OCR
Oxford Cambridge and RSA

Model Solutions

H

GCSE (9–1) Mathematics
J560/04 Paper 4 (Higher Tier)

Tuesday 6 November 2018 – Morning
Time allowed: 1 hour 30 minutes

You may use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

First name	
Last name	
Centre number	
Candidate number	

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

OCR is an exempt Charity

Turn over

Answer **all** the questions.

1 Shari buys a box of 60 candles for £125.
She sells the candles for £2.25 each.

Calculate her percentage profit.

Total Revenue : $2.25 \times 60 = 135$

% Profit: $\dfrac{change}{original} \times 100$

$= \dfrac{135-125}{125} \times 100$

$= 8\%$

.... **8** % **[4]**

2 Hector can run 400 metres in 66 seconds.   5000m

(a) Use this information to show that he could run 5 kilometres in less than 14 minutes.   **[4]**

×12·5 ⟨ 400 metres : 66 sec
        5000 metre : 825 sec ⟩ ×12·5

$825\,sec \xrightarrow{\div 60} = 13.75\,min$

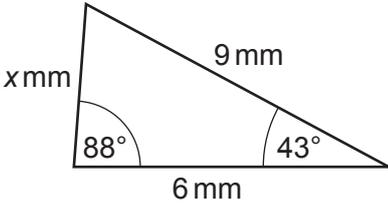Hector can run under 14min as the working shows
he can run 5km in 13.75min

(b) Hector tries to run 5 kilometres in less than 14 minutes.

Give one reason why he might not achieve this.

.... May not run at same rate ....

.... **[1]**

3 Here is Mario's answer to a question.

Question 3

x mm

9 mm

6 mm

88° 43°

Answer

$x = \sqrt{9^2 - 6^2}$

$x = \sqrt{45}$

$x = 6.708 \, (3 \, \text{d.p.})$

Work out the value of x.

Explain the error in Mario's method.

Mario has used pythagorous' theorem, but this theorem can only be used for rightangled triangles **[1]**

4 Here are the interest rates for two bank accounts.

Northern Savings Bank (NSB)	Central Alliance Bank (CAB)
2.5% per year **compound interest**	2.7% per year **simple interest**

1   2

Mia puts £6400 in each account.

Calculate the difference in value between the two accounts after 8 years.
Give your answer correct to the nearest penny.

①   2.5% interest: 100% + 2.5% = 102.5% = ×1.025

Total money after 8years   : $6400 \times 1.025^8 = £7797.78$ — 8years

②   simple 2.7% interest :   2.7% of 6400 = £172.80

Total money after 8years   : $6400 + (172.80) \times 8$ — 8years
= £7782.40

Difference: 7797.78 − 7782.40
= 15.38

£ ....15.38........ **[6]**

5 Marcin buys 7 rulers and 15 crayons for £7. ①
A ruler costs 12p more than a crayon. ②
£0.12
Find the cost of one crayon.

① $7r + 15c = 7$

② $r = c + 0.12$

Substitute r

$7(c + 0.12) + 15c = 7$
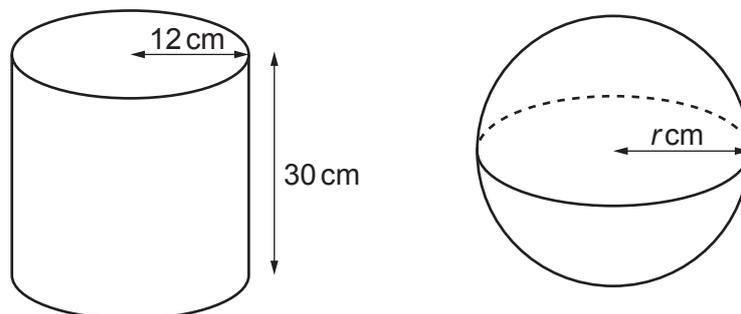
expand

$7c + 0.84 + 15c = 7$

collect like terms

$22c + 0.84 = 7$
      -0.84

$22c = 6.16$
      $\div 22$

$c = 0.28 = £0.28 = 28p$

cost of one crayon = ...28........ p **[5]**

6 The diagram shows a cylinder and a sphere.

The cylinder has radius 12 cm and height 30 cm.
The cylinder and the sphere have the same volume.

