MODEL SOLUTIONS

H

OCR
Oxford Cambridge and RSA

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Tuesday 12 June 2018 – Morning

Time allowed: 1 hours 30 minutes

You may use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

7048300902

J56006

First name	
Last name	
Centre number	
Candidate number	

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

1 Ping chooses four numbers.

The mode of these four numbers is 8, the range is 7 and the mean is 11.

Find Ping's four numbers.

mode = 8 so at least two of the numbers are 8

sum of all 4 numbers = $4 \times 11 = 44$

$8 + 8 + x + y = 44$

x and y are the two unknown numbers

-16

$x + y = 44 - 16 = 28$

range = 7 so the highest number is $8 + 7 = 15$ either x or y

highest

lowest

$15 + y = 28$

-15

$y = 28 - 15 = 13$

8 , 8 , 13 , 15 **[3]**

2 A box contains only red, blue and green pens.
The ratio of red pens to blue pens is 5 : 9.
The ratio of blue pens to green pens is 1 : 4.

Calculate the percentage of pens that are blue.

red : blue

5 : 9

blue : green

$\div 9$ 1 : 4 $\div 9$

9 : 36

equal number of parts of blue pens to combine ratio

red : blue : green

5 : 9 : 36 so the total number of parts in the ratio is $5 + 9 + 36 = 50$

percentage of blue pens: $\frac{9}{50} \times 100$

blue parts of ratio

total parts of ratio

= 18%

18 % **[4]**

3 Asha worked out $\dfrac{326.8 \times (6.94 - 3.4)}{59.4}$.

She got an answer of 19.5, correct to 3 significant figures.

Write each number correct to 1 significant figure to decide if Asha's answer is reasonable.

$326.8 \sim 300$ $6.94 \sim 7$ $3.4 \sim 3$ $59.4 \sim 60$

$$\approx \frac{300 \times (7-3)}{60}$$

$$= \frac{300 \times 4}{60} = \frac{1200}{60} = 20$$

Asha's answer is reasonable as 19.5 rounds to 20.

.... [3]

4 (a) Show that $a^5 \times (a^3)^2$ can be expressed as a^{11}. [2]

$= a^5 \times a^{3\times 2} = a^5 \times a^6$ $(a^3)^2 = a^{3\times 2}$

$a^5 \times a^6 = a^{5+6} = a^{11}$

$a^m \times a^n = a^{m+n}$

(b) Write $\dfrac{1}{125} \times 25^9$ as a power of 5.

$\dfrac{1}{125} = \dfrac{1}{5^3} = 5^{-3}$ $a^{-n} = \dfrac{1}{a^n}$

$25^9 = (5^2)^9 = 5^{18}$ $(a^m)^n = a^{m\times n}$

$$\frac{1}{125} \times 25^9 = 5^{-3} \times 5^{18}$$

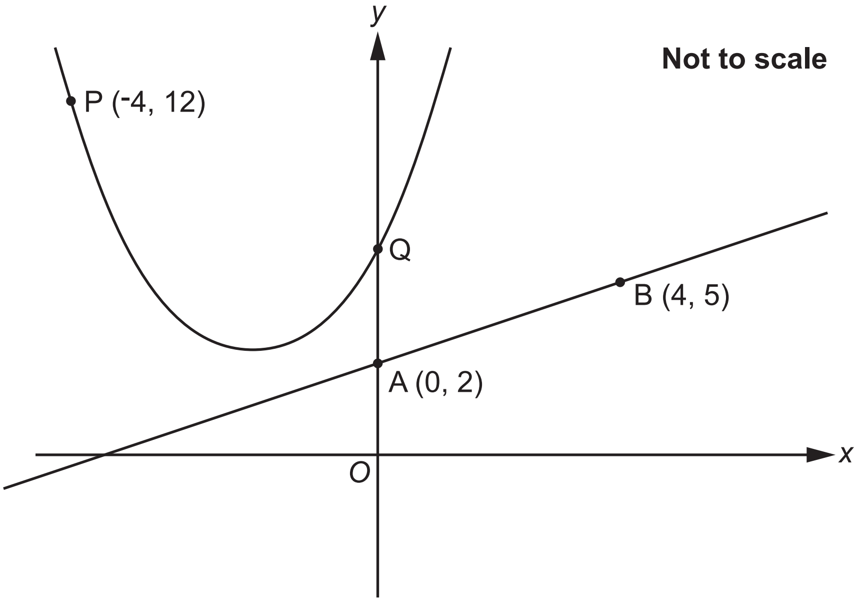
$$= 5^{-3+18}$$

$$= 5^{15}$$

$a^m \times a^n = a^{m+n}$

(b) ....5^{15}........ [3]

5 The diagram shows a straight line that passes through points A and B, and a curve that passes through points P and Q.



(a) Find the equation of the straight line.

where line crosses y-axis, where $x = 0$

$y = mx + c$

y-intercept

gradient

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{4-0} = \frac{3}{4}$$

y-intercept $= (0, 2)$ so $c = 2$

therefore: $y = \frac{3}{4}x + 2$

(a) $y = \frac{3}{4}x + 2$ **[3]**

(b) The equation of the curve is $y = x^2 + kx + 8$.

