MODEL SOLUTIONS

Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Practice Paper

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You may use:
- Geometrical instruments
- Tracing paper

Do not use:
- A calculator

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First name	
Last name	

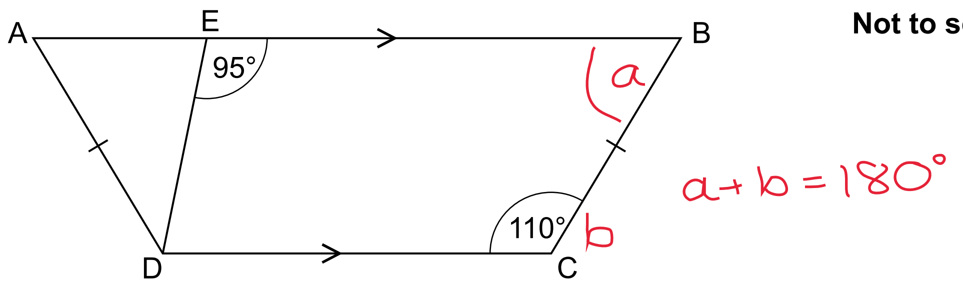
Centre number						Candidate number				

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document consists of **20** pages.

Answer **all** the questions

1 ABCD is a trapezium.
AD = BC.



Not to scale

Work out

(a) angle EBC,

EBC = 180 − 110
= 70° co-interior angles add up to 180

(a) ....70........ ° **[1]**

(b) angle ADE.

$E\hat{D}C = 180 - 95 = 85°$ co-interior angles
$A\hat{D}C = B\hat{C}D = 110°$ because AD = BC
$A\hat{D}E = A\hat{D}C - E\hat{D}C$
$= 110 - 85$
$= 25°$

(b) ....25........ ° **[2]**

2 The angles in a triangle are in the ratio 1 : 2 : 3.
Neil says

This is a right-angled triangle.

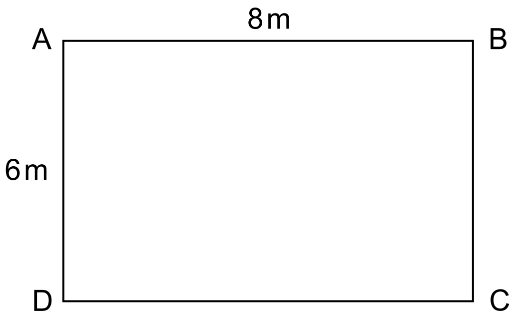
Is Neil correct?
Show your reasoning.

1 : 2 : 3 total parts of ratio = 1 + 2 + 3 = 6

angles in a triangle = 180° so 1 part of the ratio = $\frac{180}{6}$ = 30°

1 : 2 : 3
×30 ×30 ×30
30° : 60° : 90° so Neil is correct as 90° is a right angle, so triangle is right-angled.
[3]

3 ABCD is a rectangle.



Not to scale

(a) Sunita calculates the length of AC, but gets it wrong.

should be +

$8^2 - 6^2 = AC^2$

$\sqrt{28} = AC$

$\sqrt{28} = 5.29$ or $^-5.29$

$AC = 5.29$

Explain what Sunita has done wrong.

She has calculated $8^2 - 6^2$ when she should have calculated $8^2 + 6^2$. Pythagoras: $a^2 + b^2 = c^2$ **[1]**

