

OCR

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Model Solution**H****Thursday 6 June 2019 – Morning****GCSE (9–1) Mathematics****J560/05 Paper 5 (Higher Tier)****Time allowed: 1 hour 30 minutes****You may use:**

- geometrical instruments
- tracing paper

Do not use:

- a calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s) _____

Last name _____

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document consists of **20** pages.



No calculator can
be used for this
paper

Answer **all** the questions.

1 Work out $(2 \times 10^3) \times (4 \times 10^4)$, giving your answer in standard form.

$$(2 \times 4) \times (10^3 \times 10^4)$$

$$8 \times 10^7$$

$$a^b \times a^c = a^{b+c}$$

..... 8×10^7 [2]

2 (a) Simplify fully.

$$\frac{3a^8 \times 2a^5}{a^2}$$

$$\frac{(3 \times 2) \times (a^8 \times a^5)}{a^2}$$

$$\frac{a^b}{a^c} = a^{b-c}$$

$$= \frac{6 \times a^{13}}{a^2} = 6 \frac{a^{13}}{a^2} = 6a^{13-2} = 6a^{11}$$

(a) $6a^{11}$ [3]

(b) Solve.

$$\frac{6x - 10}{5} = 1$$

$$\frac{6x - 10}{5} = 1$$

$$6x - 10 = 5 \quad \left. \begin{array}{l} \times 5 \\ +10 \end{array} \right\}$$

$$6x = 15 \quad \left. \begin{array}{l} \times 5 \\ +10 \end{array} \right\}$$

$$x = 5/2 \quad \left. \begin{array}{l} \times 5 \\ +10 \end{array} \right\} \div 6$$

(b) $x = \dots\dots\dots \frac{5}{2} \dots\dots\dots$ [3]

3 Ed has a card shop.

(a) He buys a particular card for £1.20 and sells it for £1.68.

Fraction $\times 100\%$ = Percentage

Calculate his percentage profit on this card.

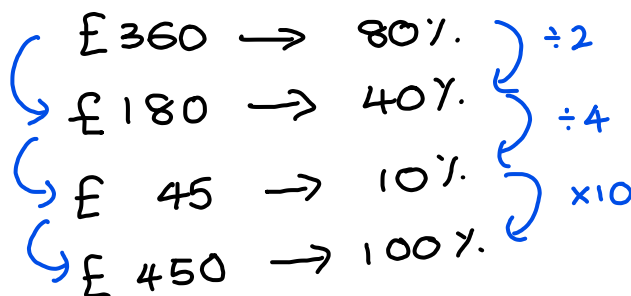
$$\text{Profit} = \text{£}1.68 - \text{£}1.20 = \text{£}0.48$$

$$\% \text{ Profit} = \frac{0.48}{1.20} = \frac{48}{120} = \frac{4}{10} \times 100\% = 40\%$$

(a) 40 % [3]

(b) Ed's profit on "Good Luck" cards in 2018 was £360. This was a decrease of 20% on his profit in 2017.

Work out Ed's profit on "Good Luck" cards in 2017.



(b) £ 450 [3]

4 (a) A sunflower grows at a rate of 4 cm each day.

How many days does it take to grow from a height of 80 cm to more than 1.06 m?

$$\begin{aligned} \text{Height increase} &= 1.06 \text{ m} - 80 \text{ cm} = 106 \text{ cm} - 80 \text{ cm} \\ &= 26 \text{ cm} \end{aligned}$$

Annotation: $1 \text{ m} = 100 \text{ cm}$ (with an arrow pointing to the 1.06 m term)

$$\text{No. of days} = \frac{26}{4} = \frac{13}{2} = 6.5 \approx 7$$

round up to nearest whole number as days are integers.

(a) 7 [3]

