OCR

Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Thursday 2 November 2017 – Morning

Time allowed: 1 hour 30 minutes

You may use:
• A scientific or graphical calculator
• Geometrical instruments
• Tracing paper

First name	**MODEL SOLUTIONS**
Last name	

Centre number						Candidate number				

INSTRUCTIONS
• Use black ink. You may use an HB pencil for graphs and diagrams.
• Complete the boxes above with your name, centre number and candidate number.
• Answer **all** the questions.
• Read each question carefully before you start to write your answer.
• Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
• Write your answer to each question in the space provided.
• If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
• Do **not** write in the barcodes.

INFORMATION
• The total mark for this paper is **100**.
• The marks for each question are shown in brackets **[ ]**.
• Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
• This document consists of **20** pages.

OCR is an exempt Charity

Turn over

2

Answer **all** the questions.

1   Donald swims 3 lengths of a swimming pool in 93 seconds.

   (a)   Use this information to show that he could swim 100 lengths in under 55 minutes.   **[4]**

3 lengths = 93 seconds

1 length = 31 seconds

100 lengths = 3100 seconds

55 minutes = 55 × 60 = 3300 seconds

3100 < 3300 → less than 55 minutes

   (b)   What assumption did you make in part **(a)**?

He swims at the same speed/rate the whole time.   **[1]**

   (c)   Donald tries to swim the 100 lengths in under 55 minutes.

   Suggest one reason why he might not achieve this.

He gets tired out as time progresses, and slows down.   **[1]**

2   **(a)**   Simplify.

   (i) $a^6 \div a^2$

$a^{6-2} = a^4$

   (a)(i) a^4 **[1]**

   (ii) $(b^5)^3$

$b^{5 \times 3} = b^{15}$

   (ii) b^{15} **[1]**

   (b)   Factorise.

$6x - x^2$
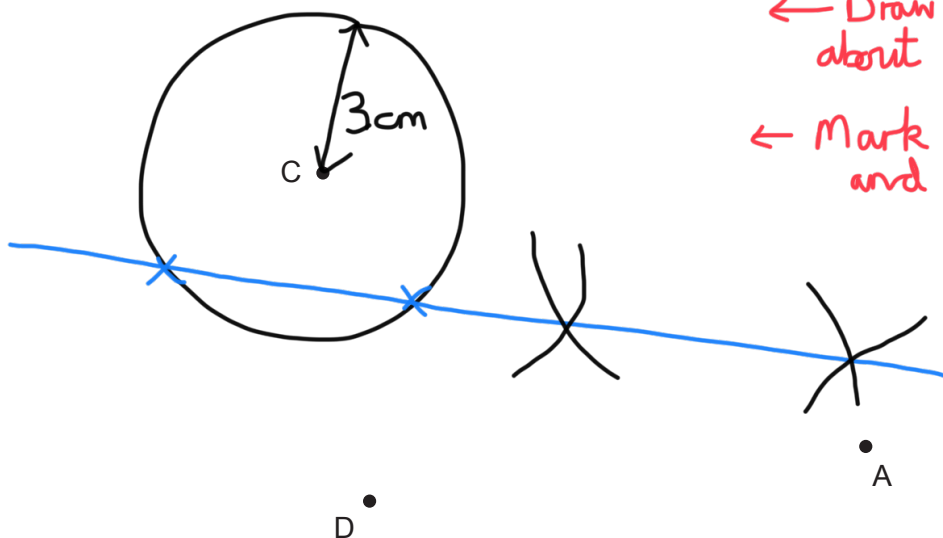
$x(6-x)$

   (b) $x(6-x)$ **[1]**

3

3 Four points A, B, C and D are shown on the scale diagram below.

B

← Draw perpendicular bisector of AB

← Draw circle of radius 3cm about C.

← Mark intersection of circle and lines.

3cm

C

D

A

Scale: 1 cm represents 5 m

(a) On the diagram, construct and mark the two points that are

- the same distance from A and B

and

- 15 m from C.