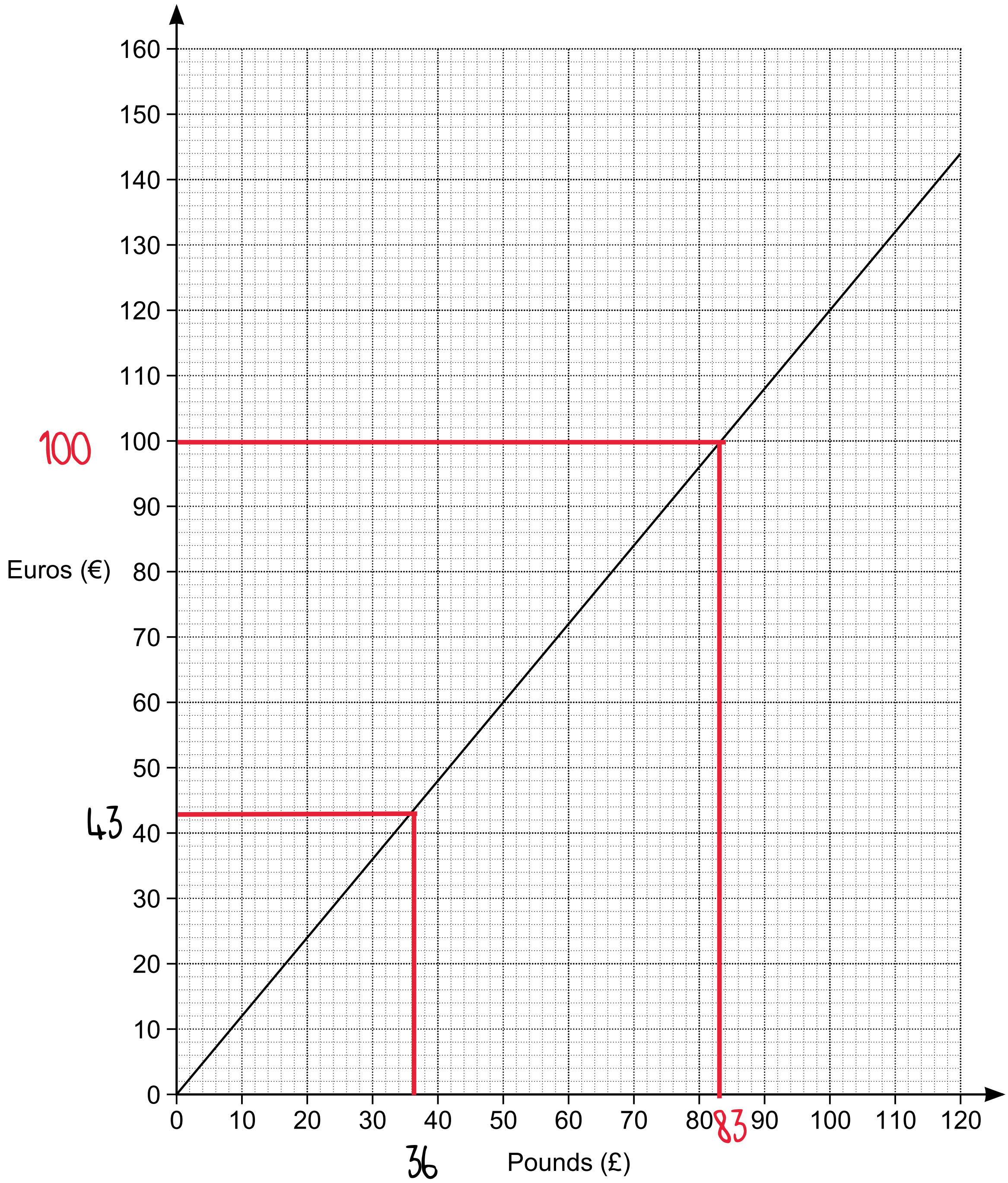
(b) Calculate the length of AC.

$AC^2 = 8^2 + 6^2$
$= 64 + 26$
$= 100$

$AC = \sqrt{100}$
$= 10$

(b) ....10........ m **[2]**

4 This is a conversion graph between pounds and euros.



(a) Convert £36 into euros.

(a) € ......43...... **[1]**

(b) **(i)** Convert €400 into pounds.

€400 not on scale so convert €100 to £s then multiply by 4.

€100 = £83

£83 X 4 = £332

(b)(i) £ ....332.... **[3]**

(ii) State an assumption that you have made in working out your answer to part **(b)(i)**.

The exchange rate is constant/stays the same. **[1]**

(c) Explain how the graph shows that the number of euros is directly proportional to the number of pounds.

