

**Tuesday 11 June 2019 – Morning**

**GCSE (9–1) Mathematics**

**model solutions**

**J560/06 Paper 6 (Higher Tier)**

**Time allowed: 1 hour 30 minutes**



**You may use:**

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

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Last name

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**INSTRUCTIONS**

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).

**INFORMATION**

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- Use the  $\pi$  button on your calculator or take  $\pi$  to be 3.142 unless the question says otherwise.
- This document consists of **24** pages.

Answer **all** the questions.

- 1 A grain of salt weighs  $6.48 \times 10^{-5}$  kg on average.  
A packet contains 0.35 kg of salt.

(a) Use this information to calculate the number of grains of salt in the packet.

1a)  $1 \text{ grain} = 6.48 \times 10^{-5} \text{ kg}$   
 $\times 5401.234 \left( \right. 5401 \text{ grains} = 0.35 \text{ kg} \left. \right) \times 5401.234 \dots$

(a) 5401 ..... [2]

- (b) Explain why your answer to part (a) is unlikely to be the actual number of grains of salt in the packet.

not every grain of sand weighs  
 $6.48 \times 10^{-5}$  kg - this is an average .....  
 and the individual grains .....  
 might instead weigh less/more. .... [1]

3

2 Tom researches the weights of plant seeds.

- One poppy seed weighs  $3 \times 10^{-4}$  grams.
- 250 pumpkin seeds weigh 21 grams.
- One sesame seed weighs  $3.64 \times 10^{-6}$  kilograms.

Write the three types of seed in order according to the weight of one seed.

Write the lightest type of seed first.

You must show how you decide.

$$1 \text{ poppy seed} : 3 \times 10^{-4} \text{ g} \\ = 0.0003 \text{ g}$$

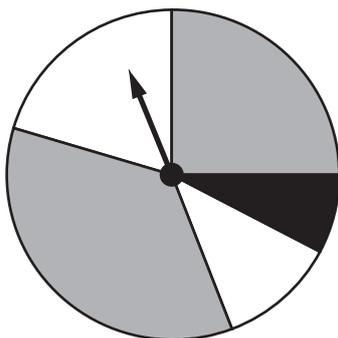
$$250 \text{ pumpkin seeds} = 21 \text{ g}$$

$$1 \text{ pumpkin seed} = 0.084 \text{ g} \left( = \frac{21}{250} \right)$$

$$1 \text{ sesame seed} : 3.64 \times 10^{-6} \text{ kg} \\ = 3.64 \times 10^{-3} \text{ g} \quad \left. \right) \div 1000 \\ = 0.00364 \text{ g}$$

poppy, sesame, pumpkin [4]  
lightest

- 3 (a) This spinner has two grey sections, two white sections and one black section.