Find the value of k.

$y = x^2 + Kx + 8$

$12 = (-4)^2 + K(-4) + 8$ substitute P(-4 , 12) into the equation

$12 = 16 - 4K + 8$

$12 = 24 - 4K$ +4K

$4K + 12 = 24$ −12

$4K = 12$ ÷4

$K = 3$

(b) $k =$ 3 **[3]**

(c) Diann draws line BQ.
She says

Triangle ABQ is isosceles.

Is Diann correct?
You must show all your working.

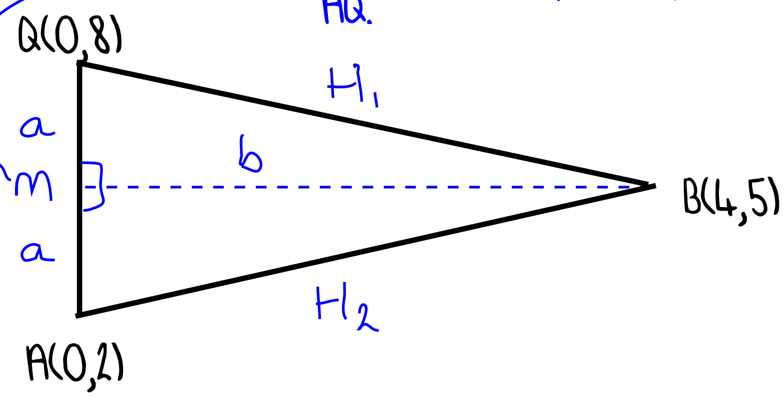
Q : (0 , 8)

y-intercept of curve : value of y when $x = 0$
$y = x^2 - 4x + 8$
$y = 0^2 - 4(0) + 8$
$y = 8$

length AQ = 8 - 2 = 6
so length a = 3

M is the midpoint of AQ.

length MB = 4
so length b = 4



$a^2 + b^2 = H_1^2$
$3^2 + 4^2 = H_1^2$
$H = \sqrt{25} = 5$

$a^2 + b^2 = H_2^2$
$3^2 + 4^2 = H_2^2$
$H = \sqrt{25} = 5$

$H_1 = H_2$ therefore ABQ is isosceles. Yes, Diane is correct. **[4]**

6 y is inversely proportional to x.
$y = 0.04$ when $x = 80$.

Find the value of y when $x = 32$.

$y \propto \frac{1}{x}$ $\quad y = \frac{K}{x}$ K is the constant of proportionality

$0.04 = \frac{K}{80}$

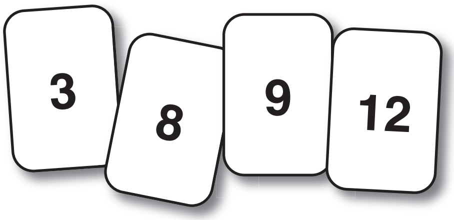
$K = 0.04 \times 80 = 3.2$ substitute $x = 80$, $y = 0.04$

$y = \frac{3.2}{x}$

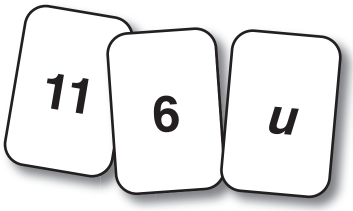
$y = \frac{3.2}{32} = 0.1$ substitute $x = 32$

$y =$ 0.1 **[3]**

7 Edsel has four number cards.



Sharon has three number cards.
u represents a number that Sharon knows.



Edsel and Sharon each pick one of their cards at random.
They calculate the **difference** between the numbers on their cards.
This is their sample space.

		Edsel			
		3	8	9	12
	6	3	2	3	6
Sharon	11	8	3	2	1
	u	11	6	r	t

Work out the values of r and t.