Straight line.

Passes through origin. **[2]**

5 Kamile sells sandwiches.

In May, she sold 400 sandwiches.
In June, Kamile sold 20% more sandwiches than in May.
In July, Kamile sold 15% fewer sandwiches than in June.

Calculate the percentage change in her sales from May to July.

June: $20\% \text{ of } 400 = 400 \times \frac{20}{100} = 400 \div 5 = 80$

Sandwiches sold $= \underbrace{400 + 80}_{\text{20\% more than May}} = 480$

July: $10\% \text{ of } 480 = 480 \div 10 = 48$

$5\% \text{ of } 480 = 10\% \text{ of } 480 \times \frac{1}{2} = 48 \times \frac{1}{2} = 24$

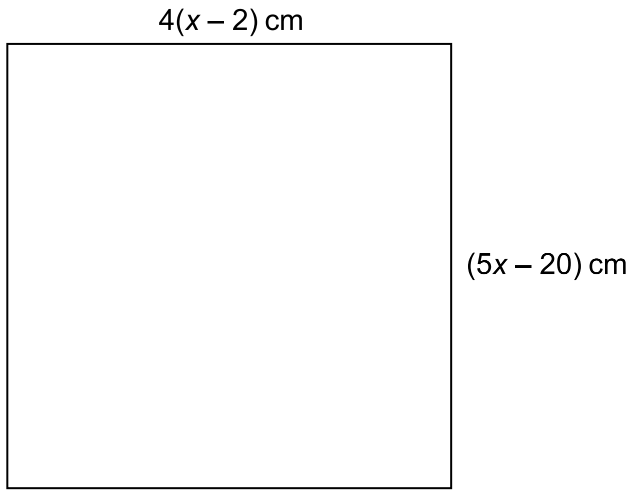
$15\% \text{ of } 480 = 48 + 24 = 72$

Sandwiches sold $= \underbrace{480 - 72}_{\text{15\% less than June}}$

$\% \text{ change} = \frac{408 - 400}{400} \times 100 = \frac{8}{400} \times 100 = 2\%$

....2.... % **[5]**

6 This is a square.



Not to scale

Work out the length of the side of the square.

$4(x-2) = 5x - 20$ ← as sides are of equal length

$4x - 8 = 5x - 20$

$-4x \qquad -4x$

$-8 = x - 20$

$+20 \qquad +20$

$12 = x$

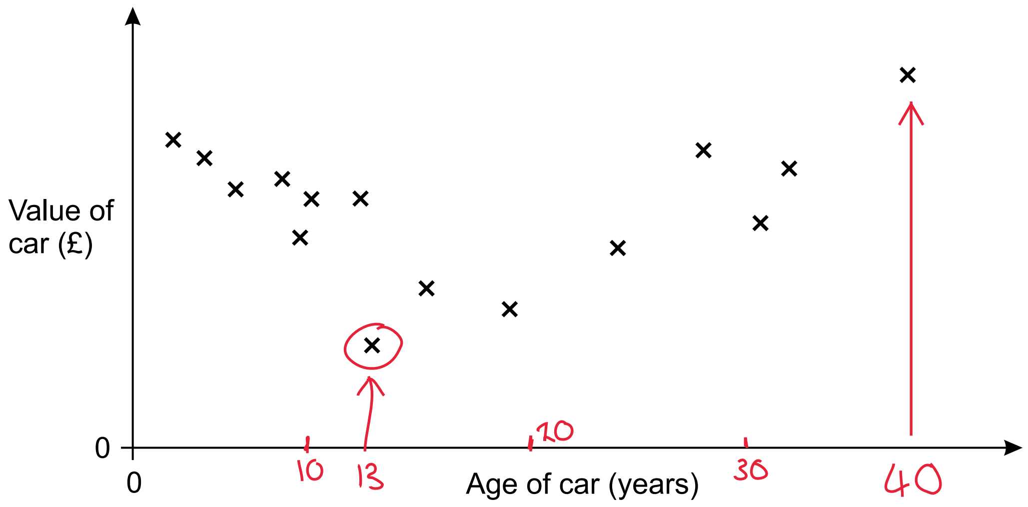
length of side: $4(x-2) = 4(12-2)$ substitute $x=12$

$= 4 \times 10$

$= 40$ cm

40 cm **[5]**

7 This scatter graph shows the values of 15 sports cars plotted against their ages.



(a) **(i)** Lewis thinks that there is **no correlation** between the ages and values of these cars.