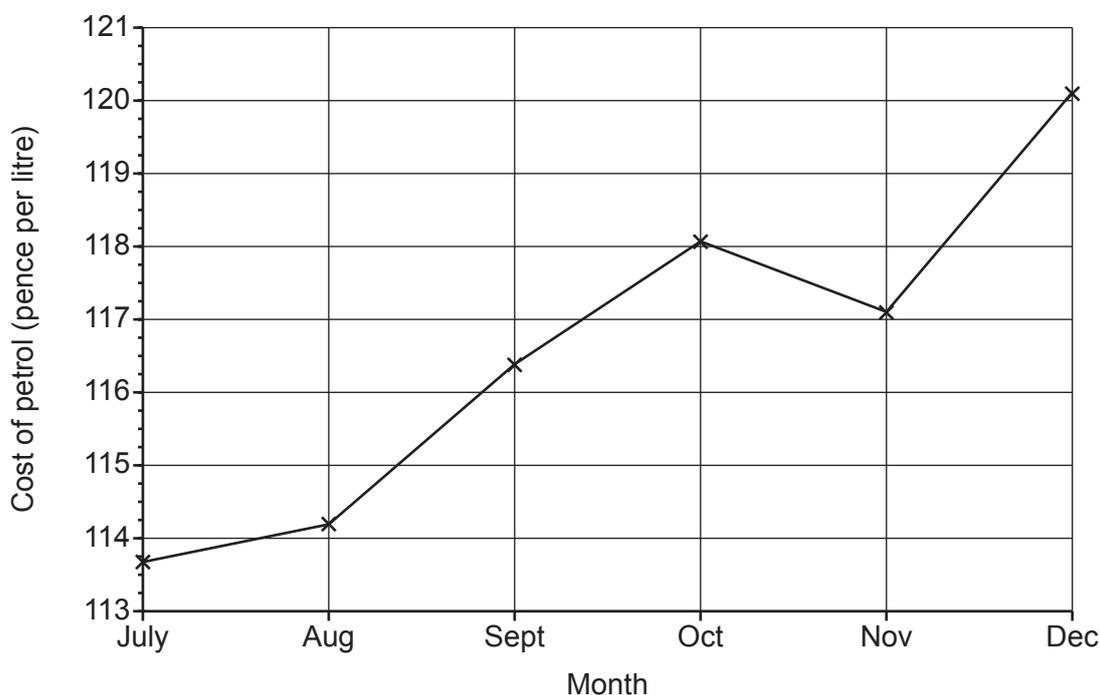
Vlad says

The probability of the spinner landing on black is  $\frac{1}{5}$ .

Explain why Vlad is not correct.

..... for the probability to be  $\frac{1}{5}$ , the .....  
 ..... black segment should measure  $72^\circ$ .....  
 ..... it is too small. .... [1]

- (b) The graph shows the cost of a litre of petrol for the last six months of 2017.



Explain why this graph is misleading.

..... the scale along the x-axis is such .....  
 ..... that the increase from month to .....  
 ..... month seems very large, even though.....  
 ..... it is a matter of pennies. .... [1]  
 ..... This is because it does not start at 0. ....

4 Sophie is organising a raffle.

- Each raffle ticket costs 50p.
- She sells 400 tickets.
- The probability that a ticket, chosen at random, wins a prize is 0.1.
- Each winning ticket receives a prize worth £3.

Sophie says

I expect the raffle to make over £100 profit.

Show that Sophie is wrong.

money made in raffle:

$$400 \times 50p = \text{£}200$$

money given away:

$$400 \times 0.1 = 40 \text{ wins}$$

(probability of winning is 0.1)

$$40 \times \text{£}3 = \text{£}120$$

profit = money made - money given away

$$= \text{£}200 - 120$$

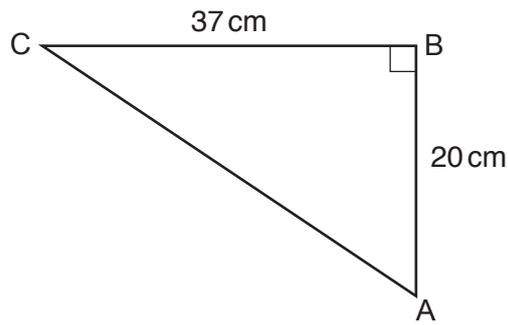
$$= \text{£}80$$

$$\text{£}80 \neq \text{£}100$$

.....  
..... [4]

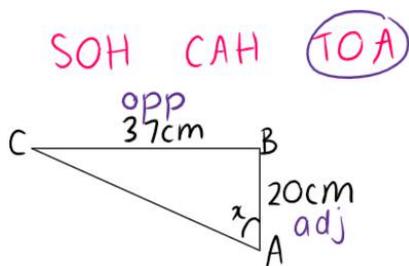
6

- 5 ABC is a right-angled triangle.  
AB = 20 cm and BC = 37 cm.



Not to scale

Calculate angle BAC.



$$\tan x = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{37}{20}$$

$$x = \tan^{-1}\left(\frac{37}{20}\right)$$

$$x = 61.6^\circ$$

∠BAC is 61.6 degrees.

61.6°

..... ° [3]

6 A bag contains some counters.

- There are 300 counters in the bag.
- There are only red, white and blue counters in the bag.
- The probability of picking a blue counter is  $\frac{23}{50}$ .
- The ratio of red counters to white counters is 2 : 1.