Work out the radius r cm of the sphere.

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Volume of cyclinder = $\pi r^2 h$

= $\pi \times 12^2 \times 30$

= 4320π

Volume of sphere = $\frac{4}{3}\pi r^3$

Equate

$\frac{4}{3}\cancel{\pi} r^3 = 4320\cancel{\pi}$

$\div \frac{4}{3}$

$r^3 = 3240$

$\sqrt[3]{}$

$r = 14.79...$

.... 14.8 .... cm **[5]**

7   The diagram shows triangle ABC.
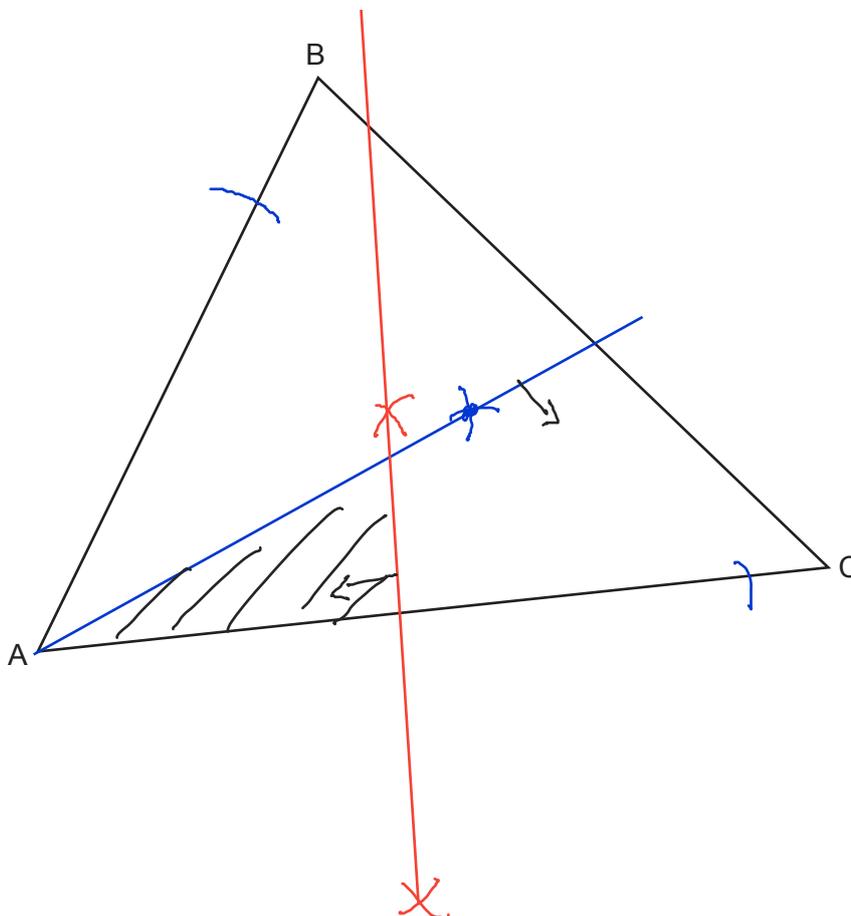
(Not to scale)

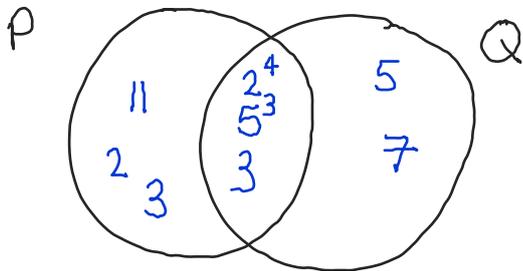**(a)** Construct the bisector of <u>angle BAC</u>.   *split angle in half*   **[2]**

(b) Construct the perpendicular bisector of AC.   *cut line in half*   **[2]**

(c) Shade the region inside triangle ABC that is

- nearer to AC than to AB
- nearer to A than to C.   **[1]**

8 **(a)** Two numbers, P and Q, are written as products of their prime factors.

$$P = 2^2 \times 3^3 \times 5^3 \times 11 \qquad Q = 2^5 \times 3 \times 5^4 \times 7$$

(handwritten: P has exponents marked 2, 3; Q has exponent 5 marked over 5)

(i) Find the lowest common multiple (LCM) of P and Q.