Is Lewis correct?
Give a reason for your answer.

A correlation is a linear relationship between the variables. (You can draw a straight line through them.)

Yes, Lewis is correct. The points do not follow the same linear pattern.
[2]

(ii) Sebastian thinks that there is a **relationship** between the ages and values of these cars.

Is Sebastian correct?
Give a reason for your answer.

Yes, Sebastian is correct. Initially, the value of the cars decreases as the age increases. After a certain point, as the cars get older, their value then increases.
[2]

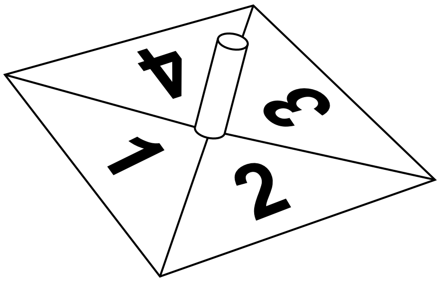
(b) The car with the highest value is 40 years old.

Estimate the age of the car with the lowest value.

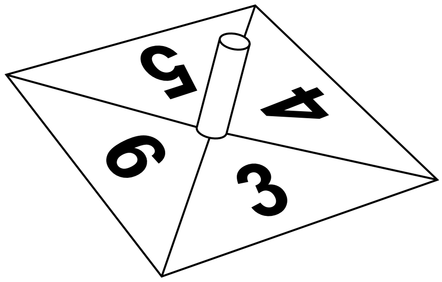
Look at graph to find lowest value.
(Answers between 11 and 14 are accepted.)

(b) ......13...... years **[2]**

8 Andrea has these two fair spinners.



Spinner A



Spinner B

(a) Andrea spins **spinner A**.

Calculate the probability that Andrea gets 2 with one spin.

$P(2) = \frac{1}{4}$

(a) $\frac{1}{4}$ **[1]**

(b) Andrea now spins **both** spinners once.
She adds the number she gets on spinner A to the number she gets on spinner B.

(i) Andrea works out the probability that the two numbers she gets add to 4.
Here is her working.

> $1 + 3 = 4$ $3 + 1 = 4$
>
> There are 4 outcomes on each spinner making 8 outcomes in total.
>
> The probability of the two numbers adding to 4 is $\frac{2}{8} = \frac{1}{4}$.

Andrea has made some errors.
Describe these errors.

There is only one way for the numbers to add to 4.
1 on spinner A and 3 on B as there is no 1 on spinner B.

The total number of outcomes is 16 not 8.
4 outcomes on spinner A X 4 on spinner B = 4 X 4 = 16 **[2]**

(ii) Find the probability that the two numbers she gets add to 6.

A	B
1	5
2	4
3	3

= 3 ways to add to 6

P(add to 6) = 3 out of 16

$= \frac{3}{16}$ ← total number of outcomes

(b)(ii) $\frac{3}{16}$ **[3]**

9 (a) Calculate.

$$2\frac{3}{8} \div 1\frac{1}{18}$$

Give your answer as a mixed number in its lowest terms.

