

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Thursday 8 June 2017 – Morning

Time allowed: 1 hour 30 minutes



You may use:

- Geometrical instruments
- Tracing paper

Do not use:

- A calculator



First name										
Last name										
Centre number						Candidate number				

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

INFORMATION

- The total mark for this paper is **100**
- The marks for each question are shown in brackets [].
- This document consists of **16** pages.

Answer **all** the questions.

1 Work out $\frac{2}{15} \times \frac{15}{22}$.

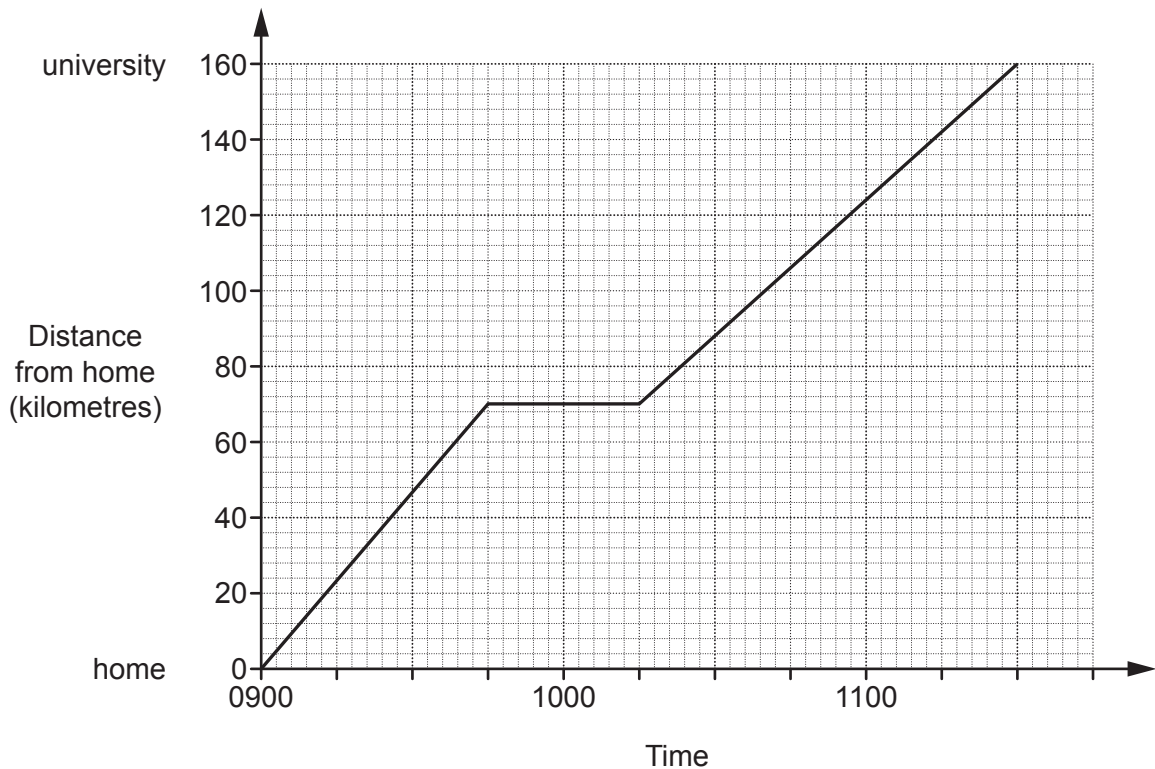
Give your answer in its lowest terms.

$$\frac{2}{15} \times \frac{15}{22} = \frac{2}{22} =$$

$$\frac{1}{11}$$

..... [2]

2 The graph shows Mia's journey from her home to university.



Calculate Mia's average speed for the whole journey.

$$\text{Avg speed} = \frac{\text{Total dist}}{\text{Total time}} = \frac{160\text{km}}{2.5}$$

$$\begin{aligned} & \text{9am to 11:30am} \\ & = 2.5 \text{ hours} \end{aligned}$$

..... 64 km/h [3]

- 3 Last year, Katie earned £16200.
Her total loan repayments were £6400.

Katie estimates that the ratio of her loan repayments to her earnings is approximately 3 : 8.

