



Oxford Cambridge and RSA

Model Solutions

H

Tuesday 21 May 2019 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes



You may use:

- a scientific or graphical calculator
- geometrical instruments
- tracing paper



Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

Candidate number

First name(s) _____

Last name _____

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **16** pages.

Answer all the questions.

1 Calculate.

$$\sqrt[3]{\frac{210}{10^2 + 5^2}}$$

Give your answer correct to 3 significant figures.

$$\sqrt[3]{\frac{210}{100 + 25}} = \sqrt[3]{\frac{210}{125}} = 1.188 \approx 1.19$$

↳ 8 > 5 round up

↑
put this
in the
calculator

..... 1.19 [3]

2 The ratio 50 grams to 1 kilogram can be written in the form 1 : n.

Find the value of n.

$$50 \text{ g} : 1 \text{ kg}$$

50 g : 1000 g

÷ 50 ↪ 50 : 1000 ↪ ÷ 50

1 : 20

n = 20 [2]

3 (a) Anne, Barry and Colin share a prize in the ratio 3 : 4 : 5.

Colin gives $\frac{1}{3}$ of his share to a charity.

What fraction of the whole prize does Colin give to the charity?

$$A : B : C$$

$$3 : 4 : 5 \rightarrow 3 + 4 + 5 = 12 \text{ total parts.}$$

↙

$$\text{charity} = \frac{1}{3} \times 5 = \frac{5}{3} \text{ parts}$$

$$\text{Charity} = \frac{5}{3} \div 12 = \frac{5}{3} \times \frac{1}{12} = \frac{5}{36}$$

(a) $\frac{5}{36}$ [3]

(b) Delia, Edwin and Freya share some money in the ratio 5 : 7 : 8.

Freya's share is £1600.

How much money did they share?

$$D : E : F$$

$$5 : 7 : 8 \rightarrow 5 + 7 + 8 = 20 \text{ total parts}$$

↙

8 parts → £1600

1 part → £200

20 parts → £4000

(b) £ 4000 [2]

4 A bus timetable shows the following information.

- A bus following route T leaves for the train station every 20 minutes.
- A bus following route A leaves for the airport every 18 minutes.
- A bus following route T and a bus following route A both leave at 8.37 am.

(a) When is the next time one of each bus is timetabled to leave at the same time?

T : 20 40 60 80 100 120 140 160 180
 A : 18 36 54 72 90 108 126 144 162 180

LCM of T and A is 180 minutes → 3 hrs
 60 min = 1 hr

8:37 am + 3:00 = 11:37 am
 (a) 11:37 am [4]

(b) Write down one assumption that was necessary to solve this problem.

All buses leave according to the timetable.
 [1]

5 Bennie is 7 years older than Ayesha.
 Chloe is twice as old as Bennie.
 The sum of their three ages is 57.

Work out the ages of Ayesha, Bennie and Chloe.

$B = A + 7 \rightarrow A = B - 7$ } write A in terms of B
 $C = 2B$

$A + B + C = 57$

$(B - 7) + B + (2B) = 57$ substitute A and C in terms of B.

$4B - 7 = 57$
 $4B = 64$
 $B = 16$

Subs $B = 16$ in :

$C = 2B ; C = 2(16) = 32$

$A = B - 7 ; A = 16 - 7 = 9$

Ayesha's age is 9

Bennie's age is 16

Chloe's age is 32 [6]

6 120 students in Year 10 and Year 11 sit a test.

- 61 of the students are in Year 10.
- 83 of the students are right-handed.
- 20 of the students in Year 11 are left-handed.

One of the students in Year 10 and one of the students in Year 11 are chosen at random.

Which one is more likely to be left-handed?

Show your working. You may use the table if you wish.

