commission $= \frac{k}{100} \times x = \frac{xk}{100}$

Cost of the painting with commission $= x + \frac{xk}{100}$

$= x\left(1 + \frac{k}{100}\right)$

Price to pay after tax $= x\left(1 + \frac{k}{100}\right) \times 1.15$

$= 1.15x\left(1 + \frac{k}{100}\right)$

Price to pay $: x(1.403)$

$= 1.403x$

$1.403x = 1.15x\left(1 + \frac{k}{100}\right)$

$\frac{1.403x}{1.15x} = 1 + \frac{k}{100}$

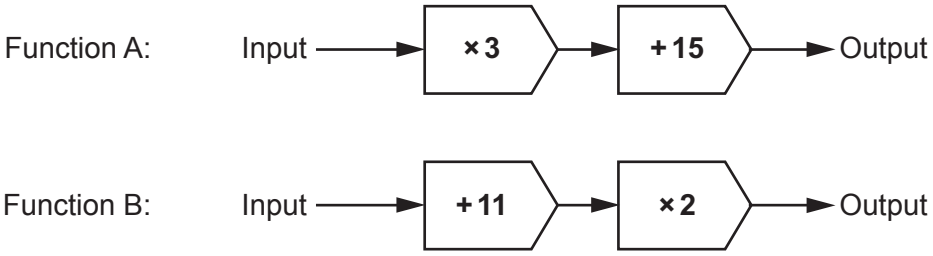
$1.22 = 1 + \frac{k}{100}$

$0.22 = \frac{k}{100}$

$\frac{22}{100} = \frac{k}{100}$ $\quad k = 22$

$k =$ 22 ..... **[3]**

11 Here are two functions.



(a) (i) Jo chooses a number, x.
She inputs x into each function.
The two outputs are equal.

Work out the value of x.

Function A : $x \times 3 + 15$

: $3x + 15$

Function B : $x + 11 \times 2$

: $2(x + 11)$

$3x + 15 = 2(x + 11)$ (Outputs are equal)

$3x + 15 = 2x + 22$

$3x - 2x = 22 - 15$

$x = 7$

(a)(i) $x =$ 7 ..... **[4]**

(ii) Explain why there is no other input that gives two outputs that are equal.

Because $3x + 15 = 2(x + 11)$ only has one solution.

..... **[1]**

(b) Here is function C.

Function C: Input → **+ *p*** → **× *q*** → Output

Kai chooses values for p and q so that if he inputs **any** number into both function A and function C, he will **always** get two outputs that are equal.

Find the value of p and the value of q.

Input = x

Function A = $x \times 3 + 15 = 3x + 15$

Function C = $x + p \times q = q(x + p)$

$3x + 15 = q(x + p)$

$3x + 15 = qx + qp$

$q = 3$

$qp = 15$

$3p = 15$

$p = 5$

(b) $p =$ 5

$q =$ 3 **[3]**

12 Students are asked to choose one subject from Option A and one subject from Option B.

Option A
Economics
Geography
History
Media Studies

Option B
Art
Drama
Engineering
German
Graphics
Music
PE

If a student chooses their subjects at random, what is the probability that both subjects have the same first letter?

Choosing both subjects with letter E :

$\frac{1}{4} \times \frac{1}{7} = \frac{1}{28}$

Choosing both subjects with letter G :

$\frac{1}{4} \times \frac{2}{7} = \frac{2}{28}$

Choosing both subjects with letter M :