Show all your construction lines. **[5]**

(b) The points A, B, C and D represent the four corners of Monty's garden.
His garden is bounded by four straight fences A to B, B to C, C to D and D to A.

Monty wants to plant a tree in his garden at a place that satisfies the two conditions in part **(a)**.

Explain why there is only one position where Monty can plant his tree.

One of the points (far left) is outside of the fence. **[1]**

Turn over

4 The scatter diagram shows the results of 17 students in their French test and their German test. Both tests are out of 100.

German test

French test
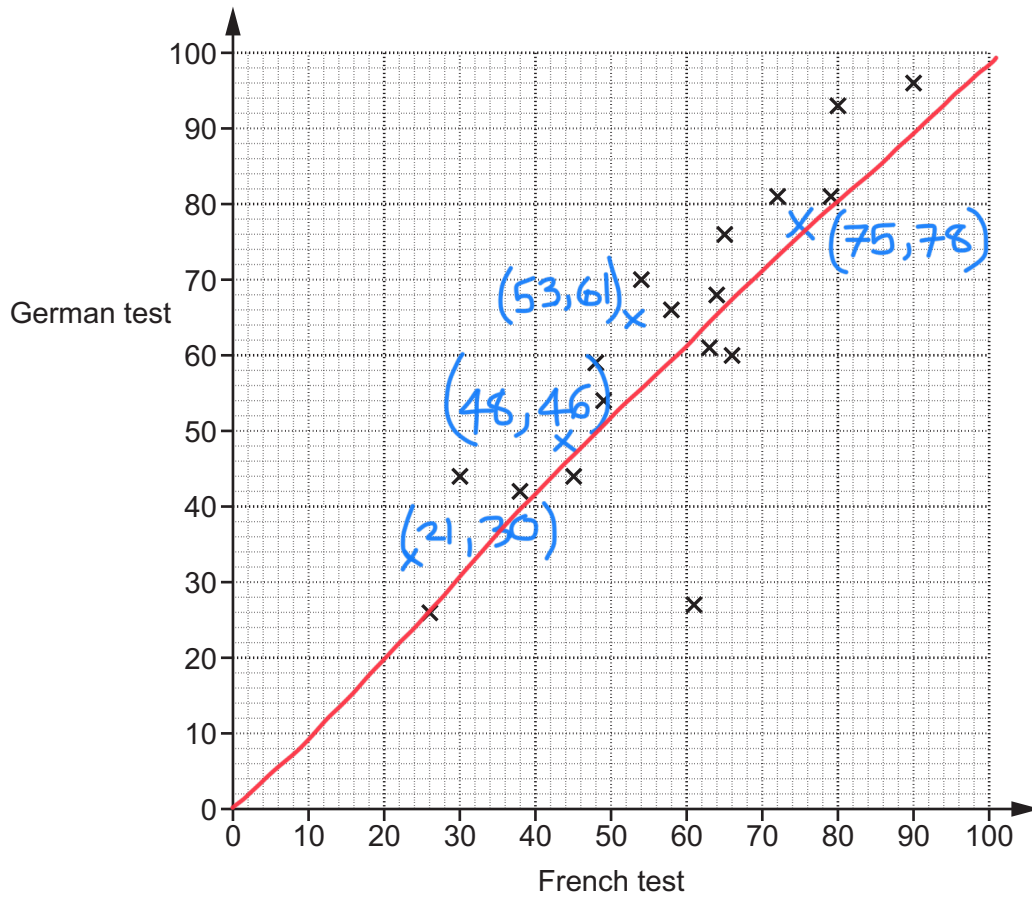
(a) Here are the results of another 4 students.

French	21	75	48	53
German	30	78	46	61

Plot these results on the scatter diagram. **[2]**

(b) Describe the type and strength of the correlation shown in this diagram.

(b) Strong, positive **[2]**

5

(c) Work out the percentage of the students whose German result was **higher** than their French result.

Draw line $y = x$ on graph (see graph.)