(handwritten) venn diagram

P and *Q* Venn diagram: P region contains 11, 2, 3; intersection contains 2^4, 5^3, 3; Q region contains 5, 7.

$$LCM = $$
$$2 \times 3 \times 11 \times 2^4 \times 5^3 \times 3 \times 5 \times 7$$
$$= 2^5 \times 3^2 \times 5^4 \times 7 \times 11$$

13 860 000

(a)(i) .... **13 860 000** .... **[2]**

(ii) The number C is written as the product of its prime factors.
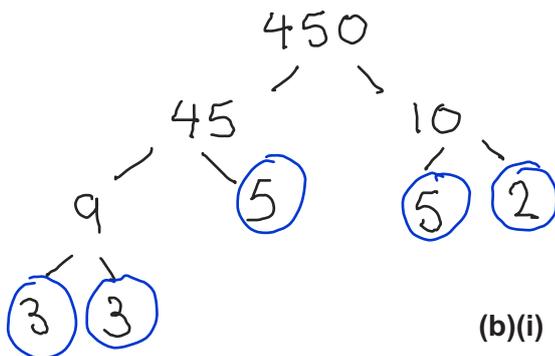
$$C = 2^3 \times 3 \times 5^2$$

Work out $P \div C$, leaving your answer as a product of powers of prime numbers.

$$\frac{P}{C} = \frac{2^5 \times 3^2 \times 5^3 \times 11}{2^3 \times 3 \times 5^2} = 11$$

$$2^{5-3} = 2^2$$

(ii) .... $2^2 \times 3 \times 5 \times 11$ .... **[2]**

(b) **(i)** Write 450 as a product of its prime factors.

(factor tree) 450 → 45, 10; 45 → 9, ⑤; 10 → ⑤, ②; 9 → ③, ③

(b)(i) .... $2 \times 3^2 \times 5^2$ .... **[3]**

(ii) Find the highest common factor (HCF) of 270 and 450.

(factor tree) 270 → 10, 27; 10 → ⑤, ②; 27 → 3^3

$$270 = 2 \times 3^3 \times 5$$
$$450 = 2 \times 3^2 \times 5^2$$

(Venn diagram: 270 region contains 3; intersection contains 2, 3^2, 5; 450 region contains 5)
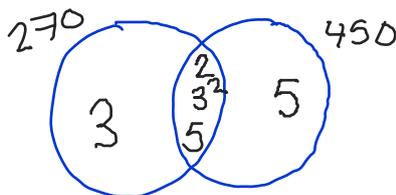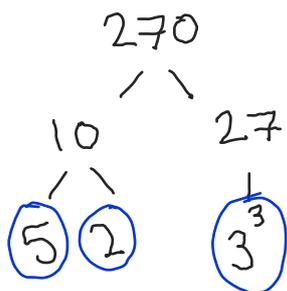
$$HCF = 2 \times 3^2 \times 5$$

(ii) .... **90** .... **[3]**

9 72 children are asked whether they have a laptop or an iPad.
- 31 have a laptop.
- 48 have an iPad.
- 12 have both.
- 5 have neither.

(a) Represent this information on a Venn diagram.