Is she correct?
Show your reasoning.

$$= 6400 : 16200$$

$$64 : 162$$

$$32 : 81$$

$$30 : 80 \quad 81 \approx 80$$

$$32 \approx 30$$

$$3 : 8$$

..... Yes, the estimation is correct [3]

- 4 Rashid invests money into an account which pays a fixed rate of compound interest each year. The value, £V, of his investment after t years is given by the formula

$$V = \underbrace{1250}_{\text{initial}} \times \underbrace{1.03^t}_{\text{multiplier}}$$

- (a) How much money did Rashid invest?

(a) £ 1250 [1]

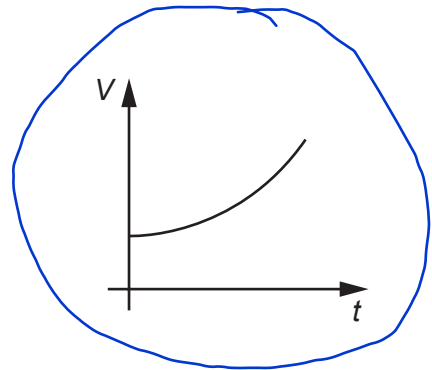
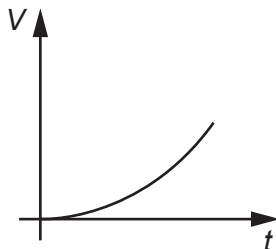
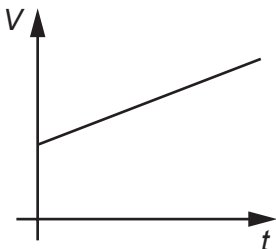
- (b) What rate of compound interest is paid each year?

$$1.03 - 1 = 0.03$$

$$= 3\%$$

(b) 3 % [1]

- (c) Circle the graph that best represents the growth in Rashid's account.



1
Exponential
Starting at
1250

[1]

5 (a) Rearrange the equation to make x the subject.

$$y = 7x - 3$$

$$+ 3$$

$$y + 3 = 7x$$

$$\div 7$$

$$\frac{y + 3}{7} = x$$

(a) $x = \frac{y + 3}{7}$ [2]

(b) Factorise.

(i) $x^2 - xy$ - x is the highest factor

$$x(x - y)$$

$x^2 \div x = x$
 $-xy \div x = -y$

(b)(i) $x(x - y)$ [1]

(ii) $x^2 + 8x + 12$

2 numbers \times to 12
 $+$ to 8 6, 2

(ii) $(x + 6)(x + 2)$ [2]

6 Jenny played four games of golf.
 For these games her modal score was 76 and her mean score was 75.
 Her range of scores was 10.

What were her scores for the four games?

Mode = 76 2 numbers are 76

Mean = $\frac{\text{Total}}{\text{Freq}}$

$$\frac{a + b + 76 + 76}{4} = 75$$

$$a + b + 152 = 300$$

$$a + b = 148 \text{ (2)}$$

Range: $b - a = 10 \text{ (1)}$

$$\begin{array}{r} a + b = 148 \text{ (1)} \\ - a + b = 10 \text{ (2)} \\ \hline 2b = 158 \\ b = 79 \end{array}$$

$$a = 79 - 10 = 69$$

..... 69

..... 76

..... 76

..... 79

[4]

7 The population of a village is in the following ratios.

- men : children = 11 : 3
- women : children = 5 : 2

(a) Find the ratio men : women.
Give your answer in its simplest form.

$$\begin{array}{l}
 \text{Men : children} \qquad \qquad \text{Women : children} \\
 11 : 3 \quad \times 2 \qquad \qquad 5 : 2 \quad \times 3 \\
 22 : 6 \qquad \qquad \qquad 15 : 6
 \end{array}$$

(a) 22 : 15 [2]

(b) There are 36 children in the village.

