MODEL SOLUTIONS

OCR
Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Practice Paper

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You may use:
- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper

First name									
Last name									
Centre number						**Candidate number**				

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions

1 **(a)** The attendance at a football match was 67 500, correct to the nearest hundred.

(i) What was the **highest** possible attendance?

67549 - highest number that rounds down to 67500

(a)(i) 67549 ..... **[1]**

(ii) What was the **lowest** possible attendance?

67450 - lowest number that rounds up to 67000

(ii) 67450 ..... **[1]**

(b) A distance, d, was given as 6.73 m, **truncated** to 2 decimal places.

Complete the error interval for the distance, d.

Truncation - cutting off the rest of the number.

6.73 $\leqslant d <$ 6.74 **[2]**

2 The population, P, of an island t years after January 1st 2016 is given by this formula.

$$P = 4200 \times 1.04^t$$

(a) What was the population of the island on January 1st 2016?

$t = 0$
$P = 4200 \times 1.04^0$
$= 4200$

(a) 4200 **[1]**

(b) Explain how you know that the population is increasing.

1.04 is greater than 1.

[1]

(c) What is the annual percentage increase in the population?

$1 - (0.8 \times 0.75) = 0.4$
$0.4 \times 100 = 40\%$

(c) 40 % **[1]**

(d) Work out the population of the island on January 1st 2021.

$P = 4200 \times 1.04^5$ 2021 is 5 years after 2016.
$= 5109.94$
$= 5109$ (can't have decimal of a person)

(d) 5109 **[2]**

3 A shop has a sale that offers 20% off all prices.
On the final day they reduce all sale prices by 25%.
Alex buys a hairdryer on the final day.

Work out the **overall** percentage reduction on the price of the hairdryer.

100 - 20 = 80%.
100 - 25 = 75%.
0.8 X 0.75 = 0.6
0.6 X 100 = 60%.
100 - 60 = 40%. overall reduction

$0.8 = \frac{80}{100} = 80\%$

40 ..... % **[6]**

4 An interior angle of a regular polygon is eleven times its exterior angle.

Work out the number of sides of the polygon.

interior angle + exterior angle = 180°
x = interior angle
x + 11x = 180
12x = 180
÷12 x = 15° ÷12
360 ÷ 15 = 24 sides

24 sides ..... **[4]**

5 **(a)** Find the *n*th term of this linear sequence.

8 11 14 17

+3 +3 +3

3n: 3 6 9 12

3n + 5 as the difference between the two sequences is + 5.

(a) 3n + 5 **[2]**

(b) Here is a quadratic sequence.

2 14 36 68

The expression for the *n*th term of this sequence is $pn^2 + qn$.

Find the value of *p* and the value of *q*.

2 14 36 68

12 22 32

10 10

second difference is 10

$10 \div 2 = 5$ so $5n^2$

2	14	36	68
− 5	20	45	80
-3	-6	-9	-12

-3 -3 -3 −3n

Find the difference between the original sequence and $5n^2$

$p = 5, q = -3$

(b) *p* = 5

q = -3 **[4]**

6 Some of the children at a nursery arrive by car.

- 40% of the children at the nursery are boys.
- 70% of the boys at the nursery arrive by car.
- 60% of the girls at the nursery arrive by car.

What is the probability that a child chosen at random from the nursery arrives by car?

100 - 40 = 60% girls

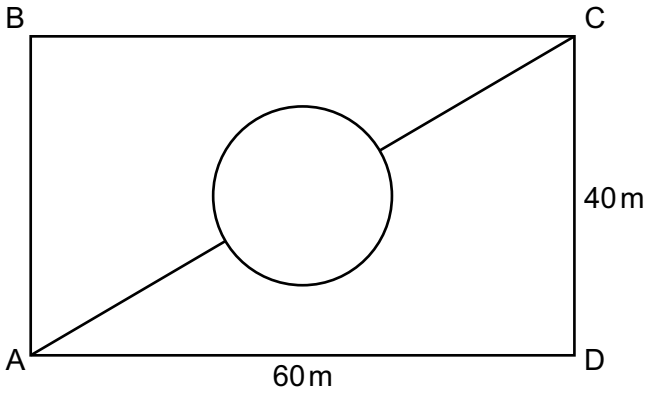
40% = $\frac{40}{100}$ = 0.4

0.4 X 0.7 + 0.6 X 0.6 = 0.64

and or

0.64 **[5]**

7 The rectangle ABCD represents a park.



Not to scale

The lines show all the paths in the park.
The circular path is in the centre of the rectangle and has a diameter of 10 m.

Calculate the shortest distance from A to C across the park, using only the paths shown.