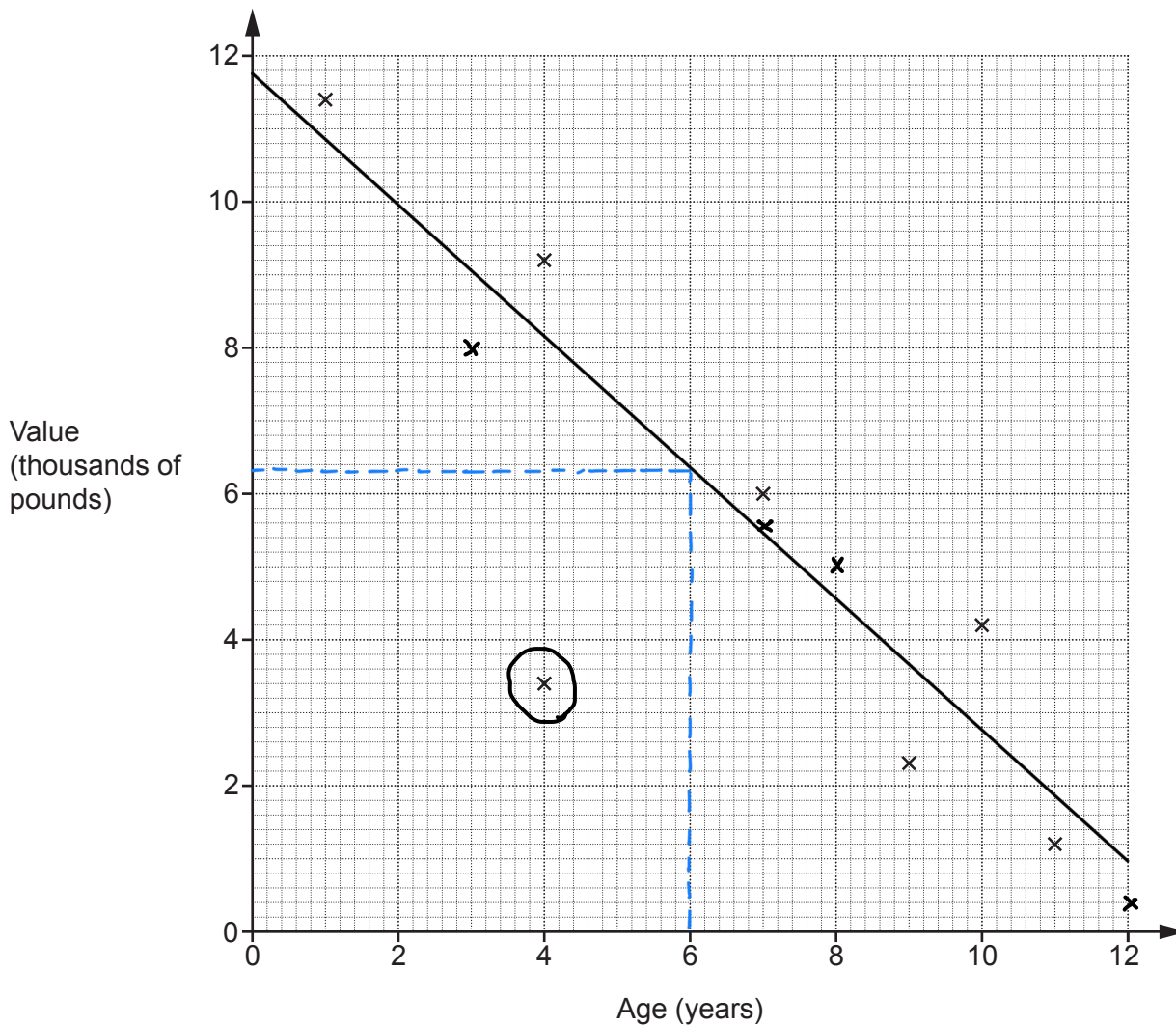
(b) If the sunflower grows at a faster rate, how would this affect your answer to part (a)?

It would take less time. [1]

5 The table shows the ages and values of 11 cars of the same model.

Age (years)	4	7	11	1	9	10	4	3	7	8	12
Value (thousands of pounds)	9.2	6.0	1.2	11.4	2.3	4.2	3.4	8.0	5.6	5.0	0.4

The points for the first 7 cars are plotted on the scatter diagram.



(a) Plot the points for the remaining 4 cars. [2]

(b) Describe the type and strength of the correlation shown in the completed scatter diagram.

Strong negative correlation. [2]

5

- (c) One car lost its value more quickly than the other cars.

On the scatter diagram, draw a circle around the point representing this car. [1]

- (d) By drawing a line of best fit, estimate the value of a car that is 6 years old.