$u - 3 = 11$

$u = 11 + 3$
$= 14$

$r = u - 9$
$= 14 - 9$
$= 5$

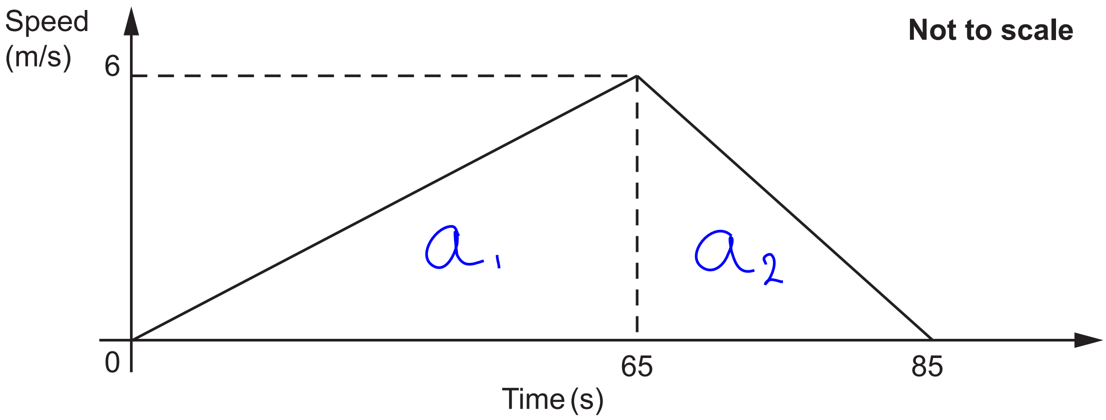
$t = u - 12$
$= 14 - 12$
$= 2$

$r =$ ....5............

$t =$ ....2............

[4]

8 The graph shows the speed of a tram as it travels from the library to the town hall.



(a) Calculate the deceleration of the tram as it approaches the town hall.

deceleration between 65 and 85 seconds

gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{85-65} = \frac{-6}{20} = -0.3$

Gradient of speed-time graph is acceleration. Negative gradient is therefore deceleration.

flip sign for deceleration

deceleration = 0.3 m/s²

(a) ....0.3........ m/s² **[2]**

(b) Calculate the distance travelled by the tram between the library and the town hall.

distance travelled = area under speed-time graph

a_1 : area $= \frac{1}{2} \times 65 \times 6 = 195$

from graph

area of a triangle $= \frac{1}{2} \times$ base $\times$ height

a_2 : area $= \frac{1}{2} \times 20 \times 6 = 60$

distance travelled = 195 + 60 = 255m

(b) ....255........ m **[3]**

(c) What was the maximum speed of the tram as it travelled between the library and the town hall?
Give your answer in **kilometres per hour**.

maximum speed = 6 m/s — 6 m/s is the highest point on graph

metres / hour : 6 × 60 × 60 = 21 600 m/h

60s in 1 min

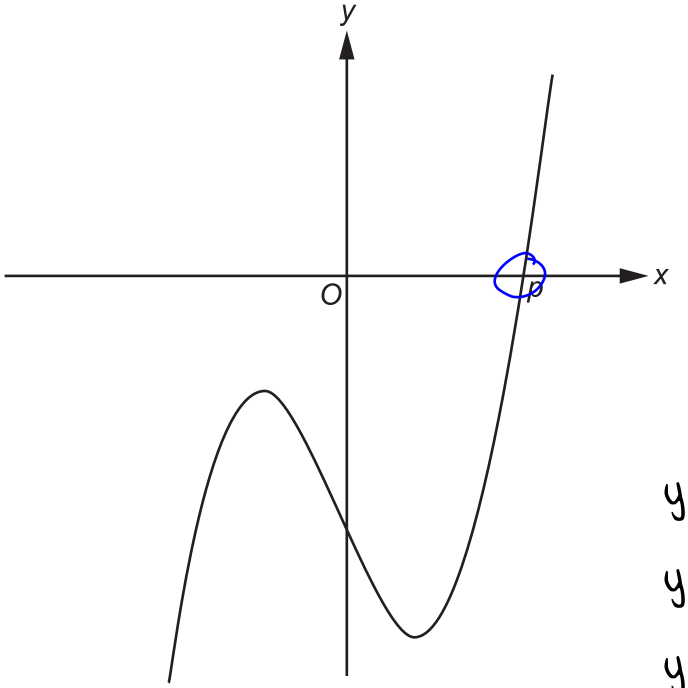
60 mins in 1 hour

kilometres / hour : 21 600 ÷ 1000 = 21.6 km/h

1000m = 1km

(c) ....21.6........ km/h **[4]**

9 The graph of $y = x^3 - 7x - 12$ is shown below.
The root of the equation $x^3 - 7x - 12 = 0$ is p.



$y = x^2 - 7x - 12$

$y = (3)^2 - 7(3) - 12$

$y = 27 - 21 - 12$

$y = -6$

(a) Calculate y when $x = 3$.

(a) $y =$ .... -6 .... **[1]**

(b) Show that $3 < p < 4$. **[2]**

$y = (4)^2 - 7(4) - 12 = 24$

$\left.\begin{array}{l} \text{when } x = 3,\ y = -6 < 0 \\ \text{when } x = 4,\ y = 24 > 0 \end{array}\right\}$ there is a change in sign so p is between 3 and 4

$x = 3$ and y is negative
$x = 4$ and y is positive

negative → positive so must cross y-axis (when $y = 0$) between 3 and 4.