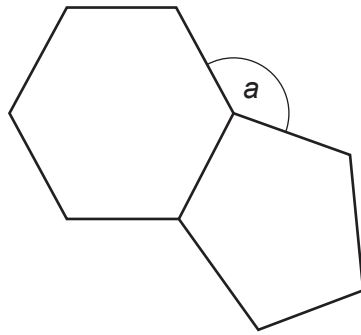
Find the total population of the village.

$$\begin{array}{l}
 \text{Ratio :} \quad \begin{array}{ccc} m & c & w \\ 22 & 6 & 15 \end{array} \\
 \quad \quad \quad \downarrow \times 6 \quad \quad \downarrow \times 6 \quad \quad \downarrow \times 6 \\
 \quad \quad \quad 132 \quad : \quad 36 \quad : \quad 90
 \end{array}$$

$$\begin{array}{r}
 132 \\
 + 36 \\
 + 90 \\
 \hline
 258
 \end{array}$$

(b) 258 [3]

- 8 Imran joins two tiles together as shown below.
One tile is a regular hexagon and the other tile is a regular pentagon.



Not to scale

Sum of external = 360
angles

- (a) Show that angle a is 132° .

$$\text{Hexagon's external angle} : 360 \div 6 = 60^\circ$$

$$\text{Pentagon's} : 360 \div 5 = 72$$

$$a = 60 + 72 = 132^\circ$$

.....
.....
..... [3]

- (b) Imran thinks that another tile in the shape of a regular polygon will fit exactly into angle a .

Is Imran correct?
Show your reasoning.

$$\text{Exterior angle} = 180 - 132 = 48$$

$$48 = \frac{360}{n}$$

$$n = \frac{360}{48}$$

$$\begin{array}{r} \overset{5}{48} \\ \times 7 \\ \hline 326 \end{array} \quad \begin{array}{r} \overset{6}{48} \\ \times 8 \\ \hline 384 \end{array}$$

No, n will not be a whole number so

no shape will fit exactly

..... [3]

- 9 George is the manager of a shoe shop. He samples 50 of his customers and asks them about the **one** style of shoe they would buy next. The table shows his results.

Style of shoe	Number of customers
Laced shoes	18
Boots	15
Sandals	8
Trainers	5
Other	4

George buys 1000 pairs of shoes with the number of each style based on his survey results.

How many pairs of sandals should he buy?

Write down any assumption you make about his sample.

$$\frac{\text{sandals}}{\text{Total}} : \frac{8}{50} \xrightarrow{\times 20} \frac{160}{1000} = 160 \text{ sandals}$$

The 50 people are representative of the whole population [3]

- 10 (a) Solve the inequality.

$$\begin{aligned} 3x - 2 &> 10 \\ + 2 & \\ 3x &> 12 \\ x &> 4 \end{aligned}$$

(a) $x > 4$ [2]

- (b) Solve.

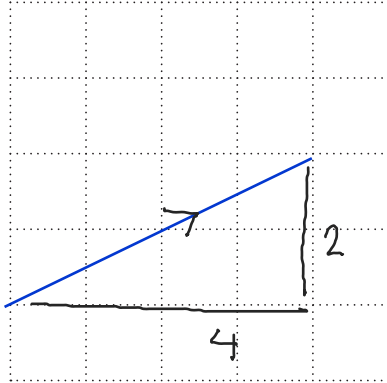
$$\begin{aligned} 6x + 2 &= 5 - 4x \\ + 4x & \\ 10x + 2 &= 5 \\ - 2 & \\ 10x &= 3 \\ \div 10 & \\ x &= \frac{3}{10} \end{aligned}$$

(b) $x = 0.3$ [3]

11 Vector $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, vector $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

(a) On each grid below, draw a vector to represent

(i) $2\mathbf{a}$, $2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ -4 right
-2 up



(ii) $\mathbf{a} + \mathbf{b}$, $\begin{pmatrix} 2 & -2 \\ 1 & +1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} = 2 \text{ up}$



[2]

(b) Emma says that if she draws vector \mathbf{a} and vector \mathbf{b} they will be the same.

Explain why this is incorrect.

\mathbf{a} is 2 right 1 up, \mathbf{b} is 2 left 1 up so they are in different directions negative [1]

(c) $\mathbf{c} = \begin{pmatrix} -12 \\ 0 \end{pmatrix}$.

Find the value k so that $k(\mathbf{a} - \mathbf{b}) = \mathbf{c}$.

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -12 \\ 0 \end{pmatrix} \div \begin{pmatrix} 4 \\ 0 \end{pmatrix} = -3$$

$-12 \div 4 = -3$ (c) $k = -3$ [2]

12 Helen delivers parcels.

On Tuesday, Helen delivered 20% more parcels than on Monday.