	Year 10	Year 11	Total
right handed	44 ^①	39 ^②	83 ^③
left handed	17 ^④	20 ^⑤	37 ^⑥
Total	61 ^⑦	59 ^⑧	120 ^⑨

⑦ given
 ③ given
 ⑤ given
 ⑨ given
 ⑥ ⑨ - ③
 ④ ⑥ - ⑤
 ① ⑦ - ④
 ② ③ - ①
 ⑧ ⑨ - ⑦

$$P(\text{Year 10 left handed}) = \frac{17}{61} = 0.279$$

left handed
AND
Year 10

Year 10

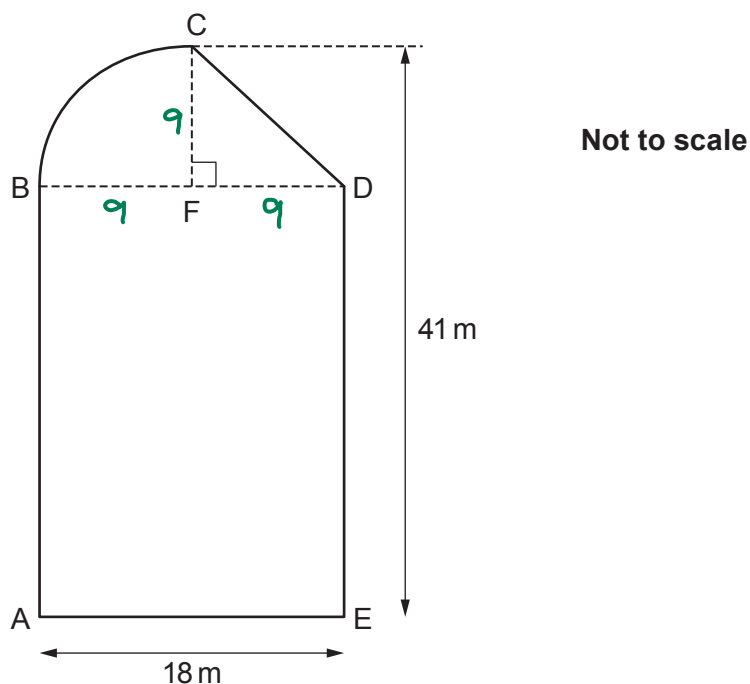
$$P(\text{Year 11 left handed}) = \frac{20}{59} = 0.339$$

left handed AND year 11

year 11

A year 11 student is more likely as 0.339 > 0.279 [6]

- 7 The diagram shows a shape ABCDE.
The shape is made from a rectangle, a right-angled triangle and a quarter of a circle.



F is the mid-point of BD.
AE = 18m and the perpendicular distance from C to AE is 41m.

Work out the **perimeter** of the shape ABCDE.

$$AB + CF = 41 \quad \Rightarrow \quad AB + 9 = 41 \quad \Rightarrow \quad AB = 32 \text{ m}$$

$$BC = \frac{1}{4} \times 2 \times \pi \times r \quad \Rightarrow \quad \frac{1}{4} \times 2 \times \pi \times 9 = \frac{9\pi}{2}$$

$$CD^2 = 9^2 + 9^2 \quad \Rightarrow \quad CD = \sqrt{81 + 81} = \sqrt{162} = 9\sqrt{2}$$