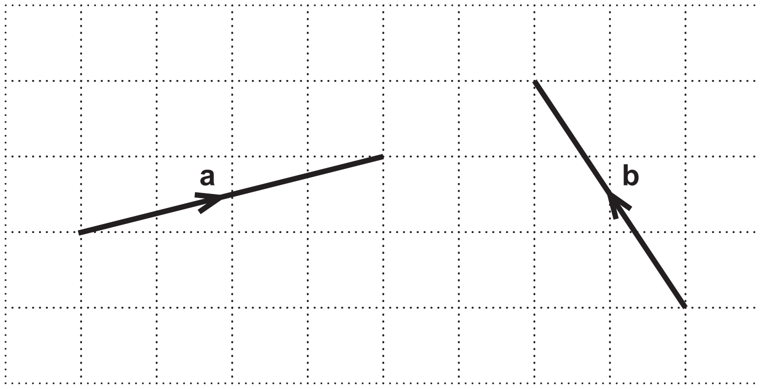
(c) Find a smaller interval that contains the value of p.
You must show calculations to support your answer.

when $x = 3.5$, $y = (3.5)^2 - 7(3.5) - 12 = 6.375 > 0$

when $x = 3.25$, $y = (3.25)^2 - 7(3.25) - 12 = -0.422 < 0$

(c) .... 3.25 .... $< p <$ .... 3.5 .... **[3]**

10 Two vectors, **a** and **b**, are shown on the 1 centimetre grid below.



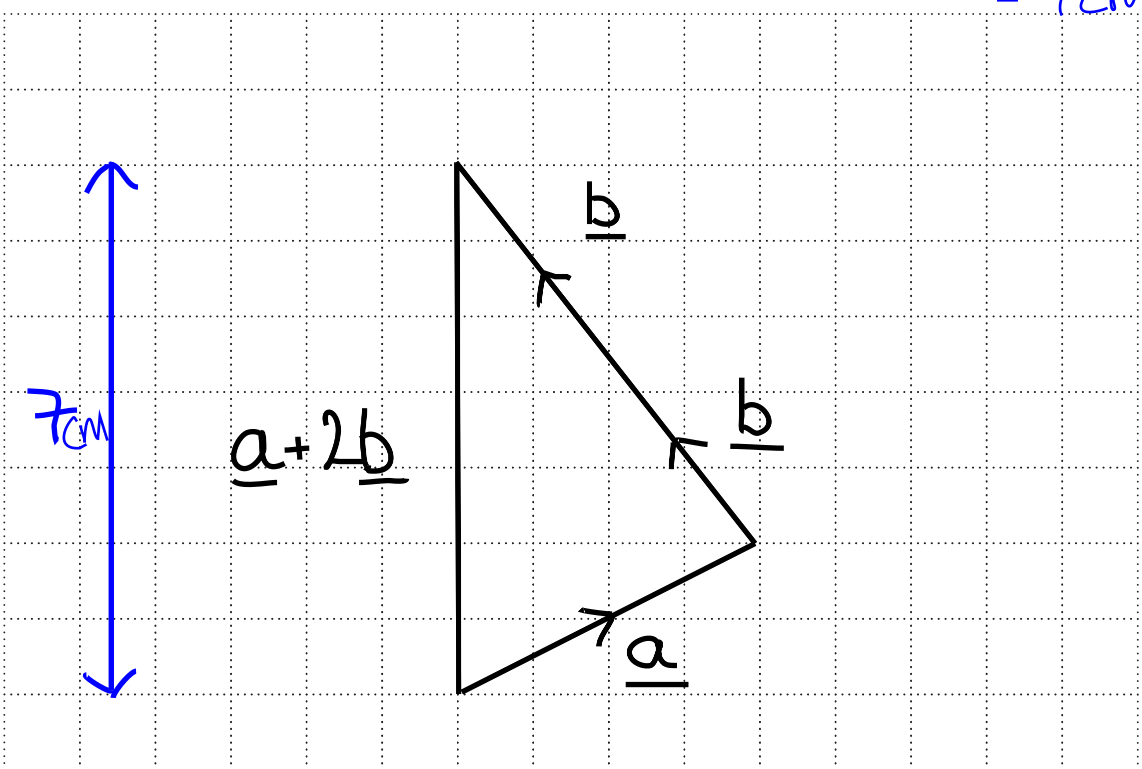
Show that the vector **a** + 2**b** has length 7 cm.
You may use the grid below.