Calculate the number of red counters in the bag.

$$6) \quad \frac{23}{50} \times 300 = 138 \text{ blue counters}$$

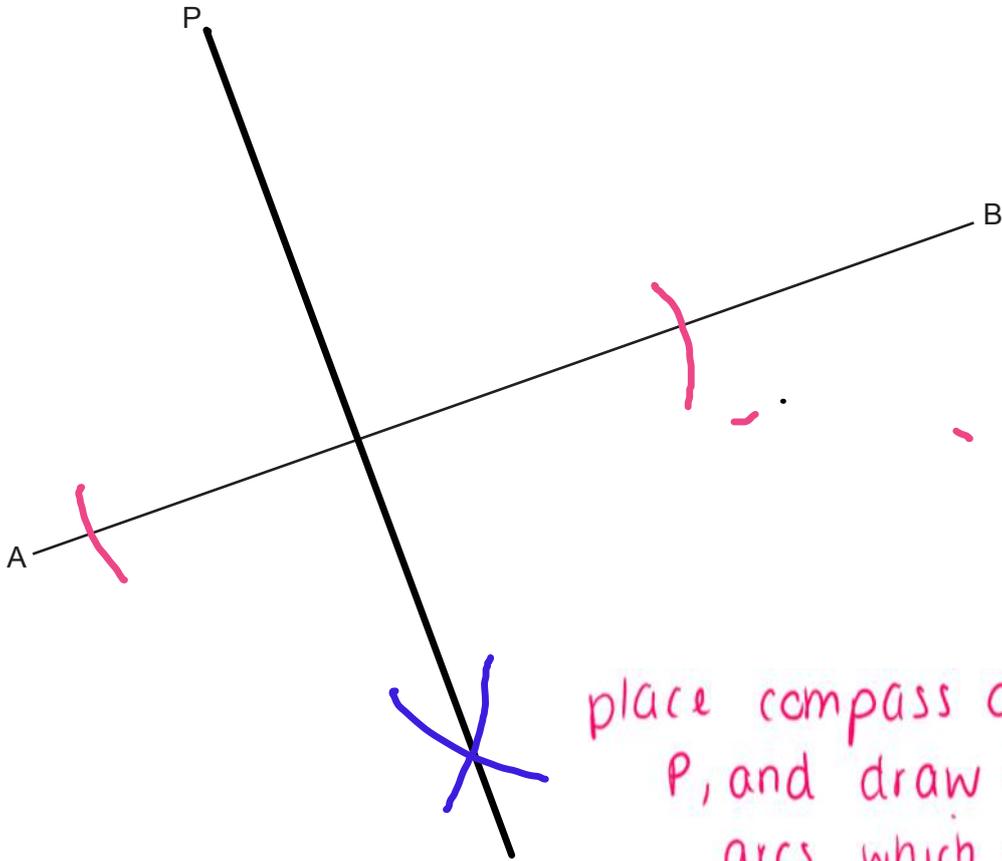
$$300 - 138 = 162 \text{ counters which are red or white}$$

$$\begin{array}{l} R : W \\ 2 : 1 \\ \times 54 \downarrow \quad \uparrow \times 54 \\ 108 : 54 \end{array} \quad \begin{array}{l} 2+1=3 \\ \frac{162}{3} = 54 \end{array}$$

**108** red counters in the bag

108..... [4]

- 7 Construct the perpendicular from the point P to the line AB.  
Show all of your construction lines.



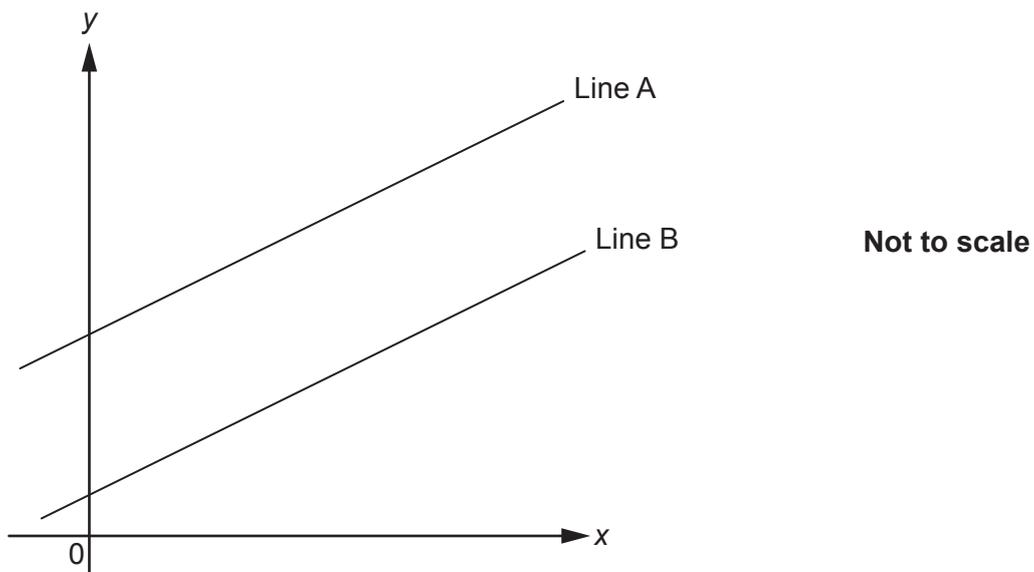
[2]