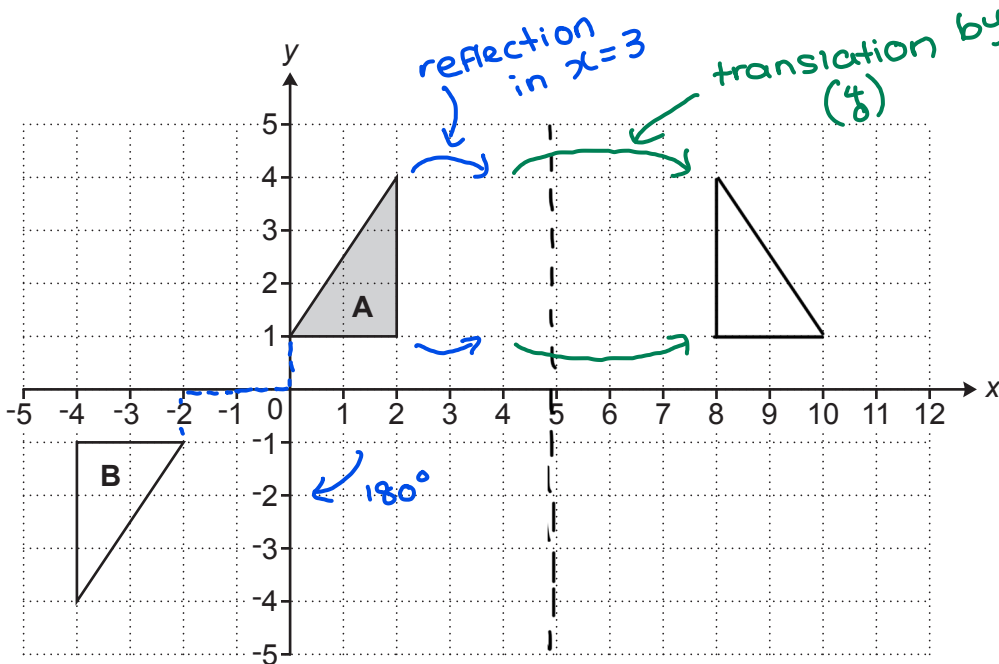
$$DE = AB = 32 \text{ m}$$

$$AE = 18 \text{ m}$$

$$32 + \frac{9\pi}{2} + 9\sqrt{2} + 32 + 18 = 108.865 \approx 109 \text{ m}$$

.....109..... m [6]

8 Triangle **A** and triangle **B** are drawn on the coordinate grid.



(a) Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

Rotation 180° about center $(-1,0)$ [3]

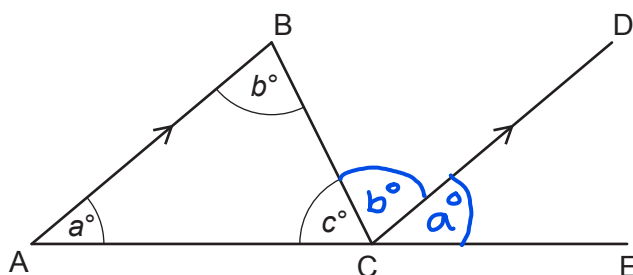
(b) Describe fully the **single** transformation that is equivalent to:

- a reflection in the line $x = 3$, followed by
- a translation by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

You may use the grid above to help you.

Reflection in the line $x=5$ [3]

- 9 The diagram shows triangle ABC.
 CD is parallel to AB.
 A, C and E lie in a straight line.
 Angles of size a° , b° and c° are shown.



Not to scale

- (a) Insert a° , b° or c° to make this statement true.
 Give a reason for your answer.

Angle DCE = a° because \widehat{BAC} is corresponding with \widehat{DCE}

..... [2]

- (b) Use the diagram and the answer to part (a) to show that the angles of a triangle add up to 180° .
 Give a reason for each statement you make. [3]

$$\widehat{BCD} = b^\circ \text{ (} \widehat{ABC} \text{ and } \widehat{BCD} \text{ are alternate angles)}$$

$$\widehat{BCA} + \widehat{BCD} + \widehat{DCE} = 180^\circ \text{ (angles in a line add up to } 180)$$

$$c^\circ + b^\circ + a^\circ = 180^\circ$$

also the sum of the angles in the Δ .

- 10 Claudia invests £25 000 at a rate of 2% per year compound interest.

Calculate the total amount of **interest** she will have earned after 5 years.
Give your answer correct to the nearest penny.

$$P \left(1 + \frac{r}{100} \right)^n$$

*P - Principal amount
r - interest rate
n - time period*

$$25000 \left(1 + \frac{2}{100} \right)^5$$

$$25000 \left(1.02 \right)^5 = £27602.020 \approx £27602.02$$

$$\begin{aligned} \text{Interest} &= £27602.02 - £25000 \\ &= £2602.02 \quad \text{£} \dots\dots\dots 2602.02 \dots\dots\dots [4] \end{aligned}$$

- 11 The area of a rectangle is 56 m², correct to the nearest m².
The length of the rectangle is 9.2 m, correct to the nearest 0.1 m.