Without grid: $a = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ 4 right 1 up $\quad b = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ 2 left 3 up

$$a + 2b = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + 2\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

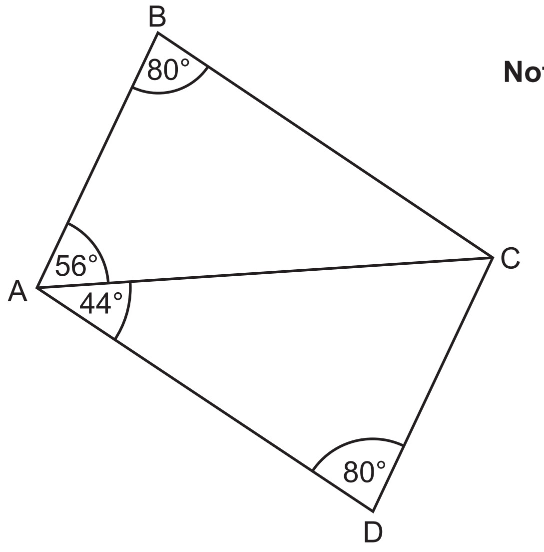
$$= \begin{pmatrix} 4 + -4 \\ 1 + 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

7 units up
= 7cm



[3]

11 The diagram below shows two triangles.



Not to scale

Prove that triangle ABC is congruent to triangle ACD.

triangle ABC:

angle $A\hat{C}B = 180 - 80 - 56 = 44°$

triangle ACD

angle $A\hat{C}D = 180 - 80 - 44 = 56°$

angles in a triangle add up to 180°

AC is common

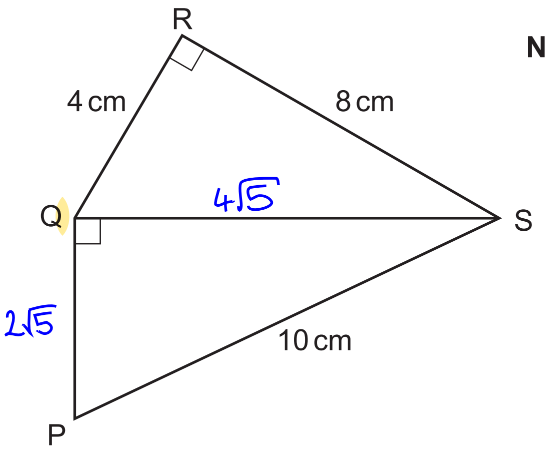
angle $A\hat{C}D$ = angle BAC

angle $D\hat{A}C$ = angle $A\hat{C}B$

Angle-Side-Angle (ASA) therefore triangle ABC is congruent to triangle ACD.

[4]

12 The diagram below shows two right-angled triangles.



Prove that triangles PQS and QRS are similar.

METHOD 1:

$QS^2 = 4^2 + 8^2 = 80$
$QS = \sqrt{80} = 4\sqrt{5}$ cm

$PQ^2 = 10^2 - (4\sqrt{5})^2 = 100 - 80 = 20$
$PQ = \sqrt{20} = 2\sqrt{5}$ cm

scale factor $= \frac{PS}{QS} = \frac{10}{4\sqrt{5}} = \frac{\sqrt{5}}{2} = 1.118$

$= \frac{QS}{RS} = \frac{4\sqrt{5}}{8} = \frac{\sqrt{5}}{2} = 1.118$

$= \frac{PQ}{QR} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2} = 1.118$

All pairs of corresponding sides have the same scale factor, therefore PQS and QRS are similar triangles.

METHOD 2:

$\tan Q\hat{S}R = \frac{4}{8}$ $\quad Q\hat{S}R = \tan^{-1}\left(\frac{4}{8}\right) = 26.6°$ $\quad \tan = \frac{O}{A}$

angle $R\hat{Q}S = 180 - 90 - 26.6 = 63.4°$ angles in a triangle add to 180°

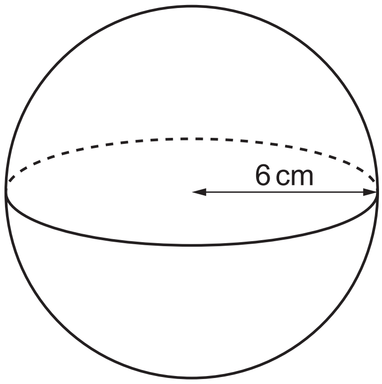
$\cos Q\hat{S}P = \frac{4\sqrt{5}}{10}$ $\quad Q\hat{S}P = \cos^{-1}\left(\frac{4\sqrt{5}}{10}\right) = 26.6°$ $\quad \cos = \frac{A}{H}$

angle $Q\hat{S}P = 180 - 90 - 26.6 = 63.4°$ angles in a triangle add to 180°

$Q\hat{S}R = Q\hat{S}P$ $\quad R\hat{Q}S = Q\hat{P}S$ $\quad Q\hat{R}S = P\hat{Q}S = 90°$

3 pairs of equal angles so the triangles are similar. **[5]**

13 (a) Calculate the volume of a sphere with radius 6 cm.