place compass at  
P, and draw two  
arcs which cross  
the line AB.

from the points,  
draw an arc  
with the compass  
set at the same  
distance each time.

draw a line from P  
to where the 2 new  
arcs intersect.

- 8 The graph shows two parallel lines, Line A and Line B.



Line A has equation  $y = 6x + 7$ .  
Line B passes through the point  $(4, 26)$ .

Find the equation of Line B.

line B:

$$\underline{\hspace{2cm}} (4, 26)$$

$$m = 6 \quad (\text{parallel to line A, so has the same gradient})$$

[sub in values we have]  $y = mx + c$

$$26 = 6(4) + c$$

$$26 = 24 + c$$

$$c = 2$$

$$\rightarrow y = 6x + 2$$

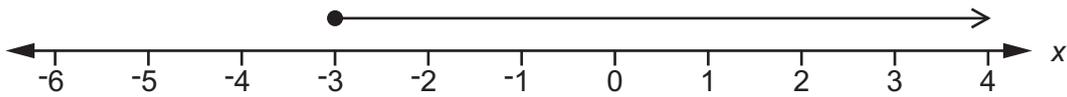
substitute back in.

$$y = 6x + 2$$

[4]

10

- 9 Martha's solution to the inequality  $8x + 5 \leq 3x - 10$  is shown on the number line.



Is her solution correct?  
Explain your reasoning.

$$8x + 5 \leq 3x - 10$$

$$8x \leq 3x - 15$$

$$5x \leq -15$$

$$x \leq -3$$

Martha's solution shows  
 $x \geq -3$ , not  $x \leq -3$ ,  
so is incorrect.

.....  
..... [4]

11

- 10 In 2017, the value of a house increased by 4%.  
In 2018, the value of the house then decreased by 3%.

Teresa says

Over the two years the value of the house increased by exactly 1% because  $4 - 3 = 1$ .

Show that Teresa is wrong.

lets say the house cost £1000 in 2016.

so, in 2017, it cost £1000 × 1.04 = £1040  
↑ 4% increase

in 2018, it cost £1040 × 0.97 = £1008.80

percentage increase =  $\frac{1008.8 - 1000}{1000} \times 100$   
↑ 3% decrease (100 - 3 = 97)

= 0.88%

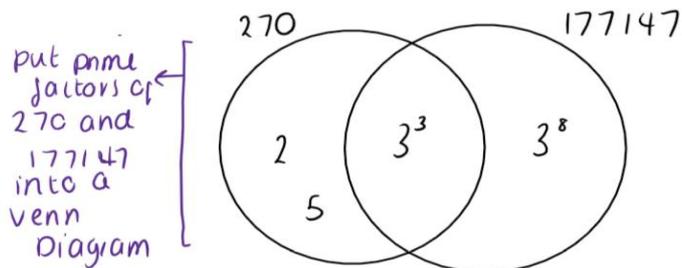
0.88% ≠ 1%

.....  
..... [6]

11 You are given that

$$270 = 3^3 \times 2 \times 5 \quad \text{and} \quad 177147 = 3^{11}$$

- (a) (i) Find the lowest common multiple (LCM) of 270 and 177147.  
Give your answer using power notation and as an ordinary number.