Calculate the smallest possible width of the rectangle.

$$\begin{aligned} \text{area} &\rightarrow 55.5 \leq a < 56.5 \\ \text{length} &\rightarrow 9.15 \leq l < 9.25 \end{aligned}$$

Smallest values that will round up (pointing to 55.5 and 9.15)
upper bound (pointing to 56.5 and 9.25)

$$\text{length} \times \text{width} = \text{area} \quad \Rightarrow \quad \frac{\text{area}}{\text{length}} = \text{width}$$

$$\text{Smallest} = \frac{\text{Smallest}}{\text{largest}}$$

$$\text{width} = \frac{55.5}{9.25} = 6$$

$$\dots\dots\dots 6 \dots\dots\dots \text{m} [4]$$

12 (a) Here are the first four terms of a sequence.



Write an expression for the n th term of this sequence.

$5n + c$: from difference of 5^*

$\therefore 5n - 6$

Subs $n=1$: $5(1) + c = -1$
 $c = -1 - 5 = -6$

(a) $5n - 6$ [2]

(b) The n th term of another sequence is given by

$$an^2 + bn$$

The third term is 9 and the sixth term is 126.

Find the value of a and the value of b .

Subs $n=3 \rightarrow 9$: $a(3)^2 + b(3) = 9$
 $9a + 3b = 9$ — ①

Subs $n=6 \rightarrow 126$: $a(6)^2 + b(6) = 126$
 $36a + 6b = 126$ — ②

① $\times 2$: $18a + 6b = 18$ — ③

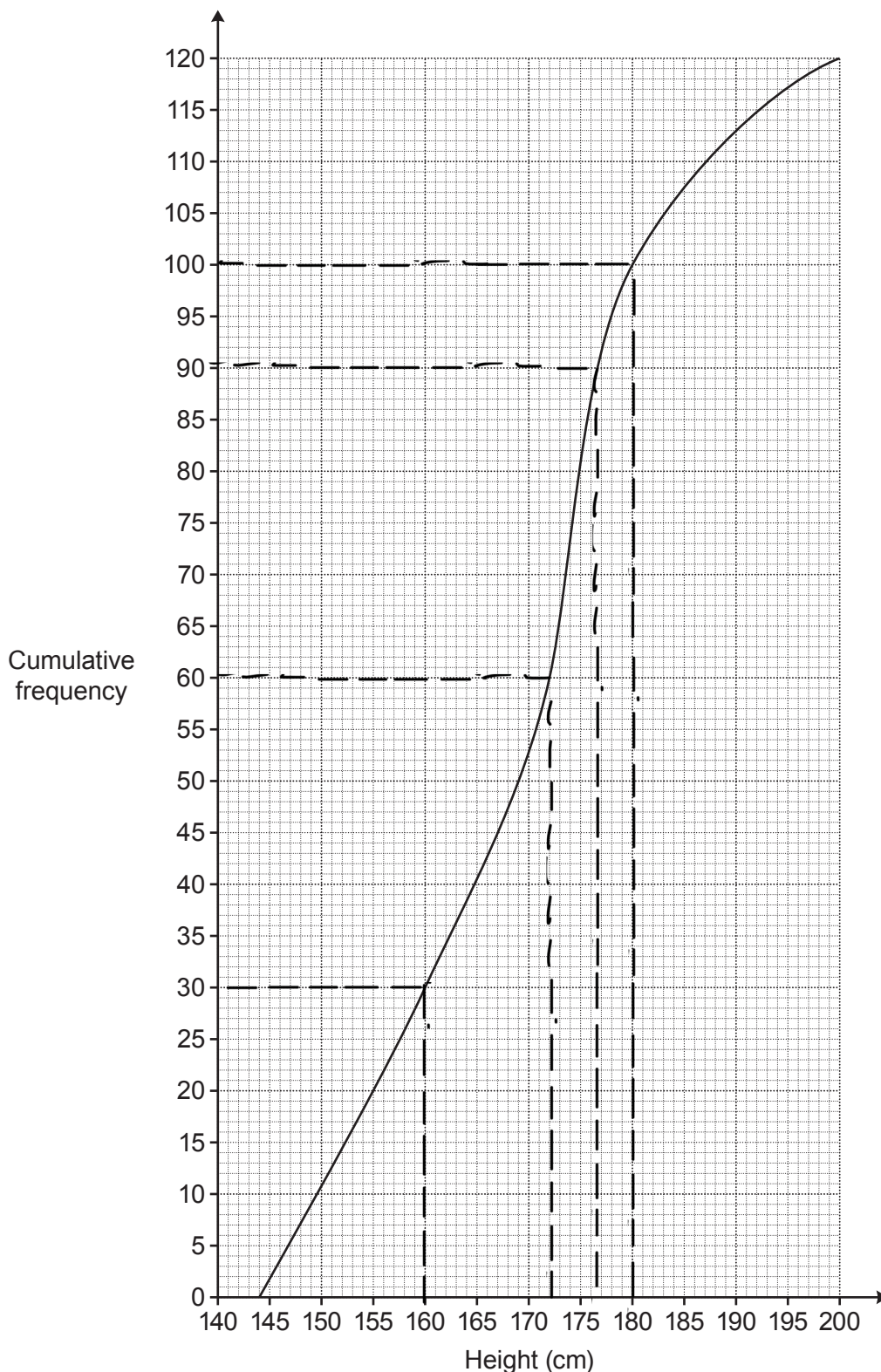
② - ③ : $18a = 108$
 $a = \frac{108}{18} = 6$