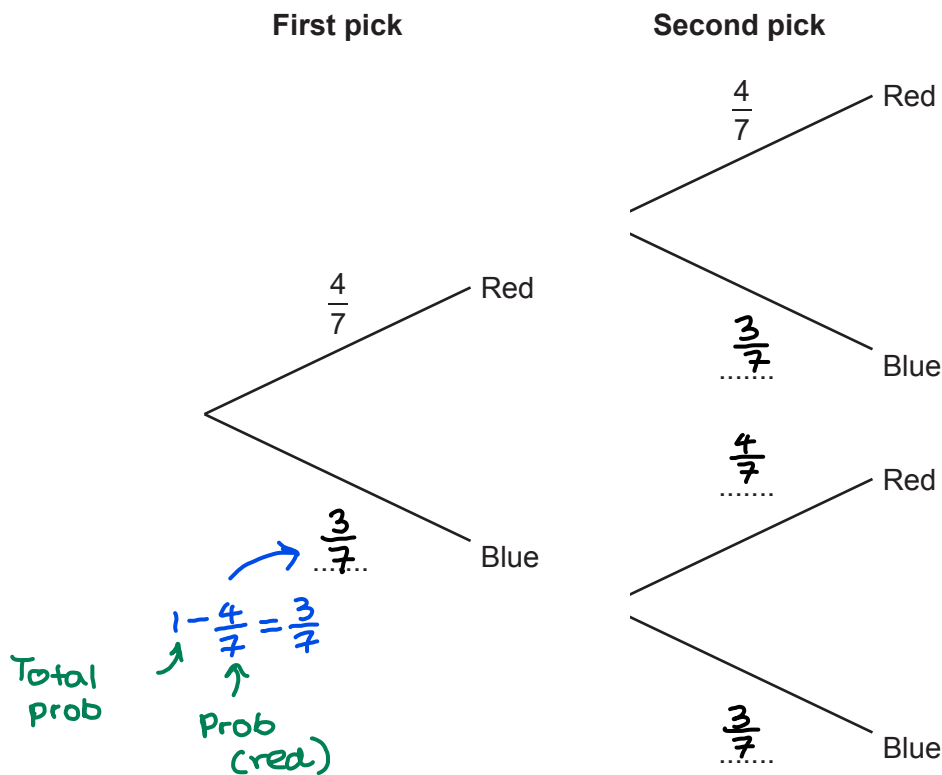
(d) £ 6300 [2]

- (e) Explain the limitations of using the equation of the line of best fit to estimate the value of a car that is 16 years old.

The current line of best fit would give a
negative value to the valuation. [1]

- 6 A bag contains 4 red counters and 3 blue counters only. Jack picks a counter at random and then replaces it. Jack then picks a second counter at random.

(a) Complete the tree diagram.



Repeat
 $P(\text{red}) = \frac{4}{7}$
 and
 $P(\text{blue}) = \frac{3}{7}$
 throughout as
 the counters
 are replaced.

[2]

- (b) Work out the probability that Jack picks two red counters.

$$P(\text{red}) \times P(\text{red})$$

└ red AND red

$$\frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$$

(b) $\frac{16}{49}$ [2]

- 7 Adam buys some theatre tickets in a sale.

The normal prices are:

£80 for each adult
£40 for each child.

In the sale, the prices are reduced by 15%.

Adam buys 2 adult tickets and 1 child ticket at the sale price.

A 2% booking fee is then added to the total cost of the tickets.



Calculate the total amount that Adam must pay.

A - Adult C - child.

$$\begin{aligned}
 \text{cost for } 2A+C &= 2(\text{£}80) + \text{£}40 \\
 \text{(pre sale and pre fee)} &= \text{£}160 + \text{£}40 \\
 &= \text{£}200
 \end{aligned}$$

$$\begin{aligned}
 \text{cost after discount} &= \left. \begin{array}{l} 100\% \rightarrow \text{£}200 \\ 1\% \rightarrow \text{£}2 \end{array} \right\} \div 100 \\
 \text{(pre fee)} & \left. \begin{array}{l} 85\% \rightarrow \text{£}170 \end{array} \right\} \times 85
 \end{aligned}$$

$$\begin{aligned}
 \text{Final cost} &= \text{£}170 + \overbrace{2\% \text{ of } \text{£}170}^{\text{fee}} \\
 &= \text{£}170 + \text{£} \frac{2}{100} \times 170 \\
 &= \text{£}170 + \text{£} 2 \times \frac{170}{100} \\
 &= \text{£}170 + \text{£} 2 \times 1.70 \\
 &= \text{£}170 + \text{£} 3.40 = \text{£}173.40
 \end{aligned}$$

£.....173.40..... [6]

8

8 Mrs Mills buys 4 packs of treats for her cats, Fluff and Tigger.

She gives Fluff $\frac{1}{6}$ of a pack each day.

She gives Tigger $\frac{1}{5}$ of a pack each day.

For how many complete days will the 4 packs of treats last?