[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times (6)^3$$ substitute r = 6

$$= \frac{4}{3} \times \pi \times 216$$

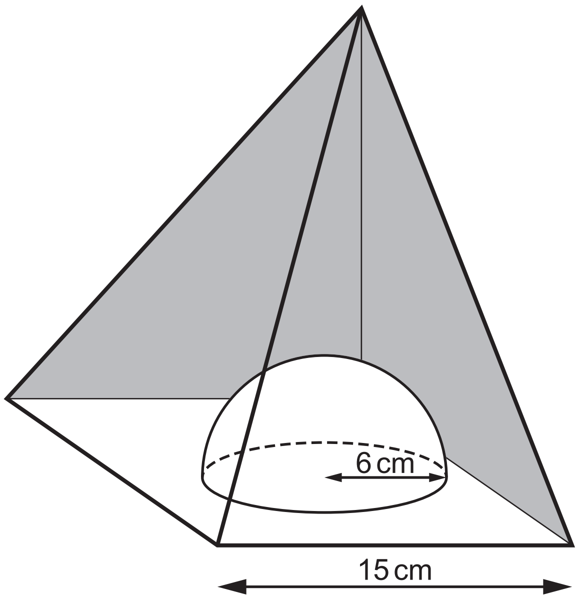
$$= 288\pi \text{ cm}^3$$

$$= 904.8 \text{ cm}^3$$

in terms of π to 1 decimal place

(a) ....904.8........ cm^3 **[2]**

(b) An ornament is made from a solid glass square-based pyramid.
The base has side length 15 cm.
A hemisphere with radius 6 cm is cut out of the base of the pyramid.
This reduces the volume of glass contained in the ornament by 30%.



Calculate the perpendicular height of the pyramid.

[The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ perpendicular height.

A hemisphere is half a sphere.]

Volume of Hemisphere:

from previous question

$V = \frac{1}{2} \times \text{volume of sphere} = \frac{1}{2} \times 288\pi = 144\pi \text{cm}$

keep in terms of pi for more exact answer

Volume of Pyramid:

volume of hemisphere = 30% of volume of pyramid

$144 = 30\%$ $\div 3$
$48 = 10\%$ $\times 10$
$480 = 100\%$ so volume of pyramid $= 480\pi$ cm

area of base $= 15 \times 15 = 225$cm

volume of pyramid $= \frac{1}{3} \times 225 \times h$
$= 75h$

h is perpendicular height

$75h = 480\pi$ so $h = \frac{480\pi}{75} = 20.106...$

$= 20.1$cm

(b) ....20.1............ cm **[5]**

14 (a) Standard bricks have dimensions 21.5 cm by 10.3 cm by 6.5 cm, correct to 1 decimal place.

A house is built using 4663 standard bricks.

Joslin says

Placed end to end, the bricks from the house would definitely reach over 1 km.

Show that Joslin's statement is correct. **[4]**

correct to 1.d.p = to nearest 0.1 = $\pm$ 0.05cm

length of brick = 21.5cm

lower bound = 21.5 - 0.05 = 21.45cm minimum possible length

21.45 × 4663 = 100 021.35 cm

100cm = 1m

= 1000.2135 m

even when using the shortest possible lengths, the bricks still reach over 1km

1000m = 1km

= 1.0002135 km which is more than 1 km

(b) A standard brick should weigh 2.8 kg, correct to 1 decimal place.
A truck can carry a maximum load of 20 tonnes.

(i) Calculate the maximum number of standard bricks that the truck should be able to carry.

$\pm$ 0.05 kg so upper bound = 2.8 + 0.05 = 2.85kg maximum possible weight

20 tonnes = 20 000 kg 1 tonne = 1000 kg

maximum number of bricks = $\frac{20\ 000}{2.85}$ = 7017.54 = 7017

the number of bricks the truck can carry if all bricks weight maximum possible weight

(b)(i) ....7017.......... **[3]**

(ii) Explain why your answer to **(b)(i)** may not be possible to achieve.

This number of bricks may not fit in the truck.

.... **[1]**

15 Ratna invests £1200 for 2 years in a bank account paying r % per year compound interest.
At the end of 2 years, the amount in the bank account is £1379.02.