$\mathcal{E}$

laptop   ipad

$31-12=$ 19   12   36 $48-12$

5
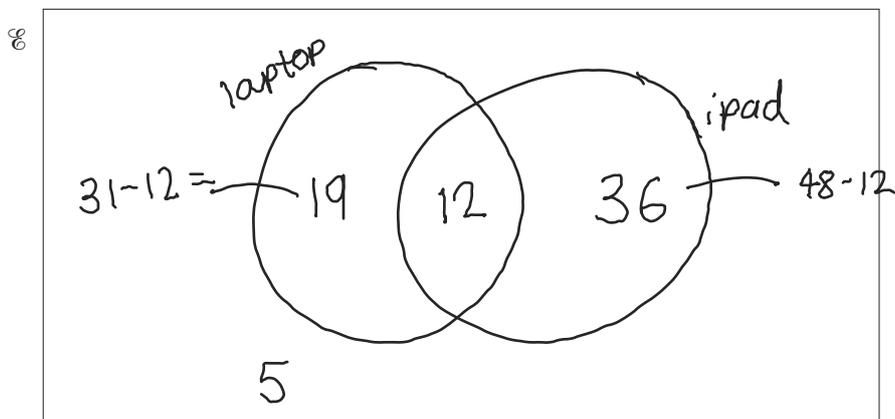
[3]

(b) One of the children is chosen at random.

Write down the probability that they have an <u>iPad but not a laptop.</u>

36

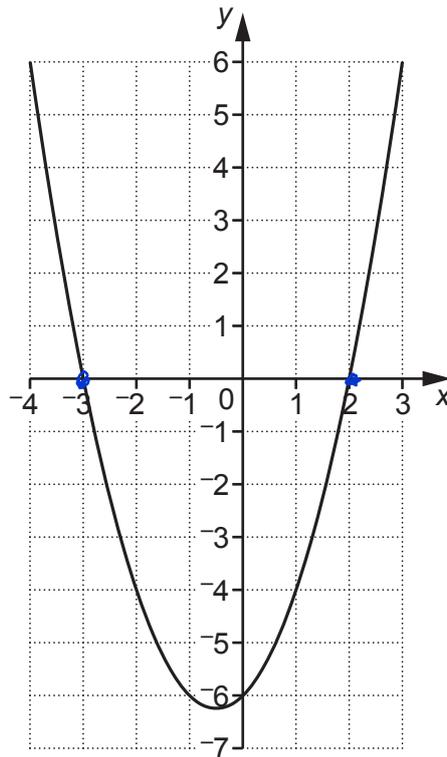$$P = \frac{36}{72} \, \text{total}$$

(b) $P = \frac{1}{2}$ **[2]**

     Turn over

10 Here is the graph of $y = x^2 + x - 6$.

Use the graph to solve the equation $x^2 + x - 6 = 0$.

Crosses x axis at.

$x =$ ....−3.................. or $x =$ ....2.................. **[2]**

11 Here are two functions.

Function A : input ⟶ ×2 ⟶ +4 ⟶ output

Function B : input ⟶ –1 ⟶ ×3 ⟶ output
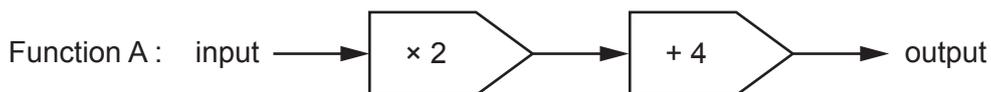
Composite function C is shown below.

Function C : input ⟶ Function A ⟶ Function B ⟶ output

(a) The output from function C is 54.

Work out the input.

$C \rightarrow$ reverse $\rightarrow$ reverse $\rightarrow$ input
$\qquad$ B $\qquad$ A

$54 \rightarrow \div 3 \rightarrow +1 \rightarrow -4 \rightarrow \div 2$
$\qquad$ 18 $\qquad$ 19 $\qquad$ 15 $\qquad$ 7·5