LCM =  $2 \times 5 \times 3^3 \times 3^8$   
 power notation =  $2 \times 5 \times 3^{11}$   
 ordinary number = 1771470

*gained by multiplying all terms in the Venn Diagram*

(a)(i) using power notation  $2 \times 5 \times 3^{11}$   
 as an ordinary number 1771470 [2]

- (ii) Write 177147000000 as a product of its prime factors.

$$\begin{aligned}
 &177147000000 \\
 &= 177147 \times 1000000 \\
 &= (3^{11}) \times (125 \times 125 \times 64) \\
 &= 3^{11} \times (5^3 \times 5^3 \times 2^6) \\
 &= 3^{11} \times 5^6 \times 2^6
 \end{aligned}$$

*express 1 million as prime factors*  
*collect like terms*

(ii)  $3^{11} \times 5^6 \times 2^6$  [3]

(b)  $3^n = 177147 \times 9^5$

Find the value of  $n$ .

$$\begin{aligned}
 3^n &= 177147 \times 9^5 \\
 3^n &= 3^{11} \times (3^2)^5 \\
 3^n &= 3^{11} \times 3^{10} \\
 3^n &= 3^{21}
 \end{aligned}$$

*LAW OF INDICES*  
 $(a^m)^n = a^{m \times n}$   
 $a^m \times a^n = a^{m+n}$

$n = 21$

(b)  $n = 21$  [3]

12 Antonio rolls two fair six-sided dice and calculates the **difference** between the scores. For example, if the two scores are 2 and 5 or 5 and 2 then the difference is 3.

(a) Complete the sample space diagram to show the possible outcomes from Antonio's dice.

		Dice 2					
		1	2	3	4	5	6
Dice 1	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

[2]

(b) Antonio rolls the two dice three times.

Calculate the probability that he gets a difference of 1 on all three rolls. Give your answer as a fraction in its lowest terms.

$$P(\text{difference } 1) = \frac{10}{36}$$

$$\frac{10}{36} \text{ and } \frac{10}{36} \text{ and } \frac{10}{36} = \frac{1000}{46656}$$

cube  $\frac{10}{36}$  as working out probability of 3 rolls

$$\frac{1000}{46656} = \frac{500}{23328} = \frac{125}{5832}$$

(b)  $\frac{125}{5832}$  [4]

13 Prove that the mean of any four **consecutive** even integers is an integer.

[4]

four consecutive integers :

must be  $2x$   
to be an  
even number

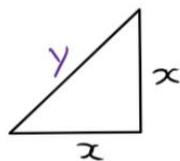
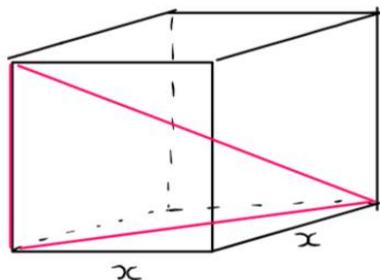
$2x, 2x+2, 2x+4, 2x+6$

$$2x + 2x + 2 + 2x + 4 + 2x + 6 = 8x + 12$$

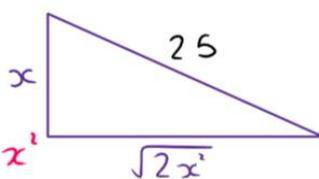
$$\frac{8x + 12}{4} = 2x + 3 \rightarrow \text{this must be an integer}$$

14 The length of the longest diagonal of a cube is 25 cm.

Calculate the total surface area of the cube.



$x^2 + x^2 = y^2$   
 $y = \sqrt{2x^2}$  → Pythagoras' theorem



$(\sqrt{2x^2})^2 + x^2 = 25^2$

$2x^2 + x^2 = 625$

$3x^2 = 625$

area of 1 face =  $x \times x = x^2$   
 cube has 6 faces

Surface area =  $6x^2$

=  $3x^2 \times 2$

=  $625 \times 2 = 1250 \text{ cm}^2$

1250

..... cm<sup>2</sup> [5]