Count number above line : 15

Total : 21

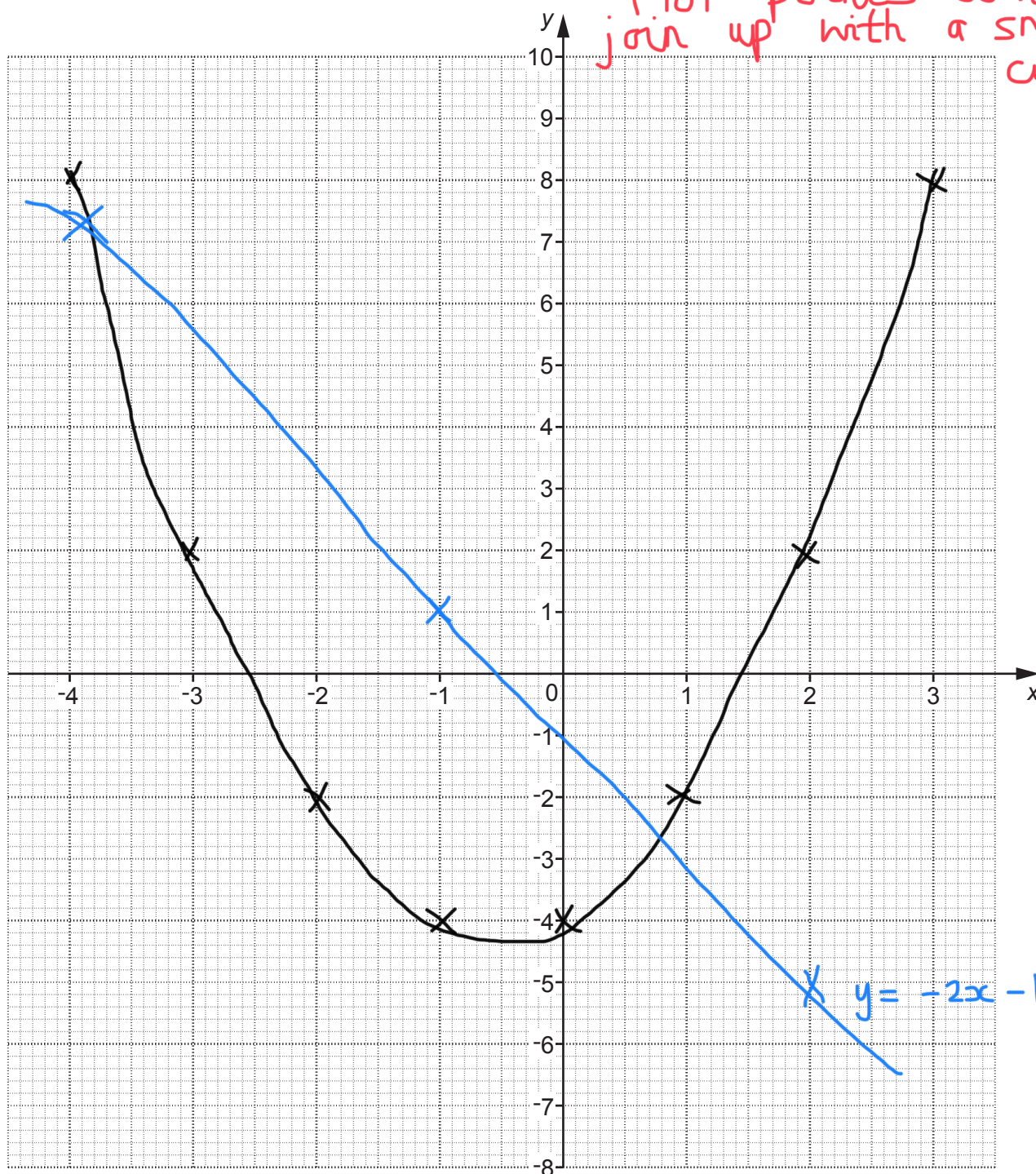Percentage : $\frac{15}{21} \times 100\% = 71.4\%$

(c) _____71.4_____ % **[4]**

Turn over

5   (a) Complete this table for $y = x^2 + x - 4$.

x	-4	-3	-2	-1	0	1	2	3
y	8	2	-2	-4	-4	-2	2	8

[2]

(b) Draw the graph of $y = x^2 + x - 4$ for $^-4 \leqslant x \leqslant 3$.