On Wednesday, Helen delivered 50% fewer parcels than on Tuesday.

On Wednesday, Helen delivered 72 parcels.

Calculate the number of parcels that Helen delivered on Monday.

Wed : 50% of Tues

72 = 1/2 of Tues

144 ^{x2} = Tues

Tues = 120% of Mon *-20% more*

144 = 120%

12 ^{÷12} = 10%

120 ^{x10} = 100%

..... 120 [5]

13 (a) Write $\frac{7}{9}$ as a recurring decimal.

$$\begin{array}{r} 0.777\ldots \\ 9 \overline{) 7.7000\ldots} \end{array}$$

(a) = $0.\overline{7}$ [1]

(b) Sally divided a two-digit number by another two-digit number. Her answer was 3.181818.....

Find two numbers that Sally could have used.

$x = 3.\overline{18}$

$$\begin{array}{r} 100x = 318.\overline{18} \\ x = 3.\overline{18} \\ \hline 99x = 315 \end{array}$$

$x = \frac{315}{99} = 315 \text{ divided by } 99$

has to be 2 digit

(b) 35 and 11 [3]

$$\begin{array}{r} 35 \\ 9 \overline{) 315} \\ \underline{27} \\ 45 \\ \underline{45} \\ 0 \end{array} \quad \begin{array}{r} 315 \\ \underline{99} \\ 216 \\ \underline{198} \\ 18 \\ \underline{18} \\ 0 \end{array} \quad \begin{array}{r} 35 \\ \underline{9} \\ 26 \\ \underline{22} \\ 4 \\ \underline{4} \\ 0 \end{array}$$

14 John has

- 8 different shirts
- 6 different hats
- 4 different scarves.

$$\begin{array}{r} 3 \\ 48 \\ \times 4 \\ \hline 192 \end{array}$$

(a) On Monday, he picks a shirt, a hat and a scarf.

Show that there are ~~192~~ different combinations he can pick.

$$8 \times 6 \times 4 = 48 \times 4 = 192 \text{ combinations} \quad [1]$$

(b) John thinks that if he picks just two of the three items of clothing there will be more than 192 combinations.

Is he correct?

Show your reasoning.

$$\begin{array}{l} \text{shirts and hats} : 8 \times 6 = 48 \\ \text{shirts and scarves} : 8 \times 4 = 32 \\ \text{hats and scarves} : 6 \times 4 = 24 \\ \hline 104 \end{array}$$

$$\begin{array}{l} \text{No there will be only 104 ways} \\ 104 < 192 \end{array} \quad [3]$$

15 (a) Simplify fully.

(i) $\sqrt{50} + \sqrt{2}$

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$5\sqrt{2} + \sqrt{2}$$

(a)(i) $6\sqrt{2} \quad [2]$

(ii) $\frac{10}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$$= \frac{10\sqrt{6}}{6} \stackrel{\div 2}{=} \frac{5\sqrt{6}}{3}$$

(ii) $\frac{5\sqrt{6}}{3} \quad [2]$

(b) There are two errors in Sam's method for finding the value of $64^{-\frac{2}{3}}$ shown below.

Find the cube root of 64 and then multiply by 2.

The cube root of 64 is 4 and then $4 \times 2 = 8$.

The negative power makes the answer negative so answer equals -8.

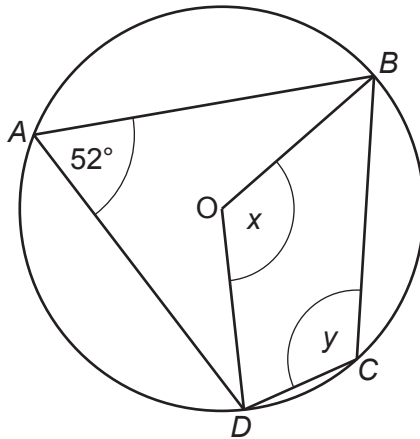
Describe these errors and then give the correct value of $64^{-\frac{2}{3}}$.

$$= (\sqrt[3]{64})^{-2} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Don't multiply by 2 instead square value
 Negative power is reciprocal

Correct value $\frac{1}{16}$ [3]

16 A, B, C and D are points on the circumference of a circle, centre O.



Not to scale

Angle BAD = 52° .

$$52 \times 2 = 104$$

(a) Work out angle x.
 Give a reason for your answer.