$\sqrt{40^2 + 60^2} = 20\sqrt{13}$ Pythagoras' Theorem

$20\sqrt{13} - 10 = 20\sqrt{13} - 10$

$\frac{1}{2} \times \pi \times 10 = 5\pi$ circumference $= \pi d$

$20\sqrt{13} - 10 + 5\pi = 77.8\text{m}$

77.8 ..... m **[6]**

8 Eddie and Caroline are going to the school play.

Eddie buys 6 adult tickets and 2 child tickets. He pays £39.
Caroline buys 5 adult tickets and 3 child tickets. She pays £36.50.

Work out the cost of an adult ticket and the cost of a child ticket.

a = adult ticket c = child ticket

$6a + 2c = 39$ [X3] → $18a + 6c = 117$ (1)

$5a + 3c = 36.5$ [X2] → $10a + 6c = 73$ (2)

Make coefficients of c the same.

$18a + 6c = 117$
$- \; 10a + 6c = 73$

$8a = 44$

$\div 8$ $a = £5.50$ $\div 8$

$6 \times 5.50 + 2c = 39$

sub a = 5.50 into (1)

$33 + 2c = 39$

-33 $2c = 6$ -33

$\div 2$ $c = 3$ $\div 2$

Adult ticket £ ..... 5.50 .....

Child ticket £ ..... 3 ..... **[5]**

9 Gavin measures the heights of 80 plants he has grown.
This table summarises his results.

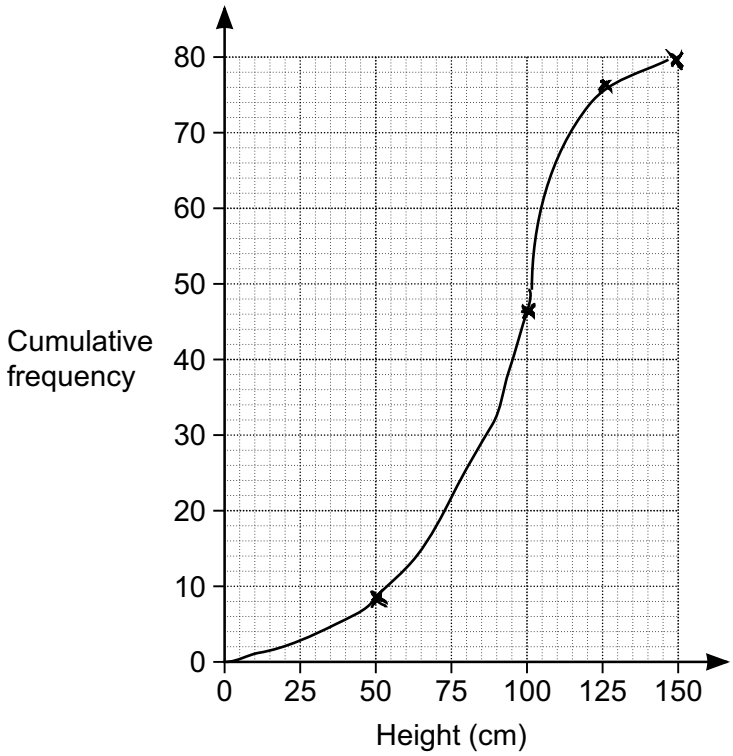
Height, h cm	$0 < h \leqslant 50$	$50 < h \leqslant 100$	$100 < h \leqslant 125$	$125 < h \leqslant 150$
Number of plants	8	38	31	3

(a) **(i)** Complete the cumulative frequency table below.

Height, h cm	$h \leqslant 50$	$h \leqslant 100$	$h \leqslant 125$	$h \leqslant 150$
Cumulative frequency	8	46	77	80
		8 + 38	46 + 31	77 + 3

[2]

(ii) Draw the cumulative frequency graph.



[2]

(b) Ted asks if Gavin has 10 plants over 120 cm in height.

Explain why Gavin cannot be certain that he has 10 plants over this height.

He does not have the heights of individual plants and these are not shown on the cumulative frequency graph.

..... **[1]**

(c) Gavin sells these 80 plants using the price list below.

Height, h cm	$h \leqslant 80$	$80 < h \leqslant 120$	$h > 120$
Price (£)	2.00	3.50	5.00

Each plant costs him 60p to grow.