Plot points and join up with a smooth curve.

$y = -2x - 1$

[3]

Roots are where curve intercepts x - axis.

7

(c) Use your graph to solve $x^2 + x - 4 = 0$.

$x = -2.5$ or $x = 1.6$

(c) $x =$ ....-2.5....... or $x =$ ....1.6......... **[2]**

(d) On the same grid, draw the graph of $y = -2x - 1$ for $-4 \leqslant x \leqslant 3$.
You may use the table if you wish.

x	$^-4$	-1	2
y	7	1	-5

[3]

(e) Use your graphs to solve the equation $x^2 + x - 4 = -2x - 1$.

Where line $y = -2x - 1$ intercepts curve

(e) $x =$ ....-3.9........ or $x =$ ...0.75... **[2]**

Turn over

6 Maria mixes white paint and red paint in the ratio 2 : 3.
She makes a total of 15 litres of paint.

How much more red paint does she need to add to the mixture so that the ratio of white paint to red paint becomes 1 : 5?

$$2 + 3 = 5 \text{ parts}$$

litres in each part : $15 \div 5 = 3$

white : $3 \times 2 = 6$

red : $3 \times 3 = 9$
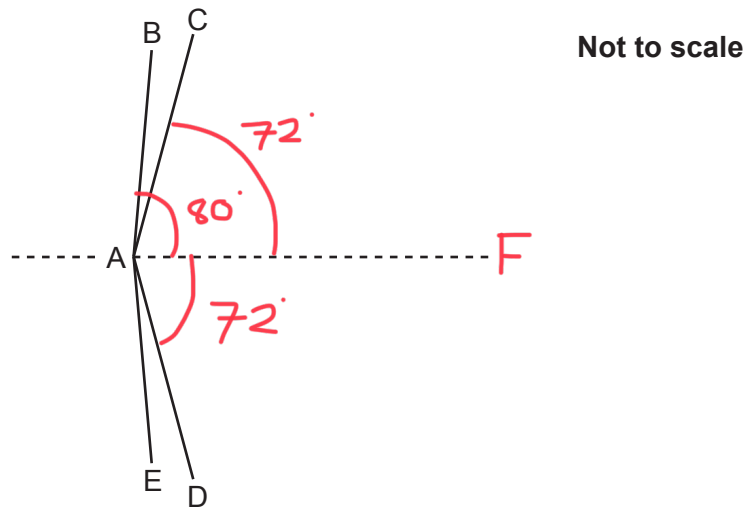
new ratio :

$\times 6 \left(\begin{array}{c} 1 : 5 \\ 6 : 30 \end{array} \right) \times 6$

$$30 - 9 = 21$$

.... **21** .... litres **[4]**

7   Angle BAE is part of a regular 18-sided polygon.
Angle CAD is part of a regular 10-sided polygon.
The dashed line through A is a line of symmetry of both polygons.

B C

Not to scale

72°

80°

A ------------------------------- F

72°

E D

Work out angle BAC.

18-sided exterior:

$360 \div 18 = 20$

$\rightarrow$ interior $= 180 - 20 = 160°$

$B\hat{A}F = 160 \div 2 = 80°$

20-sided exterior:

$360 \div 10 = 36°$

$\rightarrow$ interior $= 180 - 36 = 144°$

$\rightarrow CAF = 144 \div 2 = 72°$

Difference: $80 - 72 = 8°$

8 ....° **[5]**

8 Diners choose one starter and one main from the options given in the table below. Vegetarian dishes are indicated with a (v).

Starter	Main
Cheese salad (v)	Steak and chips
Prawn cocktail	Fish and chips
Mozzarella sticks (v)	Tomato pizza (v)
	Pork chops
	Nut cutlet (v)

(a) Work out the fraction of all the meal combinations which have at least one vegetarian option.