$x = 104^\circ$ reason Angle at circumference is half the angle at the origin [2]

(b) Work out angle y.
 Give a reason for your answer.

$$180 - 52$$

$y = 128^\circ$ reason in cyclic quadrilaterals opposite angle add to 180 [2]

17 (a) Simplify.

$$\frac{x^2 - 16}{x^2 - 3x - 4}$$

Difference of 2 squares

Factorise x to -4
 $+$ to -3 $-4, 1$

$$= \frac{(x+4)(x-4)}{(x+1)(x-4)}$$

$$(a) \frac{x+4}{x+1} \dots \dots \dots [4]$$

(b) $(x+3)(x-4)(x+5)$ is identical to $x^3 + ax^2 - 17x + b$.Find the value of a and the value of b .

$$(x+3)(x-4) = x^2 + 3x - 4x - 12 = x^2 - x - 12$$

$$x^2 - x - 12 (x+5)$$

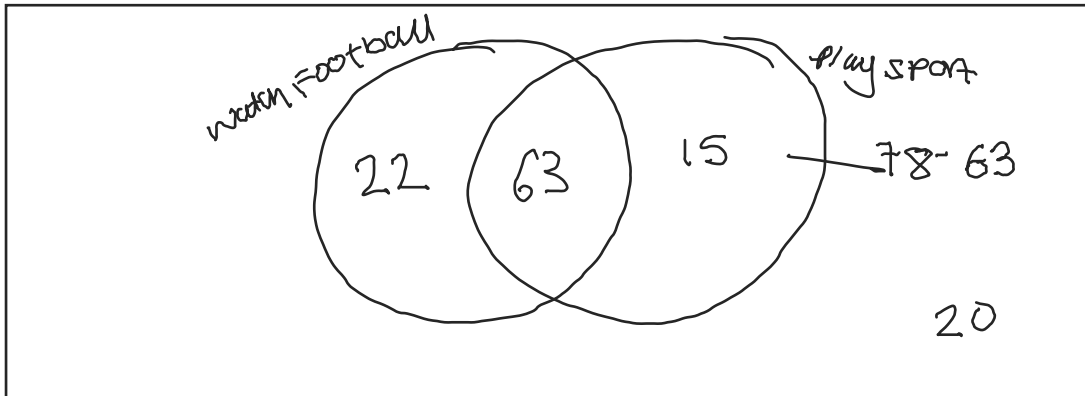
$$= x^3 + 5x^2 - x^2 - 5x - 12x - 60$$

$$= x^3 + \frac{4x^2}{a} - 17x - \frac{60}{b}$$

$$(b) \begin{aligned} a &= \frac{4}{\dots \dots \dots} \\ b &= \frac{-60}{\dots \dots \dots} \end{aligned} [2]$$

- 18 In a group of 120 adults, 85 watch football, 78 play a sport and 20 do neither.

Find the probability that an adult chosen at random from those who watch football does not play a sport.



Watch Football and doesn't play:

$$120 - 20 - 78 = 22$$

$$\text{Watch Football and play} = 85 - 22 = 63$$

$$P(\text{Doesn't play} \mid \text{watches football})$$

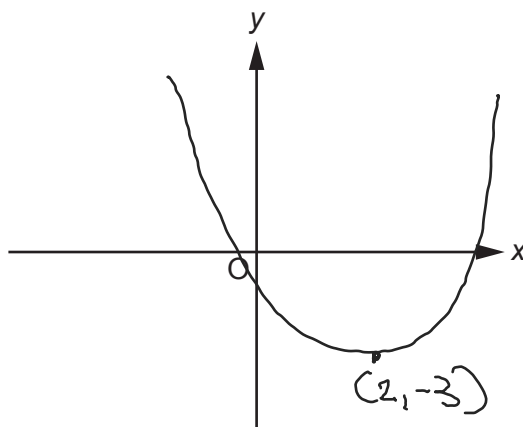
$$= \frac{22}{85}$$

$$\frac{22}{85}$$

..... [5]