$2\frac{3}{8} = \frac{16}{8} + \frac{3}{8} = \frac{19}{8}$ and $1\frac{1}{18} = \frac{18}{18} + \frac{1}{18} = \frac{19}{8}$ ← convert to improper fractions

$2\frac{3}{8} \div 1\frac{1}{18} = \frac{19}{8} \div \frac{19}{18} = \frac{19}{\cdot} \times \frac{18}{19}$ flip second fraction and multiply.

cancel 19 from top and bottom. → $= \frac{\cancel{19}^{1}}{8} \times \frac{18}{\cancel{19}_{1}} = \frac{1}{8} \times 18$

$= \frac{18}{8} = \frac{16}{8} + \frac{2}{8} = 2\frac{2}{8}$

convert back → $= 2\frac{1}{4}$

(a) $2\frac{1}{4}$ **[3]**

(b) Write $\frac{5}{11}$ as a recurring decimal.

$\frac{5}{11} = \frac{45}{99}$

$= 0.\dot{4}\dot{5}$

find equivalent fraction with only 9s on denominator. number of 9s is the number of digits in recurring part of decimal.

or $\frac{5}{11} = 5 \div 11$

$11 \overline{)5.050505}$ = 0.4545...

(b) $0.\dot{4}\dot{5}$ **[2]**

(c) Write $0.\dot{3}\dot{6}$ as a fraction in its lowest terms.

let $x = 0.\dot{3}\dot{6}$

$100x = 36.\dot{3}\dot{6}$ multiply by 100 to move one full recurring sequence past the decimal point.

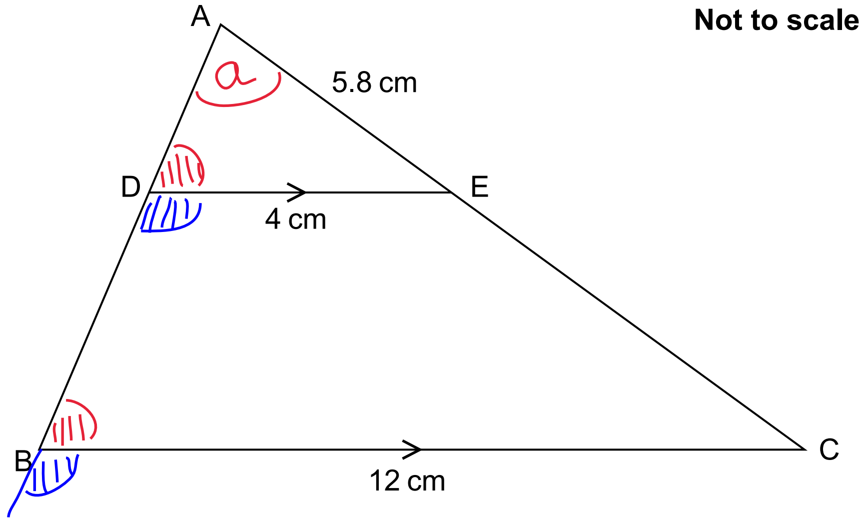
$-\quad x = 0.\dot{3}\dot{6}$

$99x = 36 \rightarrow x = \frac{36}{99} = \frac{4}{11}$

(c) $\frac{4}{11}$ **[3]**

$0.\dot{3}\dot{6} = \frac{4}{11}$

10 In the diagram BC is parallel to DE.



Not to scale

(a) Prove that triangle ABC is similar to triangle ADE. **[3]**

a is common to triangles ABC and ADE

$A\hat{D}E = A\hat{B}C$
$A\hat{E}D = A\hat{C}B$ corresponding angles are equal

3 pairs of equal angles
therefore triangle ABC is similar to triangle ADE.

(b) Calculate the length of AC.

scale factor $= \frac{BC}{DE} = \frac{12}{4} = 3$ all side pairs in same ratio.