Total options : 3×5= 15

Options with $\underline{NO}$ reggie : 1×3 = 3

Options with at least 1 reggie : 15-3= 12

Fraction : $\frac{12}{15}$

(a) ....$\frac{12}{15}$................ **[3]**

(b) Diners also choose one of 6 dessert options.

How many different three-course meal combinations are available?

Starter × Main × Dessert

3 × 5 × 6 = 90 combinations

(b) ....90.................... **[2]**

9   Here are the results of a survey of 437 people in a town.

- 62 males speak Spanish.
- 153 females do not speak Spanish.
- 280 people do not speak Spanish.

Jeff says

At least 2 out of every 5 females in the town can speak Spanish.

Is he correct?
Show clearly how you reached your decision.

Draw a 2-way table:

	Men	Women	Total
Spanish	62	95	
No Spanish	127	153	280
Total	189	248	437

total females: 248

$\frac{2}{5} \times 248 = 99.2 \approx 99$

95 speak Spanish, which is less than .... [6]

99, so he is incorrect.

10 The histogram summarises a health score for a group of people.
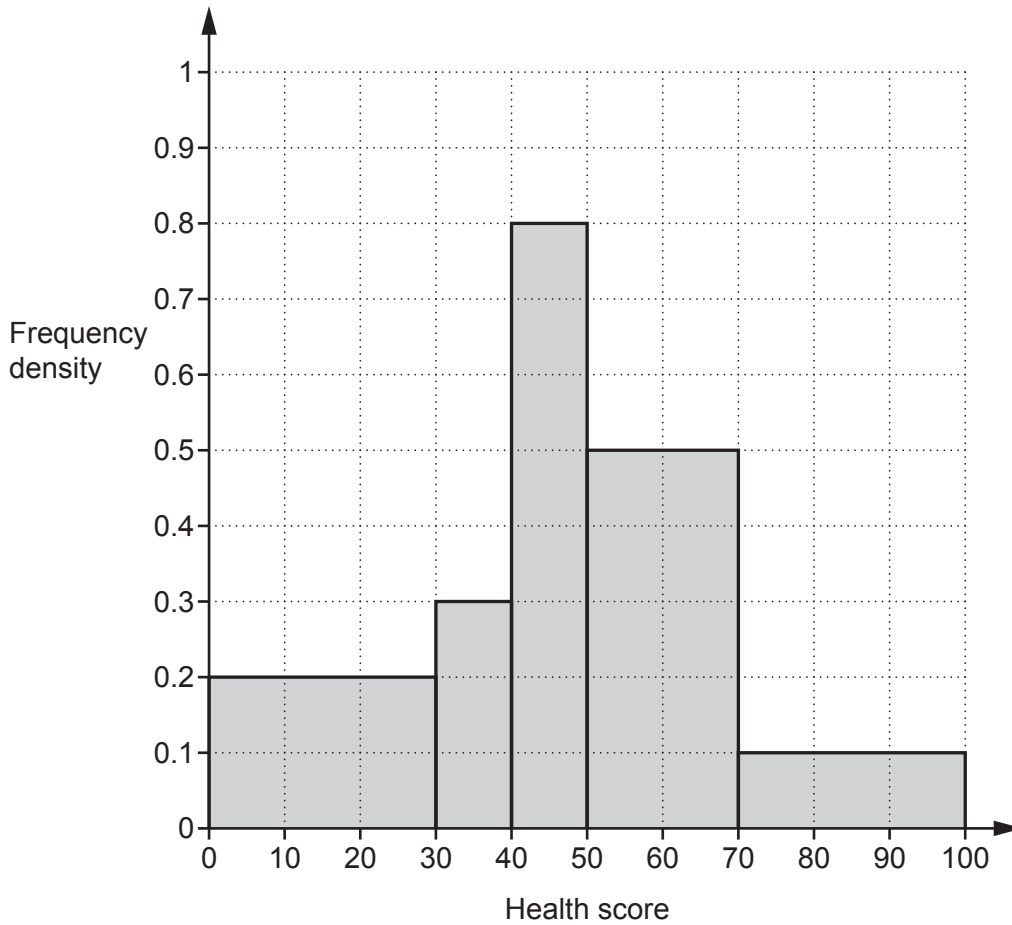
Health score

(a) Estimate the fraction of the group who had a score of 45 or more.