Subs $a=6$ in ① $9(6) + 3b = 9$
 $54 + 3b = 9$
 $3b = -45$
 $b = -15$

(b) $a =$ 6

$b =$ -15 [5]

- 13 (a) The cumulative frequency graph shows the distribution of the heights of members of a rowing club.



- (i) Find the median.

$$\frac{1}{2} \times 120 = 60^{\text{th}}$$

↑
Read from graph.

(a)(i) 172 cm [1]

(ii) Find the interquartile range.

$$\frac{3}{4} \times 120^{\text{th}} - \frac{1}{4} \times 120^{\text{th}}$$

$$176.5 - 160 = 16.5$$

(ii) 16.5 cm [2]

(iii) Calculate the percentage of the members who are at least 180 cm tall.

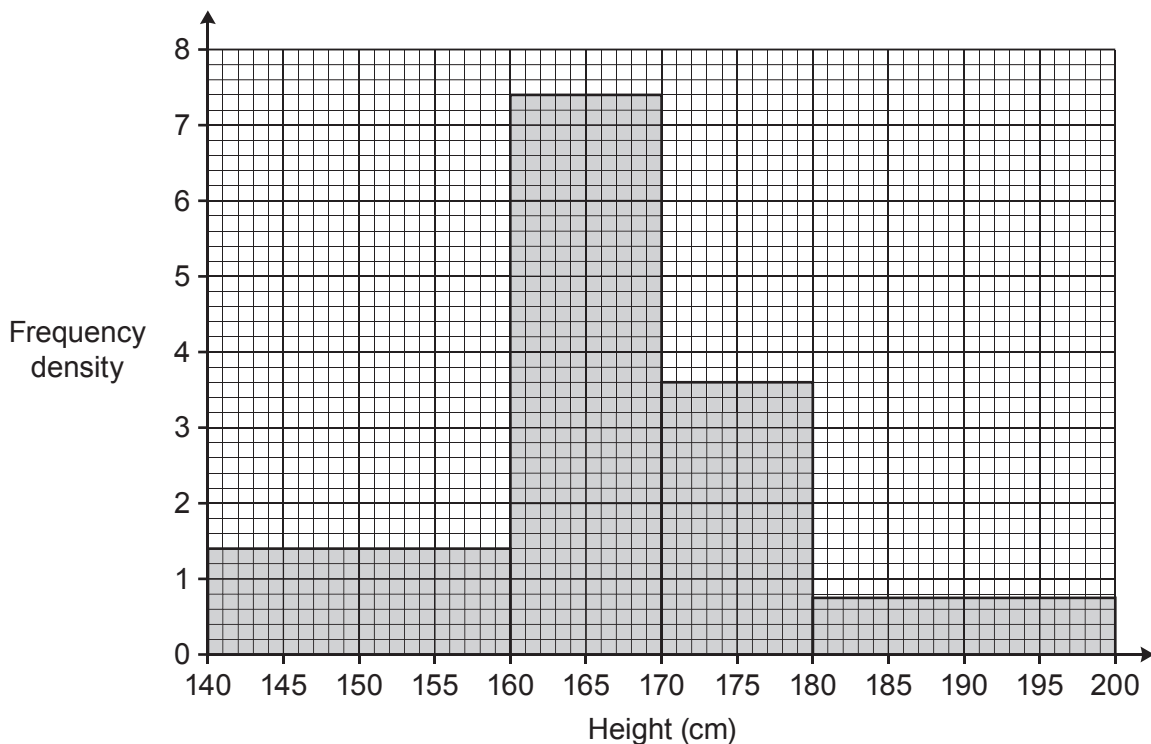
180th → 100 people → 20 people who are ≥ 180 cm