$\frac{1}{4} \times \frac{1}{7} = \frac{1}{28}$

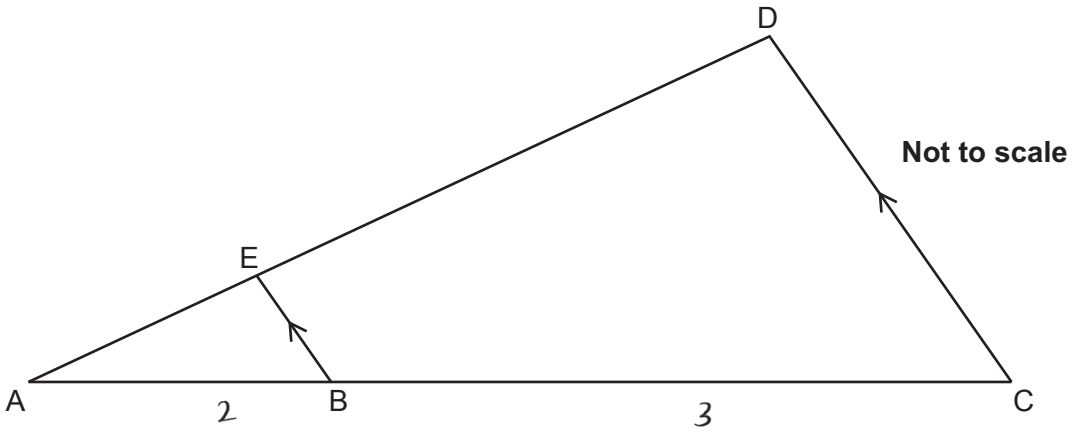
Add all the probabilities together :

$\frac{1}{28} + \frac{2}{28} + \frac{1}{28} = \frac{4}{28} = \frac{1}{7}$

$\frac{1}{7}$

..... **[3]**

13 In the diagram, AED and ABC are straight lines and BE is parallel to CD.



The ratio of length AB to length BC is 2 : 3.
Triangle ABE has an area of $8\,cm^2$.

Work out the area of triangle ACD.

AB = 2

AC = 2 + 3 = 5

Ratio AB : AC

2 : 5

linear factor from AB to AC is $5/2$

Square factor = $(5/2)^2 = \frac{25}{4}$

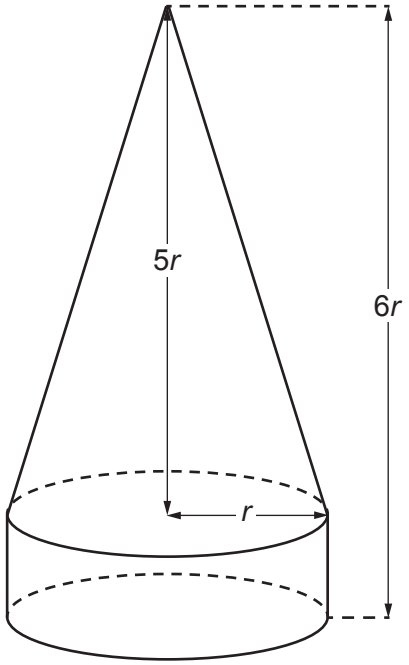
Area of ABE = $8\ cm^2$

Area of ACD = $8 \times \frac{25}{4_1}$

$= 50\ cm^2$

50 ..... cm^2 **[4]**

14 The base of a cone is fixed to the top of a cylinder to make a decoration.



The radius of the base of the cone and of the cylinder is r cm.
The cone's height is $5r$ cm.
The total height of the decoration is $6r$ cm.
The total volume of the decoration is $225\,\text{cm}^3$.

Calculate the value of r.
Show your working.