Total area : 6 + 3 + 8 + 10 + 3 = 30

Squares over 50 : 10 + 3 = 13

Squares 40 - 50 : 8   fraction: $\frac{13 + 4}{30} = \frac{17}{30}$

8 ÷ 2 = 4

(a) .... $\frac{17}{30}$ .... [4]

(b) What assumption did you make in answering part (a)?

The health scores were evenly spread in the 40 − 50 class. [1]

11 y is inversely proportional to x^2 and $y = 5$ when $x = 4$.

Find a formula linking x and y.

$$y \propto \frac{1}{x^2}$$

$$y = \frac{k}{x^2}$$

$$5 = \frac{k}{16}$$

$$80 = k$$

$$y = \frac{80}{x^2}$$

$y = \dfrac{80}{x^2}$ .... **[3]**

12 A log is 18 m long, correct to the nearest metre.
It is to be cut into fence posts which must be 80 cm long, correct to the nearest 10 centimetres.

What is the largest number of fence posts that can possibly be cut from this log?

Longest log possible : 18.5 m upper bound
Smallest fence posts : 75 cm lower bound
Most fence posts : 1850 ÷ 75 = 24.7
→ 24 (not enough for 25)

.... 24 .... **[4]**

Turn over

13 (a) Solve.

$$x^2 - 6x + 15 = 3x - 5$$

$$x^2 - 6x + 15 = 3x - 5$$
$$x^2 - 9x + 20 = 0$$
$$(x + a)(x + b) = 0 \leftarrow \begin{array}{l} a + b = -9 \\ a \times b = 20 \end{array}$$

$$(x - 5)(x - 4) = 0$$

(a) $x =$ ....**5**......... or $x =$ ....**4**......... **[4]**

(b) Expand and simplify.

$$(2x - 1)(x + 5)(3x - 2)$$

Multiply out first 2 brackets:

$$(2x - 1)(x + 5) = 2x^2 - x + 10x - 5$$
$$= 2x^2 + 9x - 5$$

Now final bracket:

$$(2x^2 + 9x - 5)(3x - 2) = 6x^3 + 27x^2$$
$$-4x^2 - 15x - 18x + 10$$

(b) $6x^3 + 23x^2 - 33x + 10$ **[4]**

14 The diagram shows a cuboid ABCDEFGH.

$a^2 + b^2 = c^2$

A, B, D, C, 8 cm, E, F, 6 cm, H, 11 cm, G
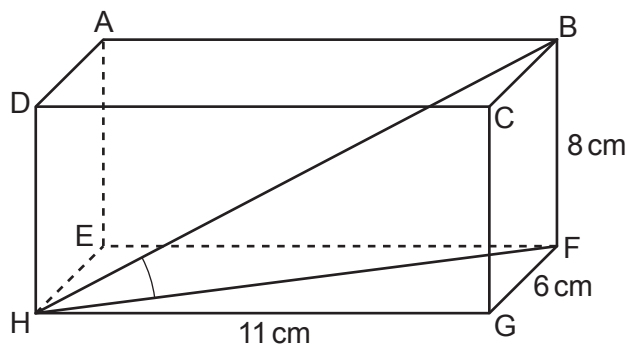
Calculate angle BHF.