$$\frac{20}{120} \times 100 = 16.67$$

$$\approx 16.7\%$$

(iii) 16.7 % [3]

(b) The histogram summarises the heights of the 153 members of a swimming club.



Which club has the greater median height?

You must show all your working.

Median of Swimming club = $\frac{n+1}{2}$ th = $\frac{153+1}{2}$ th = $\frac{154}{2}$ th = 77 th

cumulative

first bar : $20 \times 1.4 = 28$

second bar : $10 \times 7.4 = 74$

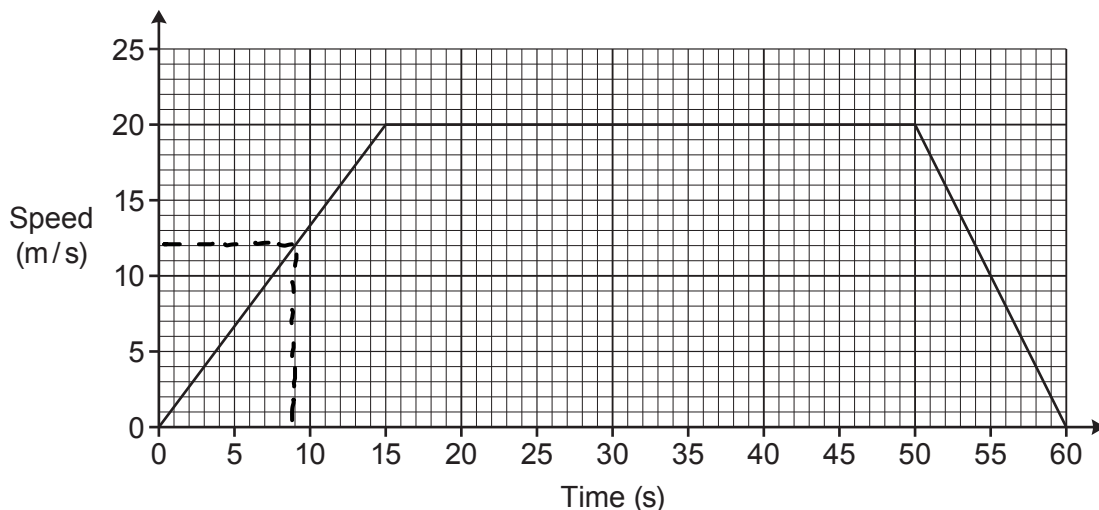
28

102 ← Median in second bar

$$172 > 160 \leq \text{Height} < 170$$

rowing club has higher median height [5]

14 The graph shows the speed of a train during the first 60 seconds of motion.



(a) What is the speed of the train after 9 seconds?

(a) 12 m/s [1]

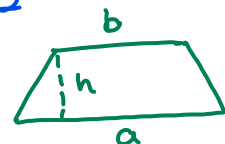
(b) What does the straight line suggest about the speed of the train over the first 15 seconds?

..... Acceleration is constant. [1]

(c) Work out the average speed of the train, in m/s, during the 60 seconds.