[The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (5r)$

$= \frac{5\pi r^3}{3}$

Volume of cylinder $= \pi r^2 h$

$= \pi r^2 (6r - 5r)$

$= \pi r^2 (r)$

$= \pi r^3$

Total volume $= \frac{5\pi r^3}{3} + \pi r^3$

$= \frac{5\pi r^3 + 3\pi r^3}{3}$

$225 = \frac{8\pi r^3}{3}$

$225 \times 3 = 8\pi r^3$

$675 = 8\pi r^3$

$r^3 = \frac{675}{8\pi}$

$r^3 = 26.857$

$r = \sqrt[3]{26.857}$

$r = 2.99$

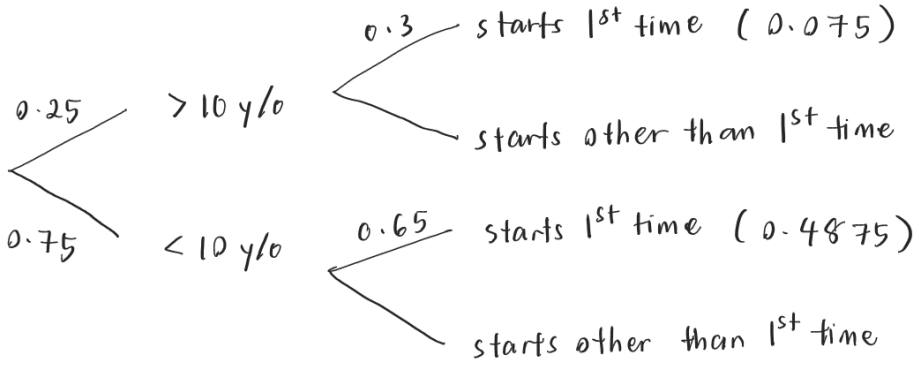
$r =$ 2.99 ..... **[5]**

15 A bus company has a large number of buses.
25% of the buses are more than 10 years old.

If a bus is more than 10 years old, the probability that it will start first time is 0.3.
If a bus is less than 10 years old, the probability that it will start first time is 0.65.

Amir is asked to drive one of the company's buses, chosen at random.

Calculate the probability that the bus starts first time.



Probability of > 10 y/o bus starts first time :

$= 0.25 \times 0.3 = 0.075$

Probability of < 10 y/o bus starts first time :

$= 0.75 \times 0.65 = 0.4875$

Probability of getting a bus that starts first time

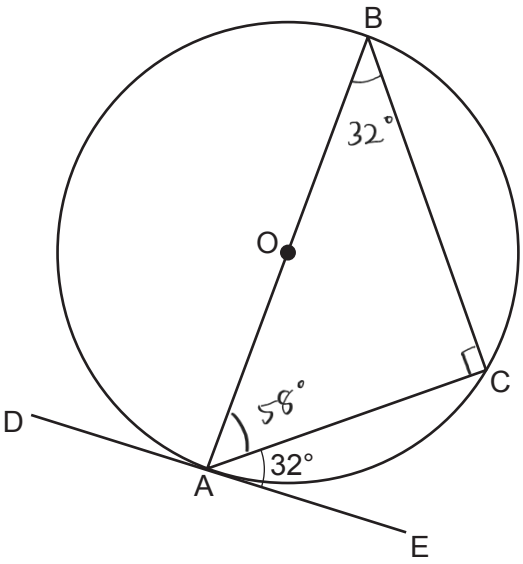
$= 0.4875 + 0.075$

$= 0.5625$

0.5625 ..... **[4]**

16 The diagram shows a circle, centre O.

Points A, B and C lie on the circumference of the circle.
Line AOB is a diameter.
Line DAE is a tangent to the circle.
Angle CAE = 32°.



Not to scale

(a) Give a reason why angle ACB is a right angle.

It is an angle in a semicircle

..... **[1]**

(b) The radius of the circle is 8 cm.

Calculate length BC.

Angle CAB = 90° − 32°
= 58°

AB = 2 × 8 = 16

$$\frac{BC}{\sin 58^\circ} = \frac{16}{\sin 90^\circ}$$

$$BC = \frac{16 \sin 58^\circ}{\sin 90^\circ} = 13.568 \approx 13.57$$

(b) 13.57 cm **[4]**

17 Here is a sequence.

3 $\quad 3\sqrt{5} \quad$ 15 $\quad 15\sqrt{5}$

(a) Work out the next term.

$15\sqrt{5} \times \sqrt{5} = 15 \times 5$

$= 75$

(a) 75 ..... **[1]**

(b) Find the *n*th term.

$a_n = a \times r^{n-1}$ → geometric sequence

$a = 3$

$r = \sqrt{5}$

n^{th} term $= 3 \times (\sqrt{5})^{n-1}$

(b) $3 \times (\sqrt{5})^{n-1}$ ..... **[3]**

18 Write $0.4\dot{1}\dot{6}$ as a fraction in its simplest form.
You must **show full working** in support of your answer.