15 Solve by factorisation.

$$5x^2 + 7x + 2 = 0$$

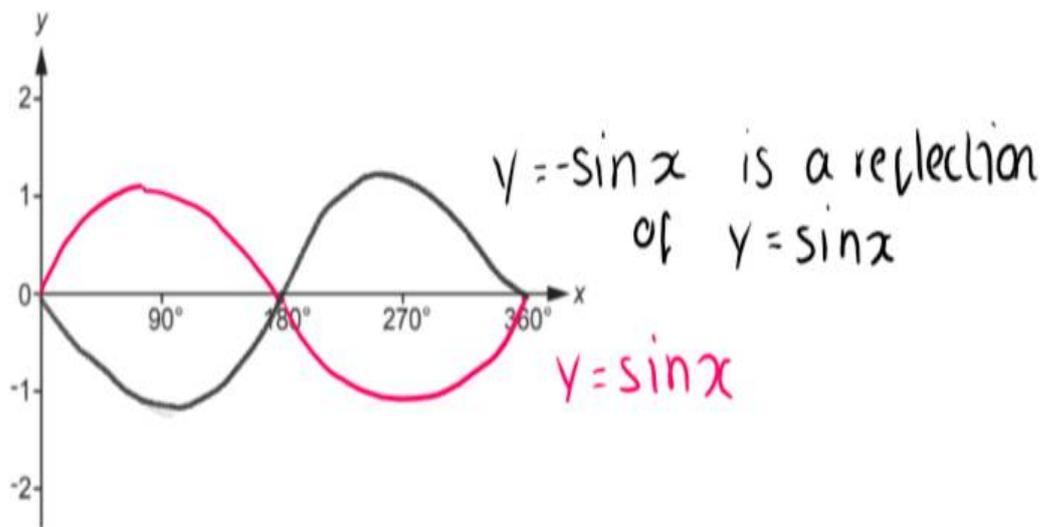
$$5x^2 + 7x + 2 = 0$$
$$(5x + 2)(x + 1) = 0$$

↓      set each bracket to zero      ↓

$$5x + 2 = 0$$
$$5x = -2$$
$$x = -\frac{2}{5}$$
$$x + 1 = 0$$
$$x = -1$$

$$x = \dots\dots\dots -\frac{2}{5} \dots\dots\dots \text{ or } x = \dots\dots\dots -1 \dots\dots\dots [3]$$

- 16 Sketch the graph of  $y = -\sin x$  for  $0^\circ \leq x \leq 360^\circ$ .



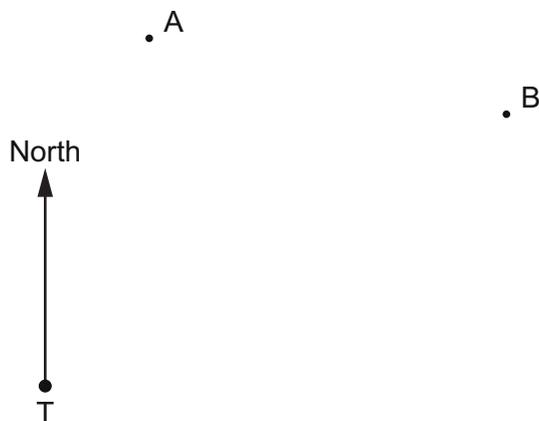
[3]

- 17 T is a radar tower.  
A and B are two aircraft.

At 3pm

- aircraft A is 3250 km from T on a bearing of 015°
- aircraft B is 4960 km from T on a bearing of 057°.

Not to scale



- (a) Aircraft A flies directly towards radar tower T at a speed of 890 km/h.

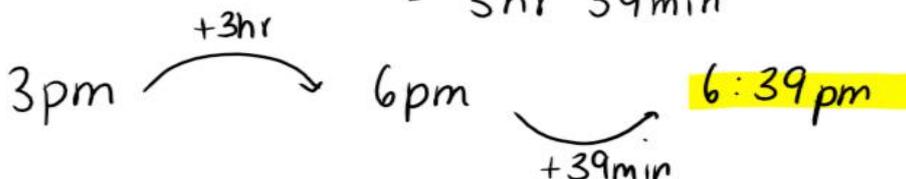
At what time will the aircraft pass over radar tower T?  
Give your answer to the nearest minute.

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \quad \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$t = \frac{3250}{890} = 3.65 \text{ hrs}$$