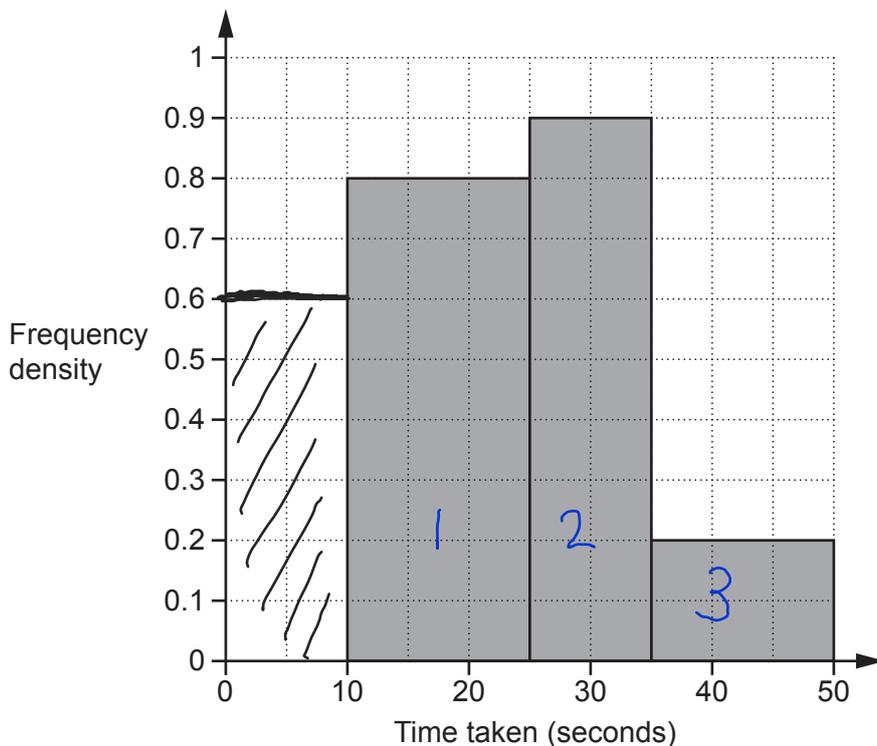**(a)** .... 7·5 .... **[2]**

(b) The input to function C is x.

Find an expression, in terms of x, for the output from function C.

$x \rightarrow \times 2 \rightarrow +4 \rightarrow -1 \rightarrow \times 3$
$\qquad 2x \qquad 2x+4 \qquad 2x+3 \qquad 3(2x+3)$

(b) .... $3(2x+3)$ .... **[2]**

Turn over

12 30 students completed a puzzle and their times were recorded.
All of the students completed the puzzle in less than 50 seconds.
The histogram shows information about some of their times.

Complete the histogram for those completing the puzzle in less than 10 seconds. **[5]**

Frequency = FD × class width

① 15 × 0.8 = 12

② 10 × 0.9 = 9

③ 15 × 0.2 = 3
                    _____
                    Total = 24

Remaining = 30 − 24 = 6 freq

$FD = \dfrac{freq}{class\ width} = \dfrac{6}{10} = 0.6$

13 Tenzin is given this question.

> Factorise fully.
>
> $$2x^2 + 6x$$

Here is his answer.

> $$2x^2 + 6x = x(2x + 6)$$

Explain why Tenzin's answer is not correct.

Because you can still factorise the bracket by 2. It should be $2x(x+3)$ **[1]**

14 y is inversely proportional to the square root of x.
y is 40 when x is 9.

Find a formula linking x and y.

$$y \propto \frac{1}{\sqrt{x}}$$
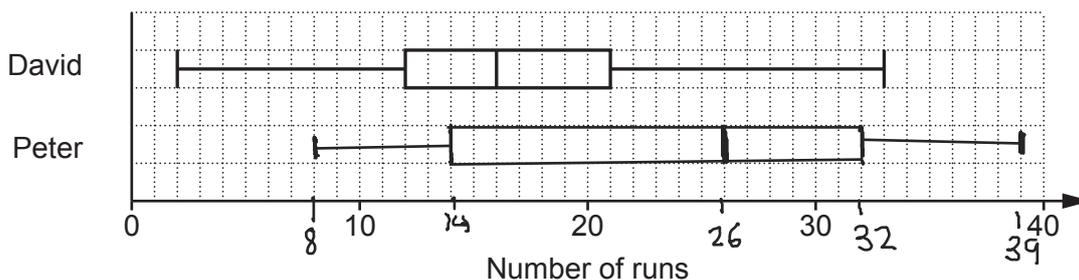
$$y = \frac{k}{\sqrt{x}}$$

$$40 = \frac{k}{\sqrt{9}}$$

$$40 \times 3 = k$$

$$k = 120$$

$$y = \frac{120}{\sqrt{x}}$$ **[3]**

15 The box plot shows the distribution of the runs scored by David in some cricket matches.

David

Peter

0   8   10   14   20   26   30   32   39 40

Number of runs

(a) Another player, Peter, has

- a median score of 26
- a highest score of 39
- a lowest score of 8
- a lower quartile of 14
- an inter-quartile range of 18.   Upper Quartile = 14 + 18 = 32