Calculate r.

final amount = initial investment X multiplier^n

n = years after investment

£1379.02 = £1200 X multiplier^2

$\frac{1379.02}{1200}$ = multiplier^2 = 1.1492

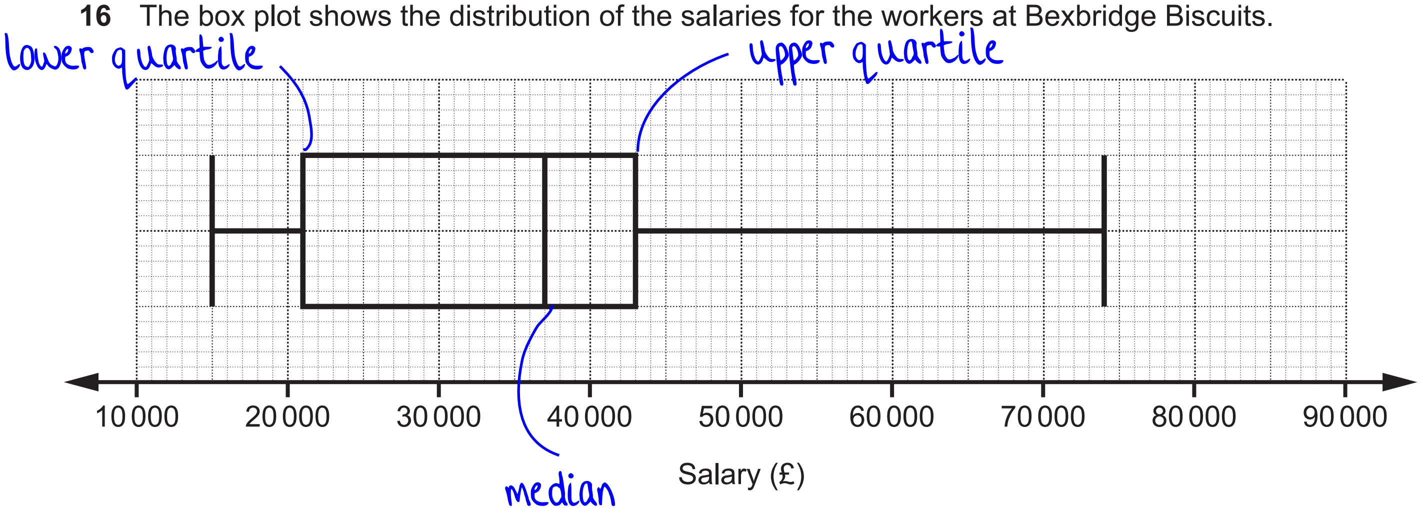
multiplier = $\sqrt{1.1492}$ = 1.072 to 3.d.p

1.072 = $\underbrace{107.2\% = 100\% + r\%}$ x100 to convert to %

r = 107.02% − 100% = 7.2%

r = ...7.2...... **[4]**

16 The box plot shows the distribution of the salaries for the workers at Bexbridge Biscuits.



(a) State the median salary.

(a) £ ....37000........ **[1]**

(b) Find the interquartile range.

Upper Quartile = £43000

IQR = upper quartile - lower quartile

Lower Quartile = £21000

interquartile range = £43000 - £21000
= £22000

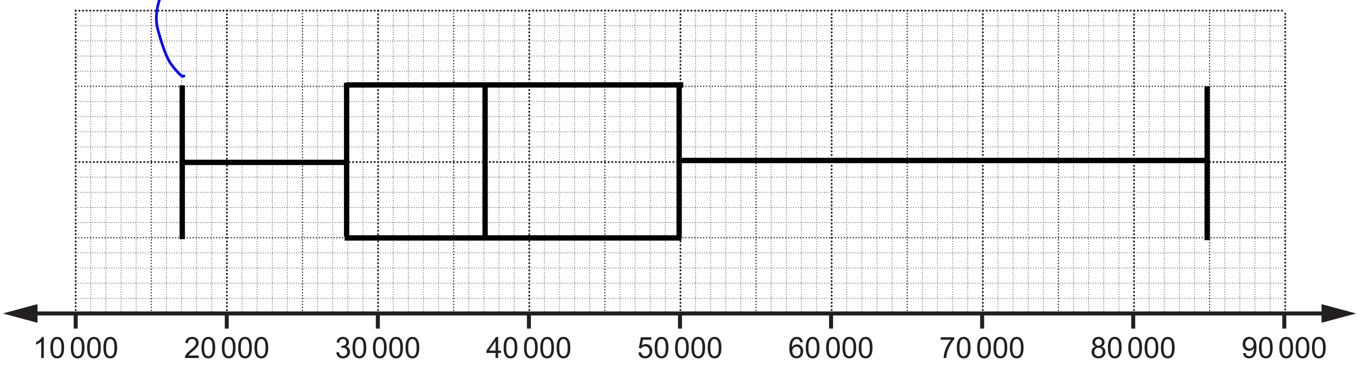
(b) £ ....22000........ **[2]**

(c) The following salary information is true for workers at Camford Cookies.