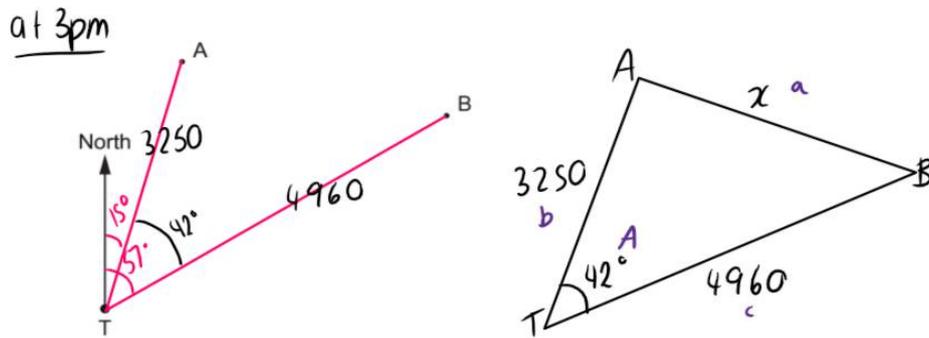
} 0.65 \times 60 \text{ mins} = 39 \text{ mins}

$$= 3 \text{ hr } 39 \text{ min}$$



(a) ..... 6:39pm ..... [4]

(b) Calculate the distance that was between aircraft A and aircraft B at 3pm.



cosine rule :  $a^2 = b^2 + c^2 - 2bc \cos A$

$$x^2 = 3250^2 + 4960^2 - (2 \times 3250 \times 4960 \times \cos 42)$$

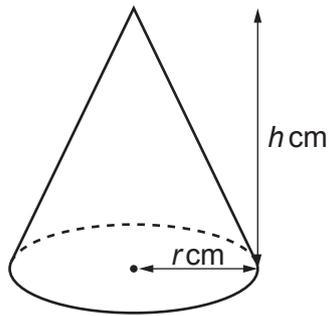
$$x^2 = 35164100 - 23958989.17$$

$$x^2 = 11205110.83$$

$$x = 3347.4 \text{ km}$$

(b) ..... 3347.4 ..... km [4]

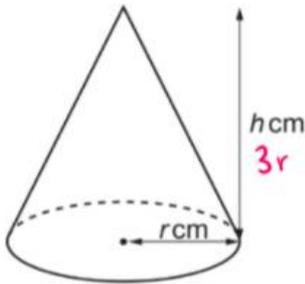
- 18 A cone has radius  $r$  cm and height  $h$  cm.



The height is three times the radius.  
The volume of the cone is  $2100 \text{ cm}^3$ .

Calculate the radius of the cone.

[The volume  $V$  of a cone with radius  $r$  and height  $h$  is  $V = \frac{1}{3}\pi r^2 h$ .]



height is three times radius

$$\text{so, } h = 3r$$

$$V = \frac{1}{3} \pi r^2 h$$

sub in values we have

$$2100 = \frac{1}{3} \times \pi \times r^2 \times 3r$$

$$2100 = \frac{3r^3 \pi}{3}$$

$$2100 = r^3 \pi$$

$$\frac{2100}{\pi} = r^3$$

$$\sqrt[3]{\frac{2100}{\pi}} = r \longrightarrow r = 8.74359\dots$$

$$r = 8.74 \text{ cm}$$

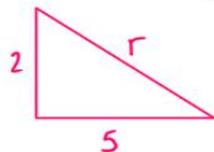
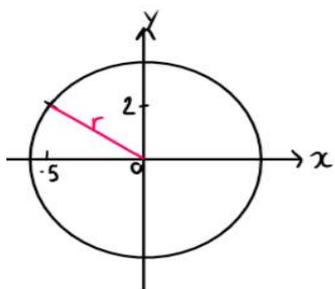
8.74

..... cm [4]

19 The point  $(-5, 2)$  lies on the circumference of a circle, centre  $(0, 0)$ .

(a) Find the equation of the circle.

equation of circle with origin as centre  
 $x^2 + y^2 = r^2$



Pythagoras' theorem  
 $a^2 + b^2 = c^2$

$$2^2 + 5^2 = r^2$$

$$4 + 25 = r^2$$

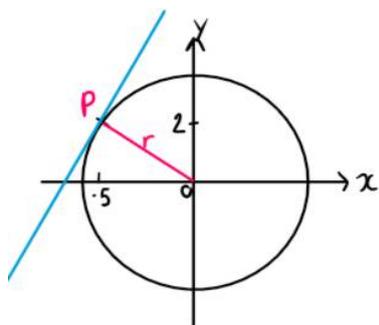
$$29 = r^2$$

sub into equation of the circle

Eqn of circle:  $x^2 + y^2 = 29$

(a)  $x^2 + y^2 = 29$  [4]

(b) Work out the gradient of the tangent to the circle at  $(-5, 2)$ .



let the point  $(-5, 2)$  be P

gradient of line OP:

$$m = \frac{y_1 - y_2}{x_1 - x_2} \quad (-5, 2) \quad (0, 0)$$

$$m = \frac{2 - 0}{-5 - 0} = -\frac{2}{5}$$

gradient of tangent is the negative reciprocal of the line OP.