Show the distribution of Peter's scores as a box plot on the diagram above. **[2]**

(b) Decide whether David or Peter best satisfies each of these questions.
Give a reason for each of your decisions.

(i) Who scored more runs on average?

............Peter........... because ....larger median..

.... **[1]**

(ii) Whose scores were more consistent?

............David........... because ....smaller interquartile.................

.................range.. **[1]**

16 Solve by factorisation.

$$2x^2 - 19x - 33 = 0$$

X to $2x \cdot -33 = -66$ Split: $2x^2 + 3x \,|\, -22x - 33 = 0$

+ to -19 $x(2x+3) \,|\, -11(2x+3)$

                $-22, 3$

$2x + 3 = 0$ $x - 11 = 0$

$2x = -3$ OR $x = 11$ $(2x+3)(x-11) = 0$

$x = \dfrac{-3}{2}$

$x = \underline{\quad -3/2 \quad}$ or $x = \underline{\quad 11 \quad}$ **[3]**

17 Here are the first four terms of a quadratic sequence.

$$2 \qquad 15 \qquad 34 \qquad 59$$

The nth term is $an^2 + bn + c$.

Find the values of a, b and c.

$2 , 15 , 34 , 59$

$+13 \quad +19 \quad +25$

$+6 \qquad +6 \qquad = 3n^2$
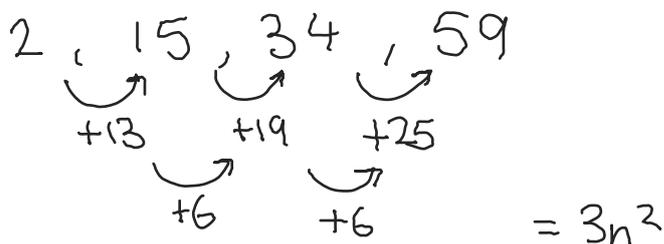
Seq	2	15	34	59
$3n^2$	3	12	27	48

$-5 , -1 , 3 , 7 , 11$

$-4 \quad +4$

Find nth term

Difference 4

$\times n$ n

$+0^{th}$ term -5

$= 4n - 5$

$\dfrac{3n^2}{a} \;\; \dfrac{+4n}{b} \;\; \dfrac{-5}{c}$

$a = \underline{\quad 3 \quad}$

$b = \underline{\quad 4 \quad}$

$c = \underline{\quad -5 \quad}$ **[4]**

Turn over

18 P is the point (0, ⁻1) and Q is the point (5, 9).

Find the equation of the line through P that is perpendicular to the line PQ.

$$\text{gradient} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{gradient of PQ}: \quad \frac{9 - ^-1}{5 - 0} = \frac{10}{5} = 2$$

perpendicular = negative reciprocal

$$m = \frac{-1}{2}$$

$$y = -\tfrac{1}{2}x + c$$

Substitute P coordinate

$$-1 = 0 + c$$

$$c = -1$$

$$y = {}^{-1}\!/_2 \, x - 1 \quad \text{[5]}$$

19 Two cylinders, A and B, are mathematically similar.

Cylinder A has volume $2400\,cm^3$ and height $12\,cm$.
Cylinder B has volume $750\,cm^3$.