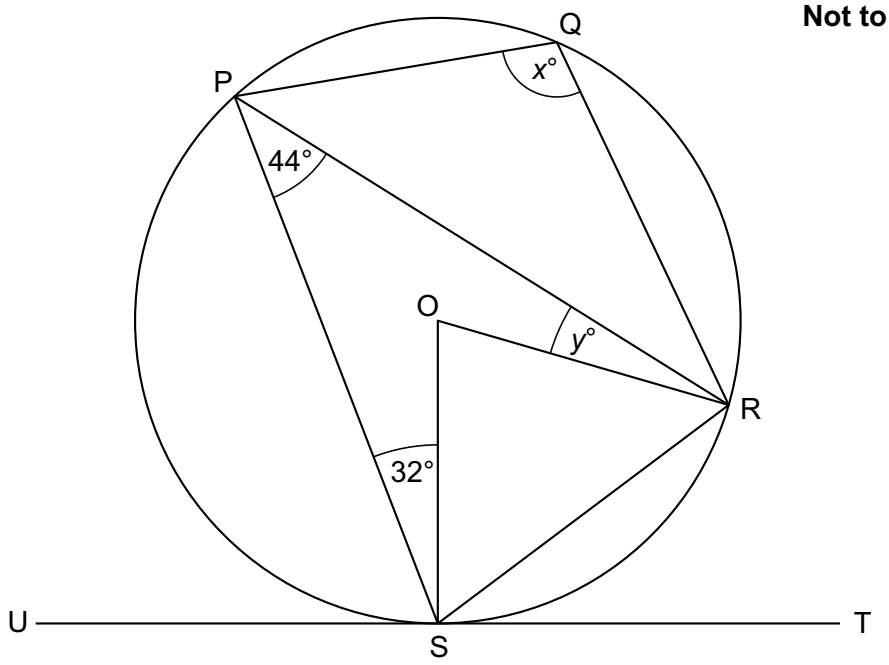
Estimate the total profit Gavin will receive when he sells all these plants.

30 X 2 (69 - 30) X 3.5 + (80 - 69) X 5 = £251.50

251.50 - 80 X 0.6 = £203.50

(c) £ ....203.50.... **[6]**

10 The diagram shows a circle, centre O.
Points P, Q, R and S lie on the circumference of the circle.
UST is a tangent to the circle.
Angle RPS = 44° and angle PSO = 32°.



(a) Work out the value of x.

$\angle SOR = 2 \times 44 = 88°$ — The angle at the centre is 2X the angle at the circumference.

$RO = SO$ — Isosceles triangle, both are radii.

$\angle OSR = \angle ORS = z$

$88 + 2z = 180°$ — Interior angles in a triangle sum to 180.

$-88 \quad 2z = 92° \quad -88$

$\div 2 \quad z = 46° \quad \div 2$

$\angle PSR = 32 + 46 = 78°$

$78 + x = 180°$ — Opposite angles in a cyclic quadrilateral add up to 180.

$x = 102°$

(a) x = 102° **[4]**

(b) Work out the value of y.

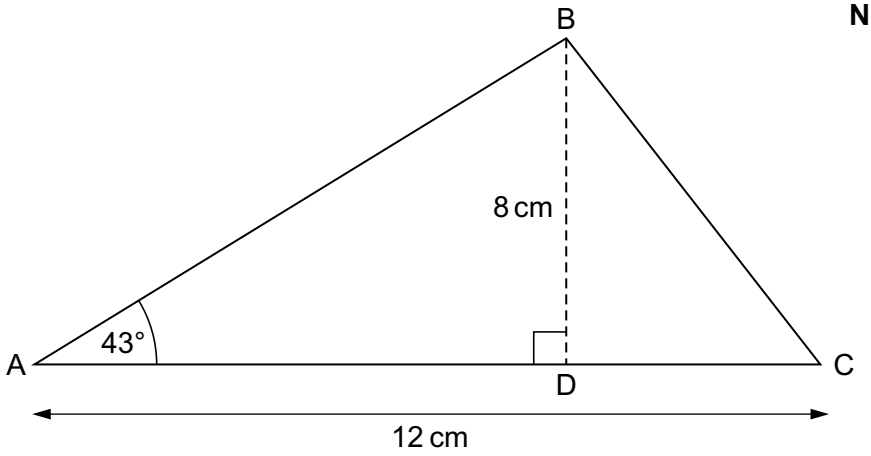
$\angle PSU = 90 - 32 = 58°$ — The angle between tangent and radius is 90.

$\angle ORS = 46$

$58 - 46 = 12°$

(b) y = 12° **[3]**

11 In the diagram, ABC is a triangle and line BD is perpendicular to AC.
Angle BAC = 43°, BD = 8 cm and AC = 12 cm.



Not to scale

Calculate angle BCA.