$\frac{AC}{AE} = 3$ $AC = 3 \times 5.8 = 17.4$ cm

$5.8 \times 3 = 17.4$

(b) ....17.4.... cm **[2]**

(c) Find the ratio

area of quadrilateral DBCE : area of triangle ABC.

ratio of lengths 1 : 3
ratio of areas 1 : 9

1^2 3^2 Scale factor $k \to k^2$

area of quadrilateral DBCE = area of ABC − area of ADE
= 9 − 1 = 8 'parts' of ratio.

area of quadrilateral DBCE : area of triangle ABC
8 : 9

(c) ....8.... : ....9.... **[3]**

11 Evaluate.

$$16^{-\frac{3}{2}}$$

$16^{-\frac{3}{2}} = \left(\frac{1}{16}\right)^{\frac{3}{2}} = \sqrt{\frac{1}{16}}^{\,3} = \frac{\sqrt{1}}{\sqrt{16}}^{\,3} = \left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m \qquad a^{-n} = \frac{1}{a^n}$

$\frac{1}{64}$ .... **[3]**

12 **(a)** Expand and simplify.

$$(x+7)(x+2)$$

$= x^2 + 2x + 7x + 14$
$= x^2 + 9x + 14$

(a) $x^2 + 9x + 14$ .... **[2]**

(b) Factorise completely.

$$2x^2 - 6xy$$

$2 \times x \times x \qquad 6 \times x \times y = 2 \times 3 \times x \times y$

$2x$ is common

$= 2x(x - 3y)$

(b) $2x(x - 3y)$ .... **[2]**

(c) Solve.

$$x^2 + 5x = 24$$

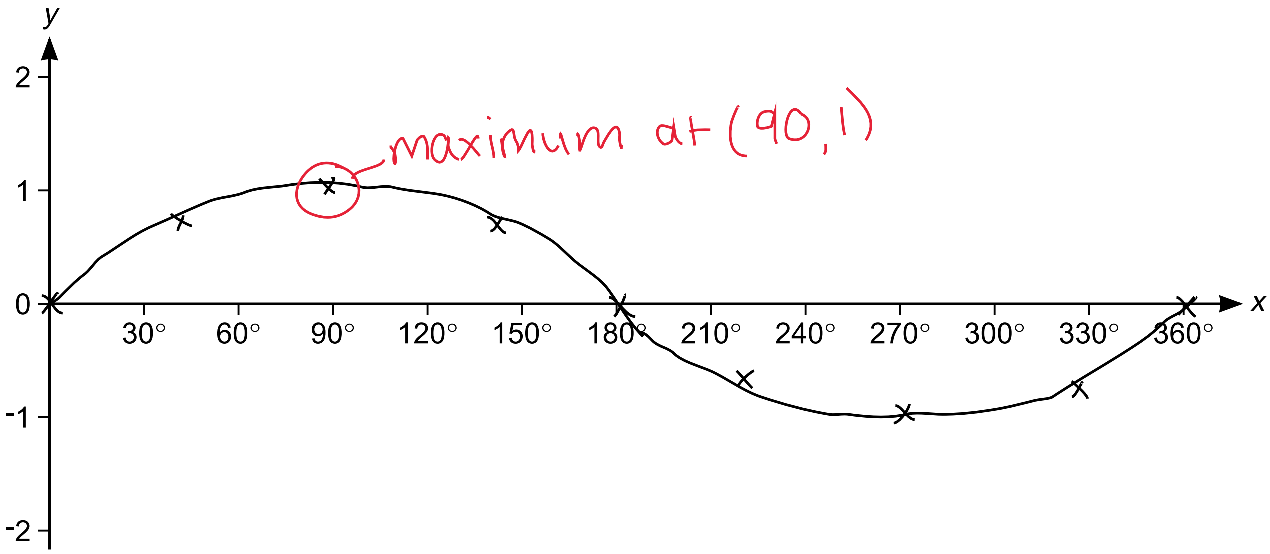
$x + 5x - 24 = 0$ rearrange to $ax^2 + bx + c = 0$
$(x+8)(x-3) = 0$ $(x+p)(x+q)$ where $pq = -24$ $p+q = 5$

$8 \times -3 = -24$
$8 + (-3) = 5$

if $x + 8 = 0$ then $x = -8$
if $x - 3 = 0$ then $x = 3$

(c) $x = 3$ and $x = -8$ .... **[3]**

13 **(a)** Sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.