Length HF:

$$HF^2 = 11^2 + 6^2$$
$$HF^2 = 157$$
$$HF = \sqrt{157} = 12.53$$

Angle $B\hat{H}F$ (θ):

H, B, 8cm O, H, θ, 12.53 cm, F, A
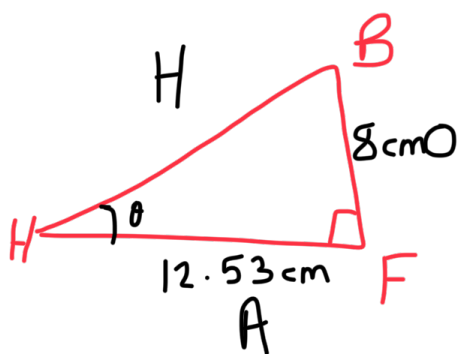
$$\tan\theta = \frac{O}{A} = \frac{8}{12.53}$$
$$\theta = \tan^{-1}(8/12.53) = 32.6°$$

32.6 .... ° **[5]**

15 Show that $\frac{(4+2\sqrt{5})}{\sqrt{5}-1}$ can be simplified to $\frac{3\sqrt{5}+7}{2}$. **[4]**

$$\frac{(4+2\sqrt{5})}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(4+2\sqrt{5})(\sqrt{5}+1)}{(\sqrt{5}+1)(\sqrt{5}-1)}$$

$$= \frac{14+6\sqrt{5}}{4} = \frac{7+3\sqrt{5}}{2}$$

$$= \frac{3\sqrt{5}+7}{2}$$

16 (a) Write $x^2 - 6x + 20$ in the form $(x-a)^2 + b$.

$$x^2 - 6x + 20$$

$$\left(x+\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + b \qquad (x-3)^2 - 9 + 20$$

$$= (x-3)^2 - 9 + 20 \qquad = (x-3)^2 + 11$$

$$a=3 \qquad b=11$$

(a) $(x-3)^2 + 11$ .... **[3]**

(b) Write down the turning point of the graph of $y = x^2 - 6x + 20$.

$$(x+p)^2 + q : (-p, q)$$

(b) (3 , 11) **[2]**

17 ABC and ACD are triangles.

A

18 cm

78°

Not to scale

46° D

14 cm

81°

B   C
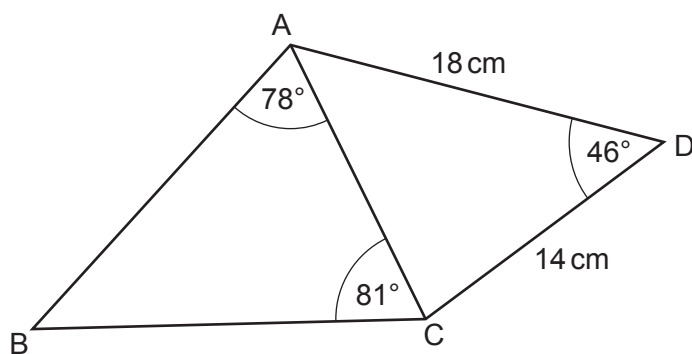
$$a^2 = b^2 + c^2 - 2bc\cos A$$

(a) Show that AC = 13.0 cm, correct to 3 significant figures. **[4]**

Using triangle ACD and cosine rule:

$$AC^2 = 18^2 + 14^2 - (2 \times 18 \times 14 \times \cos 46)$$

$$AC^2 = 324 + 196 - 350.1$$

$$AC^2 = 169.9$$

$$AC = 13.03 \, cm = 13.0 \, cm \, (3 \, s.f)$$

(b) Calculate BC.

Using triangle ABC and sine rule.

A

78°

21°   81°

B   C
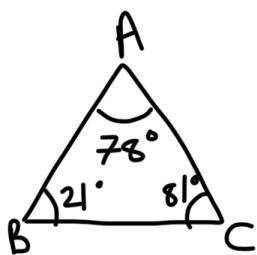
$$A\hat{B}C = 180 - 78 - 81 = 21°$$

$$\frac{13}{\sin 21} = \frac{BC}{\sin 78}$$

$$BC = \frac{13 \sin 78}{\sin 21} = 35.5 \, cm$$

(b) ....35.5................. cm **[3]**

18 Solve this equation algebraically.
Give your solutions correct to 2 decimal places.

$$3x^2 + 5x - 1 = 0$$

$$3x^2 + 5x - 1 = 0$$

$$a = 3$$
$$b = 5$$
$$c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{37}}{6}$$

$x =$ ....0.18............ or $x =$ ....−1.85........... **[4]**

END OF QUESTION PAPER

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ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).