$\tan\theta = \frac{O}{A}$

$\tan 43 = \frac{8}{OD}$

$\frac{8}{\tan 43} = OD$

$OD = 8.58\text{cm}$

$12 - 8.58 = 3.42\text{cm}$

$= CD$

$\tan \angle BCA = \frac{8}{3.42}$

$\angle BCA = \tan^{-1}\left(\frac{8}{3.42}\right)$

$= 66.8$

66.8 ..... ° **[6]**

12 Show that $k = \frac{4 + 3j}{5 - j}$ can be rearranged to $j = \frac{5k - 4}{3 + k}$. **[4]**

$k(5 - j) = 4 + 3j$ $\times(5-j)$

Expand $k(5-j)$

$5k - kj = 4 + 3j$ $+kj$

$5k = 4 + 3j + kj$

-4

$5k - 4 = 3j + kj$

Factorise

$5k - 4 = j(3 + k)$

$\div(3+k)$

$j = \frac{5k - 4}{3 + k}$

13 **(a)** y is directly proportional to $\sqrt{x}$.
y is 75 when $x = 100$.

Find a formula linking x and y.

$y = k\sqrt{x}$

$75 = k\sqrt{100}$ $\sqrt{100} = 10$

$75 = 10k$

$\div 10$ $\quad \div 10$

$\frac{15}{2} = k$ so $y = \frac{15}{2}\sqrt{x}$

(a) $y = \frac{15}{2}\sqrt{x}$ **[3]**

(b) y is inversely proportional to x^2 and $y = 3$ when $x = 12$.

Show that $y = 27$ when $x = 4$. **[3]**

$y = \frac{k}{x^2}$

$3 = \frac{k}{12^2}$ $12^2 = 144$

$3 = \frac{k}{144}$

$\times 144$ $\quad \times 144$

$432 = k$

$y = \frac{432}{x^2}$

$= \frac{432}{4^2}$ $4^2 = 16$

$= \frac{432}{16}$

$= 27$

14 **(a)** Write $x^2 + 10x + 29$ in the form $(x + a)^2 + b$.

$$\left(x + \frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 + 29 = (x + 5)^2 - 25 + 29$$

$$= (x + 5)^2 + 4$$

(a) $(x + 5)^2 + 4$ **[3]**

(b) Write down the coordinates of the turning point of the graph of $y = x^2 + 10x + 29$.

$(x + 5)^2 + 4$

$x - 1$ $\hookrightarrow (-5, 4)$

(b) (-5 , 4) **[1]**

15 **(a)** Complete the table for $y = x^3 - 6x - 5$.

x	0	1	2	3	4
y	-5	-10	-9	4	35

[2]

(b) **(i)** Between which two **consecutive integers** is there a solution to the equation $x^3 - 6x - 5 = 0$?
Give a reason for your answer.

A solution lies between $x =$ 2 and $x =$ 3

because there is a change in the sign from negative to positive.

[2]

(ii) Choose a value of x between the two values you gave in part **(b)(i)**.
Calculate the corresponding value of y.

$x = 2.5$

$2.5^2 - 6 \times 2.5 - 5 = -\frac{35}{8} = y$

(b)(ii) $x =$ 2.5

$y =$ $-\frac{35}{8}$ **[2]**

(iii) State a smaller interval in which the solution lies.

(iii) $2.5 < x < 3$ **[1]**

16 Solve these simultaneous equations algebraically.

$y = x - 3$ (1)

$y = 2x^2 + 8x - 7$ (2)

$x - 3 = 2x^2 + 8x - 7$ Sub (1) into (2)

$+3$ $+3$

$x = 2x^2 + 8x - 4$

$-x$ $-x$

$0 = 2x^2 + 7x - 4$

$0 = (2x - 1)(x + 4)$

so $x = 0.5, -4$

$y = 0.5 - 3$ sub into (1)

$= -2.5$

$y = -4 - 3$ sub into (1)

$= -7$

$x =$ 0.5 , $y =$ -2.5

$x =$ -4 , $y =$ -7 **[6]**

17 **(a)** Show that $\sqrt{396}$ can be written as $6\sqrt{11}$. **[2]**

$\sqrt{396} = \sqrt{36 \times 11}$

$= \sqrt{36} \times \sqrt{11}$

$= 6 \times \sqrt{11}$

$= 6\sqrt{11}$

(b) **Without** using a calculator, show that $\frac{4+2\sqrt{2}}{2-\sqrt{2}}$ can be simplified to $6+4\sqrt{2}$. **[6]**

$\frac{4+2\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{8+4\sqrt{2}+4\sqrt{2}+4}{4-2}$

Rationalize the denominator.

$= \frac{12+8\sqrt{2}}{2}$

$= 6+4\sqrt{2}$

PLEASE DO NOT WRITE ON THIS PAGE