[2]

(b) **(i)** Write down the coordinates of the maximum point of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

(b)(i) (90 , 1) **[1]**

(ii) Write down the coordinates of the maximum point of $y = 3 + \sin x$ for $0° \leqslant x \leqslant 360°$.

+ 3 is a translation 3 units up.

max: (90, 4)

x doesn't change

1+3=4

(ii) (90 , 4) **[1]**

(c) One solution to the equation $4 \sin x = k$ is $x = 60°$.

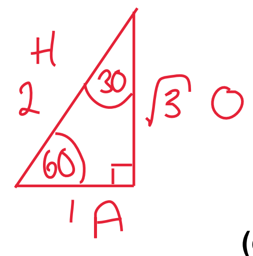
(i) Find the value of k.

$4\sin 60 = k$

$\sin 60 = \frac{O}{H} = \frac{\sqrt{3}}{2}$

$4\sin 60 = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$k = 2\sqrt{3}$



(c)(i) k = $2\sqrt{3}$ **[2]**

(ii) Find another solution for x in the range $0° \leqslant x \leqslant 360°$.

$4\sin x = 2\sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\sin 60 = \sin(180 - 60)$

$= \sin 120$

$x = 120°$

(ii) x = 120 ° **[1]**

14 Here is a sequence.

$$\underbrace{2 \qquad 2\sqrt{7}}_{\times\sqrt{7}} \qquad \underbrace{\qquad 14}_{\times\sqrt{7}} \qquad \underbrace{\qquad 14\sqrt{7}}_{\times\sqrt{7}}$$

(a) Work out the next term.

$14\sqrt{7} \times \sqrt{7} = 14 \times 7 = 98$

$$\begin{array}{r} 14 \\ \times\ 7 \\ \hline 9^28 \end{array}$$

(a) ....98.......... **[1]**

(b) Find the *n*th term.

first term = 2

common ratio = $\sqrt{7}$

nth term = $2 \times (\sqrt{7})^{n-1}$

geometric progression nth term is $a \times r^{n-1}$

common

first

(b) ....$2 \times (\sqrt{7})^{n-1}$.......... **[3]**

(c) Find the value of the 21st term divided by the 17th term.

21st term ÷ 17th term

21 - 17 = 4

21st ÷ 17th = common ratio^4

$= (\sqrt{7})^4$

$= (\sqrt{7})^2 \times (\sqrt{7})^2$

$= 7 \times 7$

$= 49$

$\times\sqrt{7}^4$

17th 18th 19th 20th 21st

$\times\sqrt{7}$ $\times\sqrt{7}$ $\times\sqrt{7}$ $\times\sqrt{7}$

$\times\sqrt{7}^2$ $\times\sqrt{7}^2$

$(\sqrt{a})^2$

$= \sqrt{a} \times \sqrt{a}$

$= a$

(c) ....49.......... **[2]**

15 Tony and Ian are each buying a new car.

There are three upgrades that they can select:

- metallic paint (10 different choices)
- alloy wheels (5 different choices)
- music system (3 different choices).

(a) Tony selects all 3 upgrades.

Show that there are 150 different possible combinations. **[1]**

10 choices of paint X 5 choices of wheels X 3 choices of music

$= 10 \times 5 \times 3$

$= 150$

(b) Ian selects 2 of these upgrades.

Show that there are 95 different possible combinations. **[3]**

metallic paint and alloy wheels: $10 \times 5 = 50$ combinations

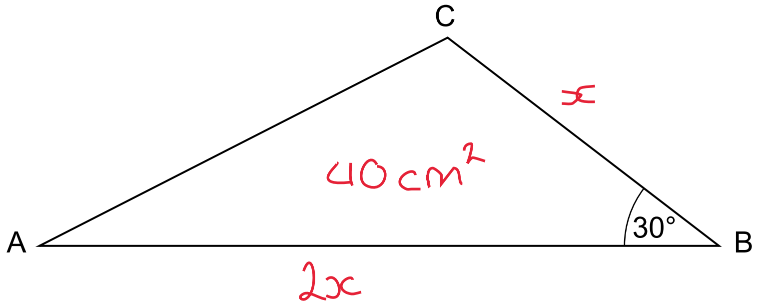
metallic paint and music system: $10 \times 3 = 30$ combinations

alloy wheels and music system: $5 \times 3 = 15$ combinations

total combinations: $50 + 30 + 15 = 95$ combinations

16 Triangle ABC has area 40 cm^2.
AB = 2BC.

Not to scale



Work out the length of BC.
Give your answer as a surd in its simplest form.