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Area of trapezium = $\frac{1}{2} \times h \times (a+b)$

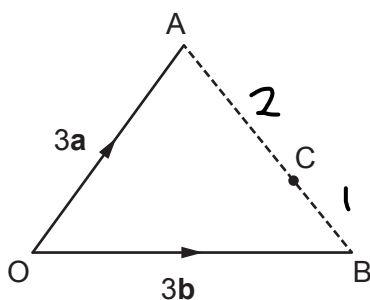


$$\begin{aligned} \text{Total distance} &= \frac{1}{2} \times 20 \times (60 + 35) \\ &= \frac{1}{2} \times 20 \times 95 = \frac{20}{2} \times 95 = 10 \times 95 = 950 \end{aligned}$$

$$\text{Avg speed} = \frac{950}{60} = 15.83 \approx 15.8$$

(c) 15.8 m/s [5]

15 The diagram shows triangle OAB and points C and D.



Not to scale

$$\vec{OA} = 3\mathbf{a} \text{ and } \vec{OB} = 3\mathbf{b}.$$

C lies on AB such that $AC = 2CB$.

D is such that $\vec{BD} = 2\mathbf{a} + \mathbf{b}$.

Show, using vectors, that OCD is a straight line.

[5]

$$\begin{aligned} \vec{OC} &= \vec{OA} + \frac{2}{3}\vec{AB} & \vec{AB} &= \vec{AO} + \vec{OB} \\ &= 3\mathbf{a} + \frac{2}{3}[-3\mathbf{a} + 3\mathbf{b}] & &= -3\mathbf{a} + 3\mathbf{b} \\ &= 3\mathbf{a} + [-2\mathbf{a} + 2\mathbf{b}] \\ &= \mathbf{a} + 2\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{CD} &= \vec{CB} + \vec{BD} \\ &= \frac{1}{3}\vec{AB} + \vec{BD} \\ &= \frac{1}{3}[-3\mathbf{a} + 3\mathbf{b}] + 2\mathbf{a} + \mathbf{b} \\ &= -\mathbf{a} + \mathbf{b} + 2\mathbf{a} + \mathbf{b} \\ &= \mathbf{a} + 2\mathbf{b} \end{aligned}$$

$\vec{OC} = \vec{CD} \quad \therefore \vec{OC} \parallel \vec{CD}$ and share point C. \therefore OCD is hence, a straight line.

16 (a) The table shows values of x and y .

x	4	16	36
y	6	3	2

Show that these values fit the relationship that y is inversely proportional to \sqrt{x} . [2]

$$y \propto \frac{1}{\sqrt{x}}$$

$$y = \frac{k}{\sqrt{x}}$$

<p>Subs $x=4, y=6$</p> $6 = \frac{k}{\sqrt{4}}$ $6 = \frac{k}{2}$ $k = 12$	<p>$x=16, y=3$</p> $3 = \frac{k}{\sqrt{16}}$ $3 = \frac{k}{4}$ $k = 12$	<p>$x=36, y=2$</p> $2 = \frac{k}{\sqrt{36}}$ $2 = \frac{k}{6}$ $k = 12$
---	--	--

(b) a is inversely proportional to b^2 and $a = 3.75$ when $b = 4$.

all have same k .
 \therefore They fit the relationship

Find a formula linking a and b .

$$a \propto \frac{1}{b^2} \quad a = \frac{k}{b^2}$$

Subs $a = 3.75 \quad b = 4$

$$3.75 = \frac{k}{4^2}$$

$$3.75 = \frac{k}{16}$$

$$k = 16 \times 3.75 = 60$$

$$\therefore a = \frac{60}{b^2}$$

(b) $a = \frac{60}{b^2}$ [3]

17 Show that $(a^3)^{-\frac{1}{3}} \times (a^2)^{\frac{1}{2}} = 1$.

[3]