This is because OP and the tangent are perpendicular.

So, gradient of tangent =  $\frac{5}{2}$

$$\left[ \frac{5}{2} \times -\frac{2}{5} = -1 \right]$$

(b)  $\frac{5}{2}$  [2]

- 20 (a) Show that the equation  $x^4 - x^2 - 9 = 0$  has a solution between  $x = 1$  and  $x = 2$ . [3]

$$x^4 - x^2 - 9 = 0$$

if  $x = 1$ ,  $(1)^4 - (1)^2 - 9 = \underline{-9}$  negative

if  $x = 2$ ,  $(2)^4 - (2)^2 - 9 = \underline{3}$  positive

There is a sign change between  $x = 1$  and  $x = 2$ ,  
so the solution must be between this.

- (b) Find this solution correct to 1 decimal place.  
Show your working.

let  $x^2 = y$

$$x^4 - x^2 - 9 = 0 \rightarrow y^2 - y - 9 = 0$$

$$a = 1 \quad b = -1 \quad c = -9$$

quadratic formula:

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - (4 \times 1 \times -9)}}{2 \times 1}$$

$$y = 3.541... \text{ or } y = -2.541...$$

we took  $y = x^2$

$$x^2 = 3.541...$$

$$x^2 = -2.541...$$

$$x = 1.881...$$

$$= 1.9 \text{ (1dp)}$$

ignore this solution  
as any value of  $x^2$   
would not be a negative  
number

(b)  $x = 1.9$  ..... [4]

- 21 Toy building bricks are available in two sizes, small and large. The small and large bricks are mathematically similar.

A small brick has volume  $8 \text{ cm}^3$  and width  $2.1 \text{ cm}$ .  
 A large brick has volume  $15.625 \text{ cm}^3$ .

Calculate the width of a large brick.

small block  
 width:  $2.1 \text{ cm}$   
 volume:  $8 \text{ cm}^3$

large block  
 width: ?  
 volume:  $15.625 \text{ cm}^3$

(V.S.F)  
 volume scale factor  
 $= \frac{15.625}{8} = \frac{125}{64}$   
 length scale factor

$V.S.F = (L.S.F)^3$   
 $\frac{125}{64} = (L.S.F)^3$   
 $L.S.F = \frac{5}{4}$

small block width  $\times$  L.S.F = large block width  
 $2.1 \times \frac{5}{4} = 2.625 \text{ cm}$

.....  $2.625$  ..... cm [4]

Turn over for question 22

- 22 At the start of 2018, the population of a town was 17 150.  
At the start of 2019, the population of the town was 16 807.

It is assumed that the population of the town is given by the formula

$$P = ar^t$$

where  $P$  is the population of the town  $t$  years after the start of 2018.

$P = ar^t$   
at the start of 2018,  $P = 17150$   $t = 0$   
 $17150 = a \times r^0$  anything to the power of 0 is 1.  
 $17150 = a \times 1$   
 $a = 17150$

- (a) Write down the value of  $a$ .

(a) 17150 ..... [1]

- (b) Show that  $r = 0.98$ . at start of 2019,  $P = 16807$  and  $t = 1$  [1]

$P = 17150 \times r^t$   
 $16807 = 17150 \times r^1$  [ $r^1 = r$ ]  
 $\frac{16807}{17150} = r$   
 $r = 0.98$

- (c) Show that the population is predicted to be less than 16 000 at the start of 2022. [2]

start of 2022,  $t = 4$   
 $P = 17150 \times 0.98^t$   
 $P = 17150 \times 0.98^4$   
 $= 15818.6 \dots$   
hence  $P < 16000$

- (d) Use the formula to work out what the population might have been at the start of 2017.

start of 2017,  $t = -1$  as one year BEFORE the start of 2018  
 $P = 17150 \times 0.98^t$   
 $P = 17150 \times 0.98^{-1}$   
 $P = 17500$

(d) 17500 ..... [2]

END OF QUESTION PAPER

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