$x = 0.41616\ldots$

$1000x = 416.16\ldots$

$10x = 4.1616\ldots$

$1000x - 10x = 416.16\ldots - 4.16\ldots$

$990x = 412$

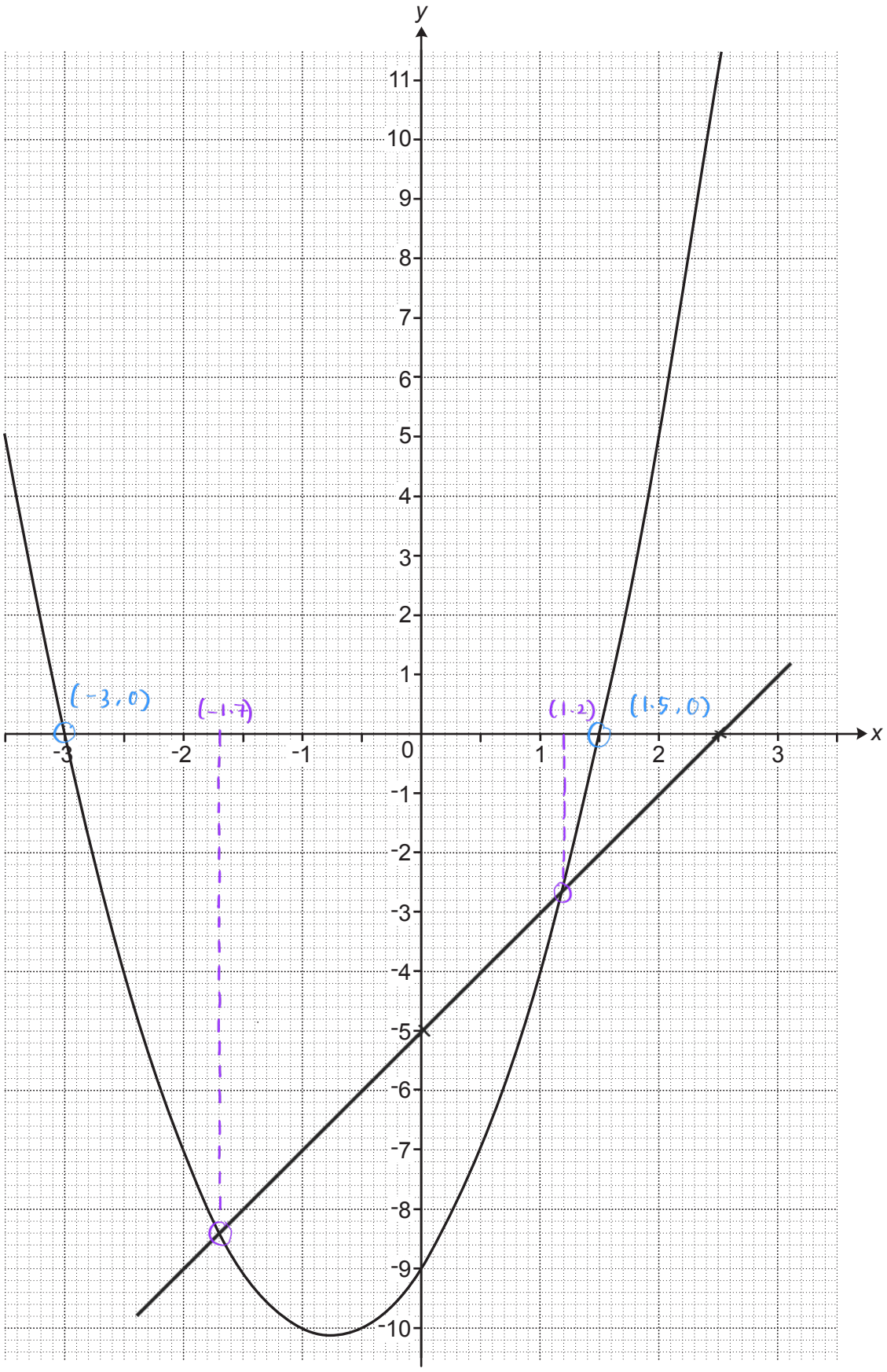
$x = \frac{412}{990}$

$x = \frac{206}{495}$

$\frac{206}{495}$

..... **[3]**

19 The graph of $y = 2x^2 + 3x - 9$ is drawn below.



(a) Use the graph to solve $2x^2 + 3x - 9 = 0$.