Find the height of cylinder B.
Give your answer correct to an appropriate degree of accuracy.

volume
  scale factor   : $\dfrac{2400}{750}$ $= \div 3.2$
  (A to B)

linear scale factor $= \sqrt[3]{3.2}$

$= \div 1.47\ldots$

height of B $= 12 \div 1.47\ldots$

$= 8.143\ldots$

to (3sf) 8.14

...................8.14...................cm **[5]**

20 The diagram shows a right-angled triangular prism ABCDEF.

(diagram of prism ABCDEF with A, D, E, F, B, C labelled; 18 cm, 25 cm, 40 cm; angle x at F)
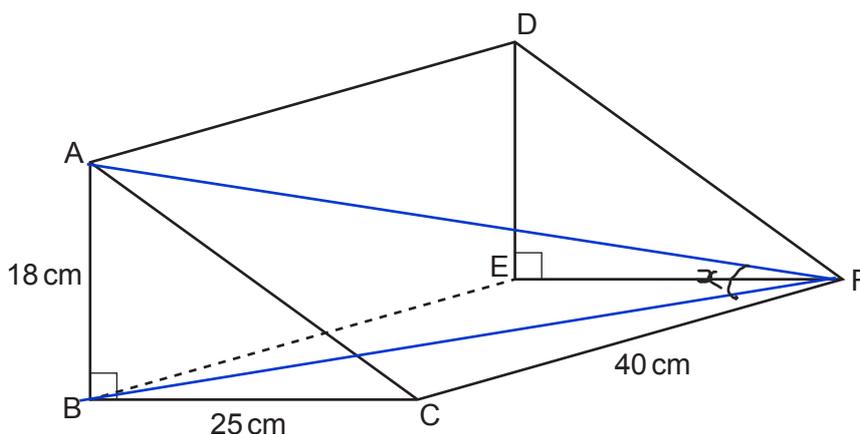
Calculate angle AFB.

Pythag : $a^2 + b^2 = c^2$

$BF^2 = BC^2 + CF^2$

$= 25^2 + 40^2 = 2225$

$BF = \sqrt{2225} = 47.1699\ldots$ cm

(right triangle diagram: A, 18, B, 47.1699, F, angle θ)

$\tan\theta = \dfrac{OPP}{adj}$

$\tan\theta = \dfrac{18}{47.169\ldots}$

$\theta = \tan^{-1}\left(\dfrac{18}{47.169\ldots}\right)$

$= 20.886\ldots$

..............20·9.............° **[6]**

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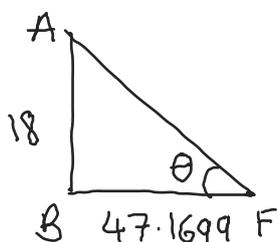
21 The number of gannets on an island is assumed to follow this exponential growth model.

$$N = 0.45 \times 1.07^x$$

N is the number of gannets, in thousands.
x is the number of years after 1st January 2010.

(a) Complete the table for $N = 0.45 \times 1.07^x$.

0.45×1.07^{10} 0.45×1.07^{20}

x	0	5	10	15	20
N	0.45	0.63	0.88	1.24	1.74

[2]

(b) Draw the graph of $N = 0.45 \times 1.07^x$.

Number of gannets, N (thousands)