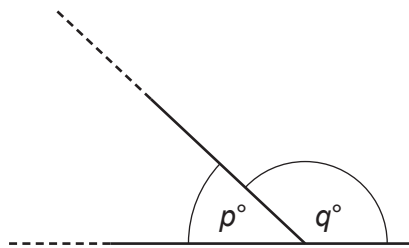
$$\begin{aligned} \text{Total consumption} &= \frac{1 \times 5}{6 \times 5} + \frac{1 \times 6}{5 \times 6} \\ \text{per day} &= \frac{5}{30} + \frac{6}{30} \\ &= \frac{11}{30} \end{aligned}$$

$$\begin{aligned} \text{No. of days} &= 4 \div \frac{11}{30} \\ \text{to last} &= 4 \times \frac{30}{11} \\ &= \frac{120}{11} \approx \frac{110}{11} \quad (\text{round down as} \\ &\quad \text{no. of Full days) \end{aligned}$$

≈ 10 full days

.....10..... [5]

- 9 An interior angle of an isosceles triangle is p° and an exterior angle is q° .



Not to scale

It is given that $q = 5p$.

- (a) Write the ratio $p : q$ in its simplest form.

for $q = 10$ $p = 2$ ($10 = 5p \rightarrow p = \frac{10}{5}$)

$\left(\begin{matrix} 2 : 10 \\ 1 : 5 \end{matrix} \right) \div 2$

(a)1..... :5..... [2]

- (b) Work out the two different possible sets of angles for the isosceles triangle.

on the instance of one angle being 30°

on the instance of two angles being 30°

$30^\circ + x + x = 180^\circ$ (angles in a triangle add up to 180).
 $2x = 180^\circ - 30^\circ$
 $x = \frac{150^\circ}{2} = 75^\circ$

$30^\circ + 30^\circ + x = 180^\circ$
 $x = 180^\circ - 60^\circ$
 $= 120^\circ$

$p + q = 180$

$p + (5p) = 180$

$6p = 180$

$p = 30$

(b) Triangle 1: ...30...°, ...75...°, ...75...°

Triangle 2: ...30...°, ...30...°, ...120...° [4]

10 (a) Write $\frac{1}{6}$ as a recurring decimal.

$$\begin{array}{r}
 0.166\dots \\
 \hline
 6 \overline{) 10} \\
 \underline{6} \\
 40 \\
 \underline{36} \\
 40 \\
 \vdots
 \end{array}$$

(a) $0.\dot{1}6$ [2]

(b) Elsa divides a two-digit number by another two-digit number. She gets the answer 0.15.

She says that there is only one possible pair of numbers that will give this answer. Is she correct? Show how you decide.

$$\begin{aligned}
 & \left. \begin{aligned} 0.\dot{1}5 &= 0.15555\dots \\ 10 \times 0.\dot{1}5 &= 1.5555 \end{aligned} \right\} - \\
 & 9 \times 0.\dot{1}5 = 1.4 \\
 & 0.\dot{1}5 = \frac{1.4}{9} = \frac{14}{90} \\
 & \qquad \qquad \qquad \xrightarrow{\times 10}
 \end{aligned}$$

..... She is correct as any other multiple of $\frac{14}{90}$ [4]
 would not be in the 2 digit range in the numerator or denominator.

11 (a) Simplify fully.

$$\sqrt{200}$$

$$\begin{aligned}
 & \sqrt{2 \times 100} \\
 & \sqrt{2} \times \sqrt{100} \\
 & \sqrt{2} \times 10
 \end{aligned}$$

(a) $10\sqrt{2}$ [2]

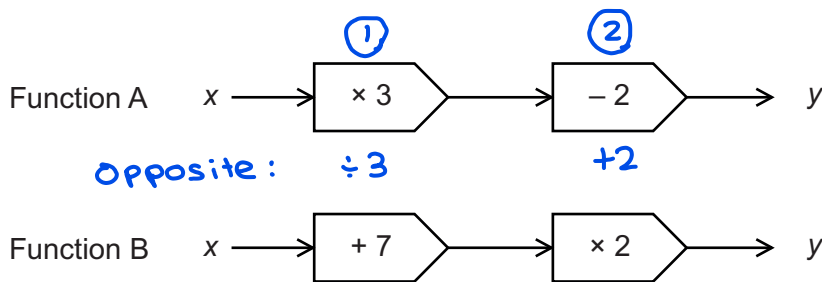
(b) Evaluate.