$$a^{3 \times -\frac{1}{3}} = a^{-1}$$

$$a^{2 \times \frac{1}{2}} = a^1$$

$$a^{-1} \times a^1 = a^{-1+1} = a^0 = 1$$

$(x^y)^z = x^{y \times z}$
 $x^y \times x^z = x^{y+z}$

18 Region R satisfies these inequalities.

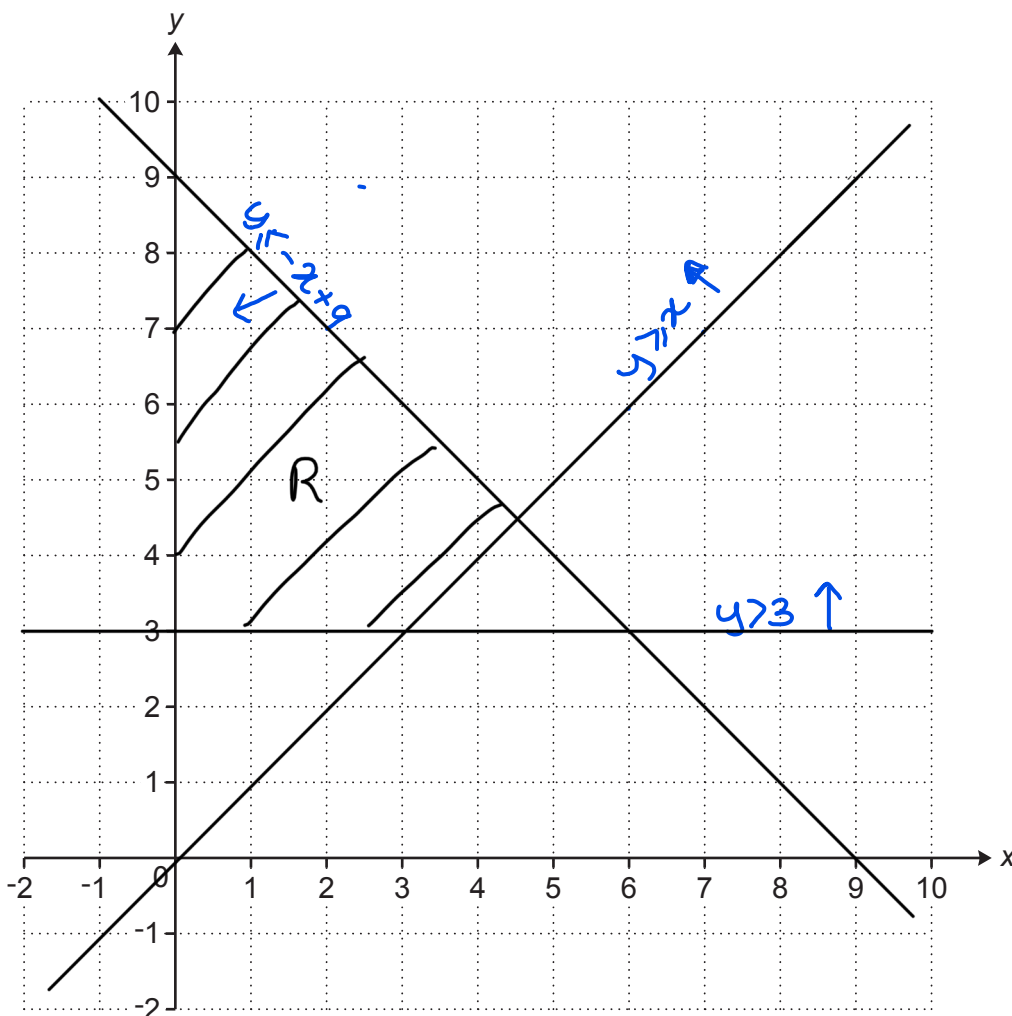
$$y > 3$$

$$y \geq x$$

$$x + y \leq 9$$

$$y \leq -x + 9$$

By drawing three straight lines on the grid, find and label the region R.



[6]

Turn over for Question 19

- 19 Solve this equation algebraically.
Give your solutions correct to 2 decimal places.

$$3x^2 + 8x - 5 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-8 \pm \sqrt{8^2 - 4(3)(-5)}}{2(3)}$$

$$\frac{-8 \pm \sqrt{64 + 60}}{6}$$

$$\frac{-8 \pm \sqrt{124}}{6}$$

$$\frac{-8 + \sqrt{124}}{6} \quad \text{or} \quad \frac{-8 - \sqrt{124}}{6}$$

$$0.5225 \quad \text{or} \quad -3.1892$$

$$\approx 0.52 \quad \quad \quad \approx -3.19$$

$$x = 0.52 \quad \text{or} \quad x = -3.19 \quad [4]$$

END OF QUESTION PAPER

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