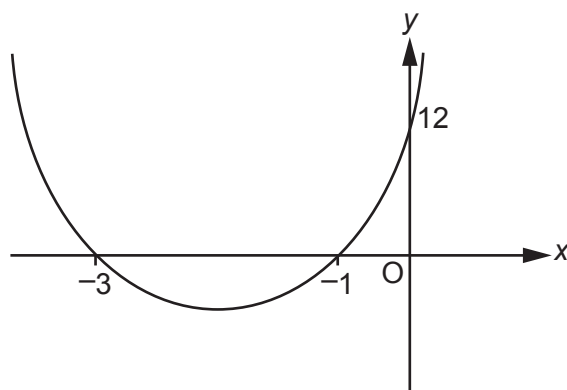
- 19 (a) Sketch the graph of $y = (x-2)^2 - 3$.
Show the coordinates of any turning points.

Turning point = $(-2, -3)$
= $(2, -3)$



[3]

- (b) The sketch shows part of a graph which has equation $y = ax^2 + bx + c$.



Not to scale

Find the values of a , b and c .

$c = 12$ y intercept

$x = -3$ $x = -1$

$$y = r(x+q)(x+p)$$

$$y = r(x+3)(x+1)$$

$$y = r(x^2 + 4x + 3)$$

We know $c = 12$

$$3r = 12$$

$$\text{so } r = 4$$

$$a = 4 \times 1 = 4$$

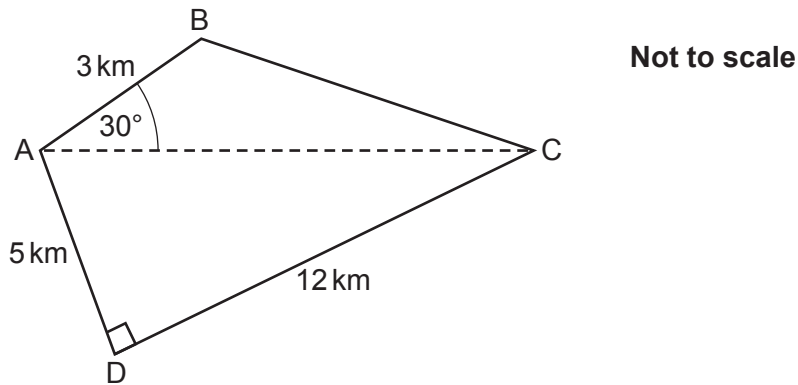
$$b = 4 \times 4 = 16$$

(b) $a = \dots 4 \dots$

$b = \dots 16 \dots$

$c = \dots 12 \dots$ [5]

20 The diagram shows some land in the shape of a quadrilateral, ABCD.



AB = 3 km, AD = 5 km, CD = 12 km and angle BAC = 30°.

The land is sold for £10 million per square kilometre.

Pythagoras: $a^2 + b^2 = c^2$

Calculate the total cost of the land.

Length AC:

$$AC^2 = 5^2 + 12^2$$

$$AC^2 = 25 + 144$$

$$AC^2 = 169$$

$$AC = 13 \text{ km}$$

Area ACD:

$$\frac{1}{2} \times 5 \times 12 = 30 \text{ km}^2$$

Area = $\frac{1}{2} ab \sin C$

Area ABC:

$$\frac{1}{2} \times 3 \times 13 \times \sin 30^\circ$$

$$= \frac{39}{4} = 9\frac{3}{4} = 9.75$$

$$\text{Area} = 30 + 9.75 = 39.75 \text{ km}^2$$

Price

$$1 \text{ km}^2 : 10 \text{ mill}$$

$$39.75 \text{ km}^2 = 397.5 \text{ mill}$$

£ 397.5 million Million [7]

21 n is an integer.

(a) Explain why $2n + 1$ is an odd number.

Because $2n$ will always be even as it is a multiple of 2. $\text{even} + 1 = \text{even} + \text{odd} = \text{odd}$ [1]
 so $2n + 1 = \text{odd}$

(b) Prove that the difference between the squares of two **consecutive** odd numbers is a multiple of 8.

consecutive odd : $(2n-1), (2n+1)$

Squares : $(2n+1)(2n+1) = 4n^2 + 4n + 1$
 $(2n-1)(2n-1) = 4n^2 - 4n + 1$
 Difference : $8n$

The difference is $8n$ which is a multiple of 8

[5]

END OF QUESTION PAPER

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