$$8^{\frac{1}{3}}$$

$$\begin{aligned}
 8 &= 2 \times 2 \times 2 = 2^3 \\
 (2^3)^{\frac{1}{3}} &= 2^{3 \times \frac{1}{3}} = 2^1 = 2
 \end{aligned}$$

(b) 2 [1]

12 Here are two functions.



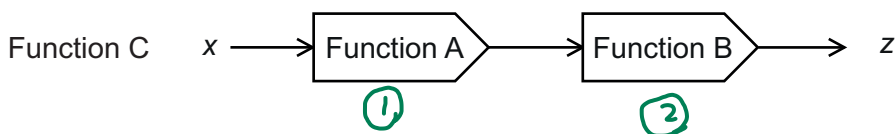
Opposite: $\div 3$ and $+2$

(a) Find an algebraic expression for the output of the **inverse** of function A when the input is x .

Input is x . $\frac{x+2}{3}$ (1) (2)
 No the opposite direction doing the opposite operation

(a) $\frac{x+2}{3}$ [2]

(b) Here is a composite function C.



Find the value x when $z = 4x$.

Write an equation for output z in terms of x .

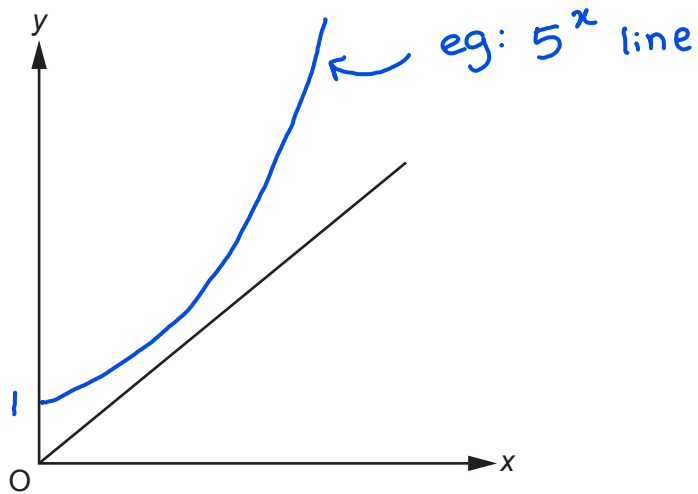
(1) $(x \times 3) - 2 \Rightarrow 3x - 2$

(2) $[(3x - 2) + 7] \times 2 \Rightarrow [3x + 5] \times 2 \Rightarrow 6x + 10 = z$

If $z = 4x$: $6x + 10 = 4x$
 $2x = -10$
 $x = -5$

(b) $x = -5$ [5]

- 13 Shirley is asked to sketch a graph of $y = 5^x$ for $x \geq 0$. She produces the following.

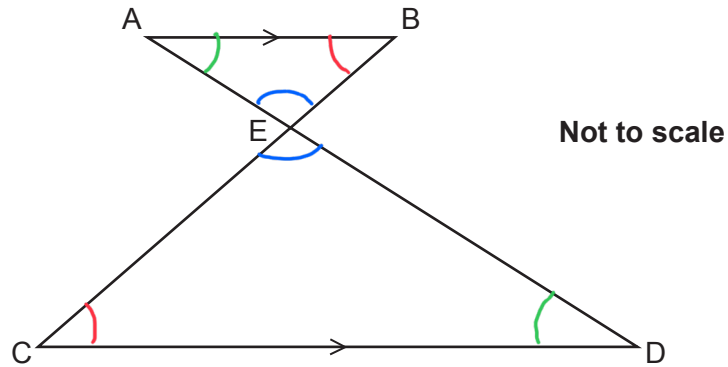


The graph has two errors.

How should they be corrected?

- 1 It should be a curve with increasing gradient.
 - 2 It should intercept y-axis at $(0, 1)$ as $5^0 = 1$
- [2]

- 14 In the diagram AB is parallel to CD.
AED and BEC are straight lines.



Prove that triangle ABE is similar to triangle CDE.

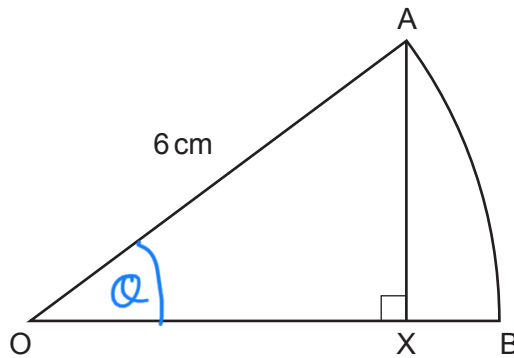
$$\hat{B}EA = \hat{C}ED \text{ (vertically opposite angles)}$$

$$\hat{D}AB = \hat{A}DC \text{ (Alternate angles)}$$

$$\hat{A}BC = \hat{D}CB \text{ (Alternate angles)}$$

[3]

- 15 OAB is a sector of a circle, centre O.
OA = 6 cm and AX is perpendicular to OB.