(a) $x =$ −3 or $x =$ 1.5 **[2]**

(b) The equation $2x^2 + x - 4 = 0$ can be solved by finding the intersection of the graph of $y = 2x^2 + 3x - 9$ and the line $y = ax + b$.

(i) Find the value of a and the value of b.

$2x^2 + 3x - 9 = ax + b$

$2x^2 + 3x - ax - 9 - b = 0$

$2x^2 + (3-a)x - 9 - b = 0$

$2x^2 + (3-a)x - 9 - b = 2x^2 + x - 4$

$3 - a = 1$ $\qquad$ $-9 - b = -4$

$3 - 1 = a$ $\qquad$ $-9 + 4 = b$

$a = 2$ $\qquad$ $b = -5$

(b)(i) $a =$ 2

$b =$ −5 **[2]**

(ii) Hence **use the graph** to solve the equation $2x^2 + x - 4 = 0$.

$y = 2x - 5$

when $y = 0$,

$0 = 2x - 5$

$x = 5/2$

(ii) $x =$ 1.2 or $x =$ −1.7 **[3]**

20 Vector $\mathbf{m} = \begin{pmatrix} 2 \\ k \end{pmatrix}$ and vector $\mathbf{n} = \begin{pmatrix} 3 \\ 11 \end{pmatrix}$.

Vector $2\mathbf{m} + \mathbf{n}$ is parallel to $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

Find the value of k.

$$2m+n = 2\begin{pmatrix} 2 \\ k \end{pmatrix} + \begin{pmatrix} 3 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 2k \end{pmatrix} + \begin{pmatrix} 3 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 2k+11 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 2k+11 \end{pmatrix} = a\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$7 = a(1)$$

$$a = 7$$

$$2k+11 = a(-1)$$

$$2k+11 = 7(-1)$$

$$= -7$$

$$2k = -7-11$$

$$= -18$$

$$k = -18/2$$

$$k = -9$$

$k =$ -9 ..... **[4]**

21 Write as a single fraction in its simplest form.

$$\frac{x}{x+2}+\frac{x+1}{x-2}-\frac{6x}{x^2-4}$$

$$= \frac{x}{x+2}+\frac{x+1}{x-2}-\frac{6x}{x^2-4}$$

$$= \frac{x(x-2)+(x+2)(x+1)}{(x+2)(x-2)}-\frac{6x}{x^2-4}$$

$$= \frac{x^2-2x+x^2+x+2x+2}{(x+2)(x-2)}-\frac{6x}{x^2-4}$$

$$= \frac{x^2+x^2-2x+x+2x+2}{(x+2)(x-2)}-\frac{6x}{x^2-2^2}$$ (factorise this)

$$= \frac{2x^2+x+2}{(x+2)(x-2)}-\frac{6x}{(x+2)(x-2)}$$

$$= \frac{2x^2+x-6x+2}{(x+2)(x-2)}$$

$$= \frac{2x^2-5x+2}{(x+2)(x-2)}$$ → factorise this

$$= \frac{2x^2-x-4x+2}{(x+2)(x-2)}$$

$$= \frac{x(2x-1)-2(2x-1)}{(x+2)(x-2)}$$

$$= \frac{(2x-1)(x-2)}{(x+2)(x-2)}$$ (cancel like terms) $$= \frac{2x-1}{x+2}$$

$\frac{2x-1}{x+2}$

..... **[6]**

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If you need extra space use the following lined pages. The question numbers must be clearly shown.