- The highest paid worker earns £85 000.
- The lowest paid worker earns 20% of the salary of the highest paid worker.
- 25% of the workers earn more than £50 000.
- 25% of the workers earn less than £28 000.
- The median salary is £37 000.

Draw a box plot to show the salaries of the workers at Camford Cookies.

lowest = 0.2 X 85000
= 17000



[3]

(d) Make two different comparisons between the distribution of the salaries at Bexbridge Biscuits and the salaries at Camford Cookies.

1: Median salaries are the same for both companies.

2: Interquartile range is the same for both companies.
Bexbridge Biscuits: 50000 - 28000 = £22000 **[2]**

17 Here is a function.

Function A: $\quad \text{Input} \longrightarrow \boxed{+\,14} \longrightarrow \boxed{\times\,5} \longrightarrow \text{Output}$

(a) The **output** of function A is *x*.

Write an algebraic expression, in terms of *x*, for the input of function A.

output $\longrightarrow \div 5 \longrightarrow -14 \longrightarrow$ input

$(x \div 5) - 14 = \text{input}$

$\frac{x}{5} - 14 = \text{input}$

(a) .... $\frac{x}{5} - 14$ .... **[2]**

(b) A number, *k*, is put into function A.
The output is also *k*.

Find the value of *k*.

$(K + 14) \times 5 = K$

$5K + 70 = K$ $\quad -K$

$4K + 70 = 0$ $\quad -70$

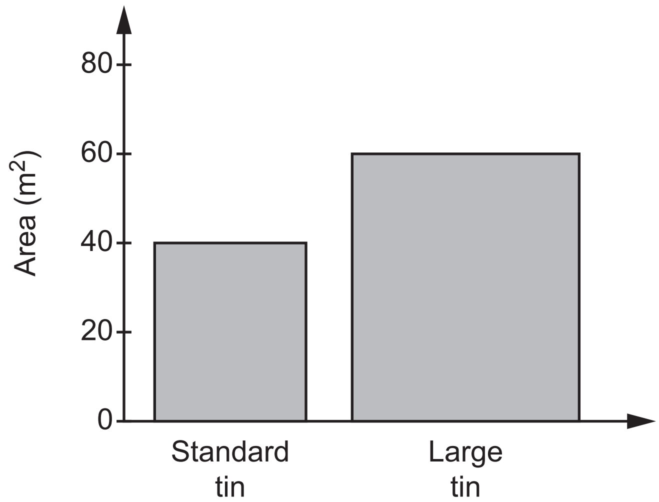
$4K = -70$ $\quad \div 4$

$K = \frac{-70}{4} = -17.5$

(b) $k =$ .... -17.5 .... **[3]**

18 Percy sells paint in standard tins and large tins.
The standard tin covers 40 m^2 and the large tin covers 60 m^2.

(a) Percy publishes this chart showing the area that can be covered with each tin of paint.



Explain why the chart is misleading.

....

The bars for standard tin and large tin are different widths ...... **[1]**

(b) The standard tin and the large tin are mathematically similar.
The **volume** of the large tin is 50% more than the volume of the standard tin.
Both tins are cylinders.
The radius of the standard tin is 10 cm.

Calculate the radius of the large tin.

÷100

50% more = 100% + 50% = 150% = 1.5 X standard tin

volume of standard tin X K^3 = volume of large tin

where K is the scale factor of lengths, K^3 for volume and K^2 for surface area

$K^3 = 1.5$ so $K = \sqrt[3]{1.5} = 1.14...$

radius of standard tin X K = radius of large tin

10cm X 1.14 = 11.4cm

radius of large tin = 11.4cm

(b) ....11.4........ cm **[4]**

19 Show that $\frac{2x^2+13x+20}{2x^2+x-10}$ simplifies to $\frac{x+a}{x-b}$ where a and b are integers. **[4]**

factorise $2x^2 + 13x + 20$

$a \times c = 2 \times 20 = 40$

$8 \times 5 = 40$
$8 + 5 = 13$

$2x^2 + 8x + 5x + 20$

$2x(x+4) + 5(x+4)$

$(2x+5)(x+4)$

factorise $2x^2 + x - 10$

$a \times c = 2 \times -10 = -20$

$5 \times -4 = -20$
$5 + -4 = 1$

$2x^2 + 5x - 4x - 10$

$2x(x-2) + 5(x-2)$

$(2x+5)(x-2)$

cancel same bracket from top and bottom

$$\frac{2x^2+13x+20}{2x^2+x-10} = \frac{(2x+5)(x+4)}{(2x+5)(x-2)} = \frac{\cancel{(2x+5)}(x+4)}{\cancel{(2x+5)}(x-2)} = \frac{x+4}{x-2}$$

in form $\frac{x+a}{x-b}$

where $a = 4$
$b = 2$

END OF QUESTION PAPER