Number of years, x, after 1st January 2010
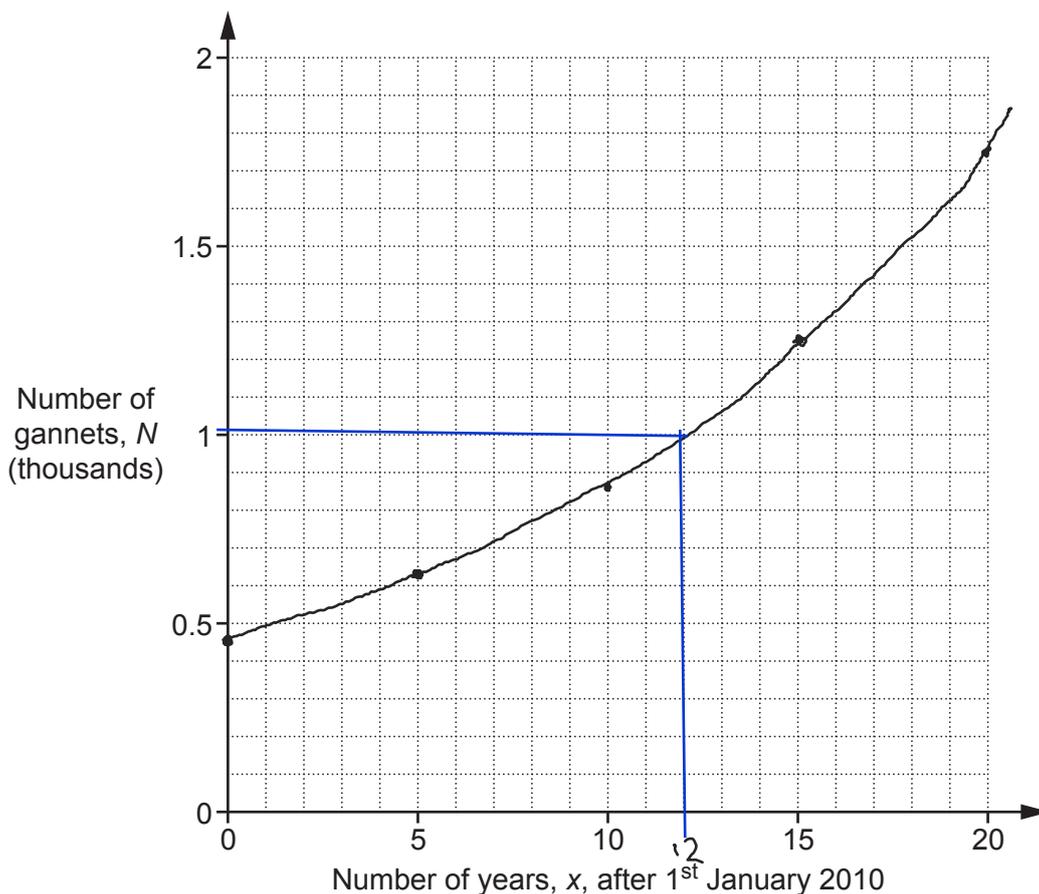
[2]

(c) Use the graph to find **the year** when the gannet population is predicted to reach 1000.

$2010 + 12 =$

(c) ....2022................... [2]

Turn over for Question 22

22 In a village the ratio of males to females is $2:1$. *[handwritten: $\frac{2}{3}$ male, $\frac{1}{3}$ female]*

[handwritten: 0.4] 40% of the people in the village are right-handed males.

[handwritten: 0.25] 25% of the people in the village are right-handed females.

Show that the proportion of females who are right-handed is greater than the proportion of males who are right-handed. **[6]**

[handwritten tree diagram:]

$\frac{2}{3}$ → male → 0.4 → Right; 1−0.4 → 0.6 → not right

$\frac{1}{3}$ → female → 0.25 → right; 1−0.25 → 0.75 → not right
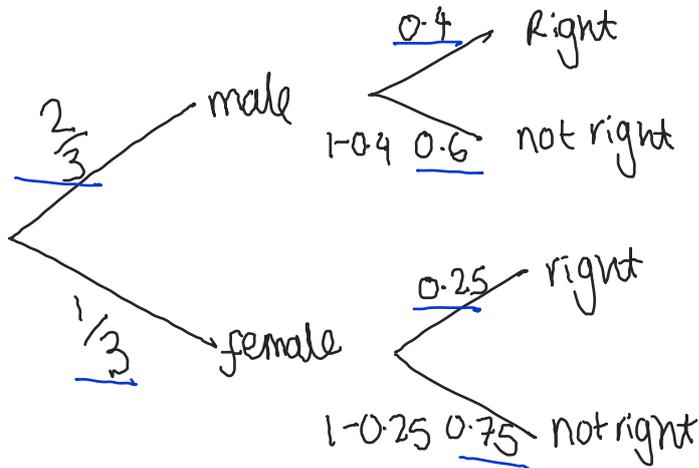
Proportion:

Female: $\dfrac{0.25}{1/3} = 0.75$

Male: $\dfrac{0.4}{2/3} = 0.6$

$0.75 > 0.6$

END OF QUESTION PAPER