Not to scale

The area of sector OAB is $6\pi \text{ cm}^2$.

Show that $AX = 3\sqrt{3} \text{ cm}$.

[6]

$$\text{Area of a sector} = \frac{Q}{360} \times \pi \times r^2$$

$$\begin{aligned} \div \pi \left(\begin{array}{l} 6\pi = \frac{Q}{360} \times \pi \times 6^2 \\ 6 = \frac{Q}{360} \times 36 \end{array} \right. & \quad \text{(Subs area as } 6\pi \text{ and radius as } 6) \\ & \quad \text{and radius as } 6) \end{aligned}$$

$$\begin{aligned} \times 10 \left(\begin{array}{l} 6 = \frac{Q}{10} \\ Q = 60^\circ \end{array} \right. & \end{aligned}$$

In $\triangle OAX$:

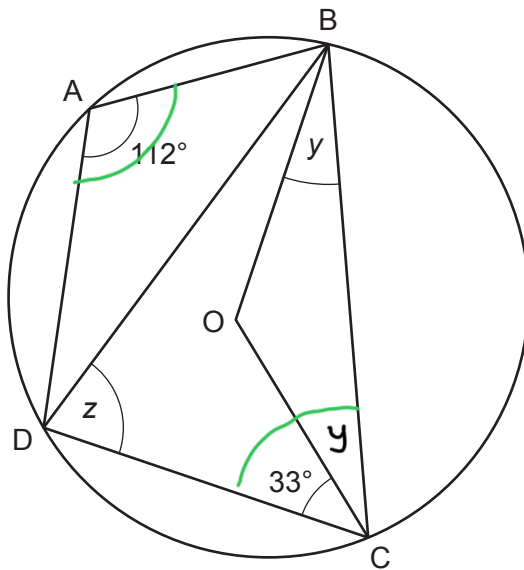
$$\sin Q = \frac{AX}{OA} \qquad \sin = \frac{O}{H}$$

$$\sin(60) = \frac{AX}{6}$$

$$\begin{aligned} \times 6 \left(\begin{array}{l} \frac{\sqrt{3}}{2} = \frac{AX}{6} \\ \frac{6}{2} \sqrt{3} = AX = 3\sqrt{3} \end{array} \right. & \end{aligned}$$

16 A, B, C and D are points on the circumference of a circle, centre O.

Angle BAD = 112° and angle DCO = 33°.



Not to scale

- (a) Show that angle $y = 35^\circ$.
Give reasons for each stage of your working.

[4]

$$\hat{D}CB = 180^\circ - 112^\circ \quad (\hat{D}AB \text{ and } \hat{D}CB \text{ are opposite angles in a cyclic quadrilateral}^*)$$

$$= 68^\circ$$

$$\hat{O}BC = \hat{O}CB = y \quad (\text{base angles are equal in an isosceles triangle})$$

$$\hat{D}CB = 33^\circ + y \Rightarrow 68^\circ = 33^\circ + y$$

$$\Rightarrow y = 68^\circ - 33^\circ = 35^\circ$$

- (b) Work out angle z .
Give reasons for your answer.

$$\hat{B}OC + \hat{O}CB + \hat{O}BC = 180^\circ \quad (\text{angles in a triangle sum to } 180^\circ)$$

$$\hat{B}OC + 35^\circ + 35^\circ = 180^\circ$$

$$\hat{B}OC = 180^\circ - 70^\circ = 110^\circ$$

$$z = \frac{1}{2} \times 110^\circ \quad (\text{Angles at the centre is twice the angle at the circumference})$$

Angle $z = 55^\circ$ because angles in a triangle sum to 180°
and Angles at the circumference is half the angle at the
centre

[3]

17 (a) Write $x^2 + 8x + 3$ in the form $(x+a)^2 - b$.

$$(x+4)^2 - 4^2 + 3 \Rightarrow (x+4)^2 - 16 + 3$$

↑ $\frac{8}{2}$ ↑ subtract the extra 4^2 ↑ write as it is ⏟ -13

↑ created when expanding $(x+4)^2$

(a) $(x+4)^2 - 13$ [3]

(b) Sketch the graph of $y = x^2 + 8x + 3$. Show clearly the coordinates of any turning points and the y-intercept.