let BC = x cm   so AB = 2 × BC = $2x$ cm

area of $\triangle = \frac{1}{2}ab\sin C$

area = $0.5 \times x \times 2x \times \sin 30°$
= $0.5 \times 2x^2 \times \sin 30°$
= $x^2 \times \sin 30$
= $0.5x^2$

$\sin 30 = \frac{1}{2}$

$0.5x^2 = 40$
$x^2 = 80$

$x = \sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

BC = $4\sqrt{5}$ cm

$4\sqrt{5}$ .... cm **[6]**

17 A solid metal sphere has radius 9.8 cm.
The metal has a density of 5.023 g/cm^3.

Lynne estimates the mass of this sphere to be 20 kg.

Show that this is a reasonable estimate for the mass of the sphere. **[5]**

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

radius = 9.8 cm $\rightarrow$ 10cm
density = 5.023 g/cm^3 $\longrightarrow$ 5 g/cm^3

round to 1sf for estimate

volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (10)^3 = \frac{4000}{3}\pi$ cm^3

$= \frac{4000}{3} \times 3 = 4000$ cm^3

$\pi = 3.14... \rightarrow \pi = 3$

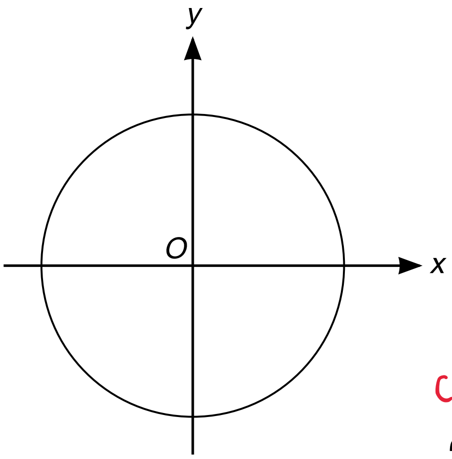
mass = 5 X 4000 = 20,000g
= 20kg

$\div 1000$

density = $\frac{\text{mass}}{\text{volume}}$ so $m = d \times V$

So the estimate is reasonable.

18 (a) The diagram shows a circle, centre *O*.



circumference $= \pi d = 2\pi r$

$2\pi r = 20\pi$

$r = 10\text{cm}$

The circumference of the circle is 20π cm.

Find the equation of the circle.

equation of a circle: $x^2 + y^2 = r^2$

centre $(0,0)$

(a) $x^2 + y^2 = 10^2$ **[4]**

(b) The line $10x + py = q$ is a tangent at the point $(5, 4)$ in another circle with centre $(0, 0)$.

Find the value of *p* and the value of *q*.

negative reciprocal

gradient of radius $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{5-0} = \frac{4}{5}$ gradient of tangent $= -\frac{5}{4}$

$m_1 \times m_2 = -1$ for perpendicular lines.

gradient of line $10x + py = q$ is $-\frac{5}{4}$

$10x + py = q$

$py = q - 10x$ rearrange for $y = mx + c$

$y = -\frac{10x}{p} + \frac{q}{p}$ $y = mx + c$ where $m = -\frac{10}{p}$

$-\frac{10}{p} \cancel{\times} -5p$

$8 = p$

cross multiply

$(5, 4)$ $10(5) + 8(4) = q$ ← substitute

$50 + 32 = q$

$82 = q$

(b) $p = 8$

$q = 82$ **[4]**

$p = 8, \quad q = 82$

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