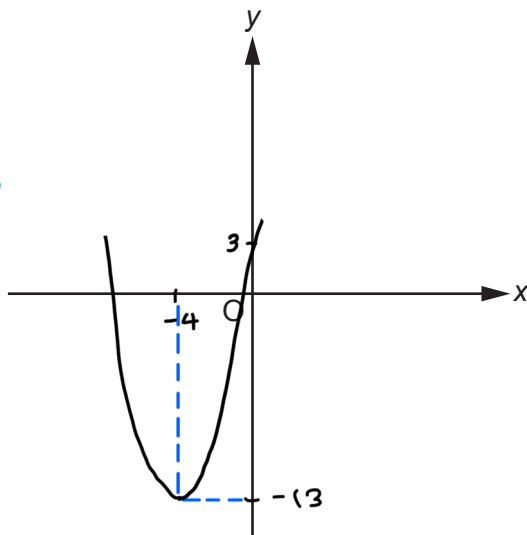
$$(x+4)^2 - 13$$

↓ ↑ $y = -13$

$$x+4=0$$

$$x = -4$$

$$\therefore (-4, -13)$$



$$y = x^2 + 8x + 3$$

↓

y-intercept
@ $y = 3$

[4]

18 21 people travelled to a meeting.

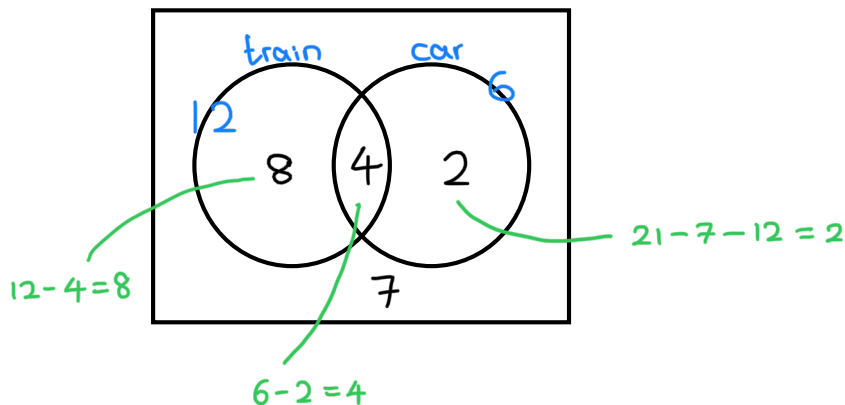
- 12 used a train.
- 6 used a car.
- 7 did not use a train or a car.
- Some used a train and a car.

Two people are chosen at random from those who used a train.

Find the probability that both these people also used a car.

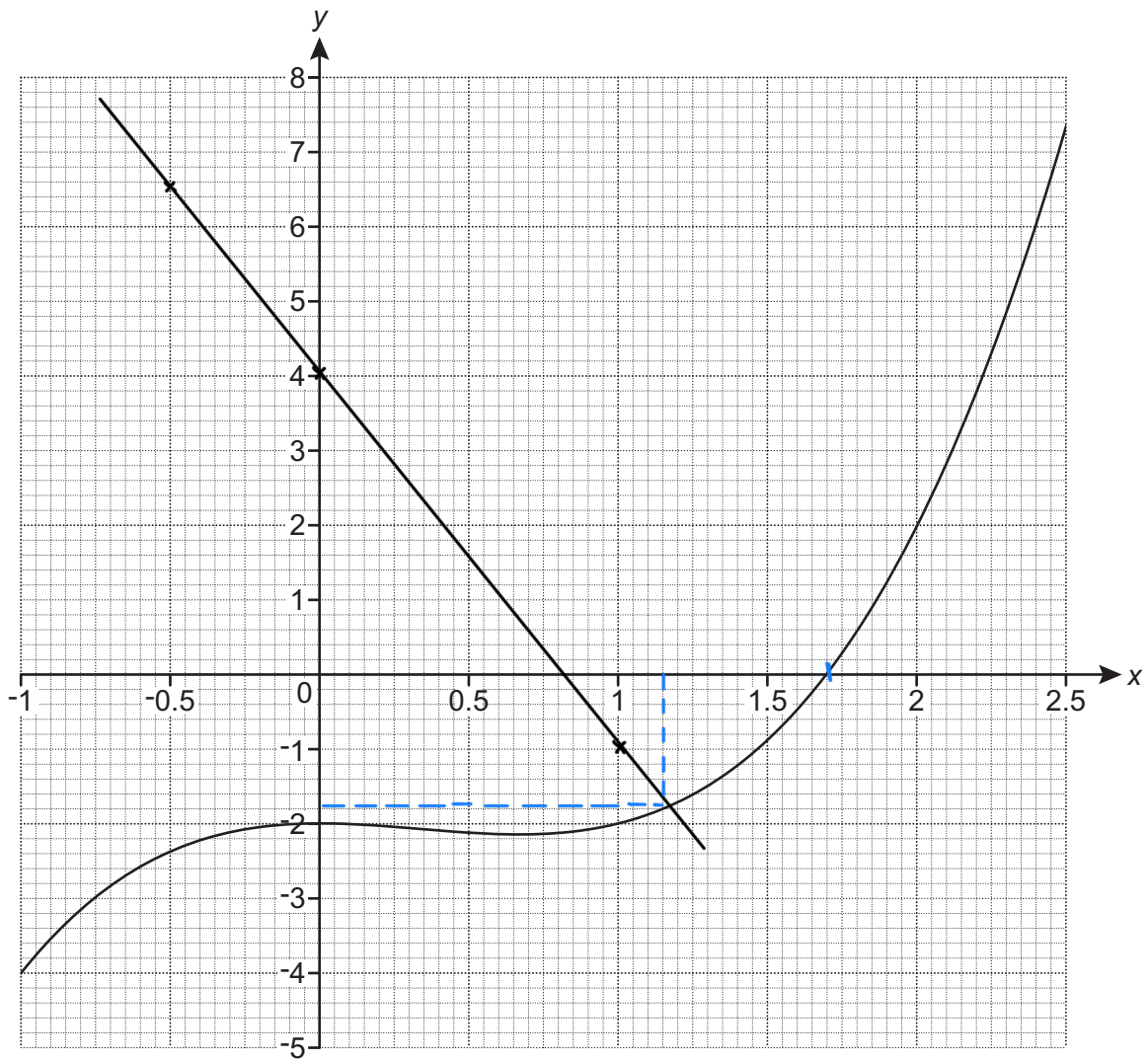
train AND car $\rightarrow \frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$

decreased by 1 because one was chosen before.



$\frac{12}{132}$ [6]

- 19 The graph of $y = x^3 - x^2 - 2$ is drawn on the grid.



- (a) Use the graph to solve $x^3 - x^2 - 2 = 0$.
Give your answer correct to 1 decimal place.

$$y = x^3 - x^2 - 2$$

$$\underbrace{x^3 - x^2 - 2}_y = 0$$

$$y = 0$$

take x at $y=0$

$$x = \dots\dots\dots 1.7 \dots\dots\dots [1]$$

(b) The equation $x^3 - x^2 + 5x - 6 = 0$ can be solved by finding the intersection of the graph of $y = x^3 - x^2 - 2$ and the line $y = ax + b$.

(i) Find the value of a and the value of b .

$$\begin{aligned}
 x^3 - x^2 + 5x - 6 &= 0 \\
 x^3 - x^2 + 5x - 2 - 4 &= 0 \\
 x^3 - x^2 - 2 + 5x - 4 &= 0 \\
 \underbrace{x^3 - x^2 - 2}_y &= \underbrace{-5x + 4}_{\text{the other line}}
 \end{aligned}$$

$$\begin{aligned}
 y &= -5x + 4 \\
 \uparrow \quad \uparrow \\
 a \quad b
 \end{aligned}$$

(b)(i) $a = \dots\dots\dots -5 \dots\dots\dots$

$b = \dots\dots\dots 4 \dots\dots\dots$ [2]

(ii) Hence, **use the graph** to solve the equation $x^3 - x^2 + 5x - 6 = 0$.
Give your answer correct to 1 decimal place.

x	$y = -5x + 4$
1	$-5(1) + 4 = -5 + 4 = -1 \quad (1, -1)$
-0.5	$-5(-0.5) + 4 = 2.5 + 4 = 6.5 \quad (2, 6.5)$
0	$-5(0) + 4 = 0 + 4 = 4 \quad (0, 4)$

$1.15 \approx 1.2$

(ii) $x = \dots\dots\dots 1.2 \dots\dots\dots$ [3]

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

A large rectangular area with a vertical solid line on the left side and horizontal dotted lines across the rest of the page, providing space for writing answers.



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