Write your name here

Surname
MODEL ANSWERS

Other names

Centre Number

Candidate Number

Pearson Edexcel
Level 1/Level 2 GCSE (9–1)

Mathematics

Paper 3 (Calculator)

Higher Tier

Wednesday 8 November 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
1MA1/3H

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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6/6/6/7/2/2/

P 4 9 3 8 4 A 0 1 2 0

Pearson

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The table shows information about the heights of 80 children.

Height (h cm)	Frequency
$130 < h \leqslant 140$	4
$140 < h \leqslant 150$	11
$150 < h \leqslant 160$	24
$160 < h \leqslant 170$	22
$170 < h \leqslant 180$	19

4
15 ↩+11
39 ↩+24
61 ↩+22 ← 40.5 fall between 39 and 61
80 ↩+19

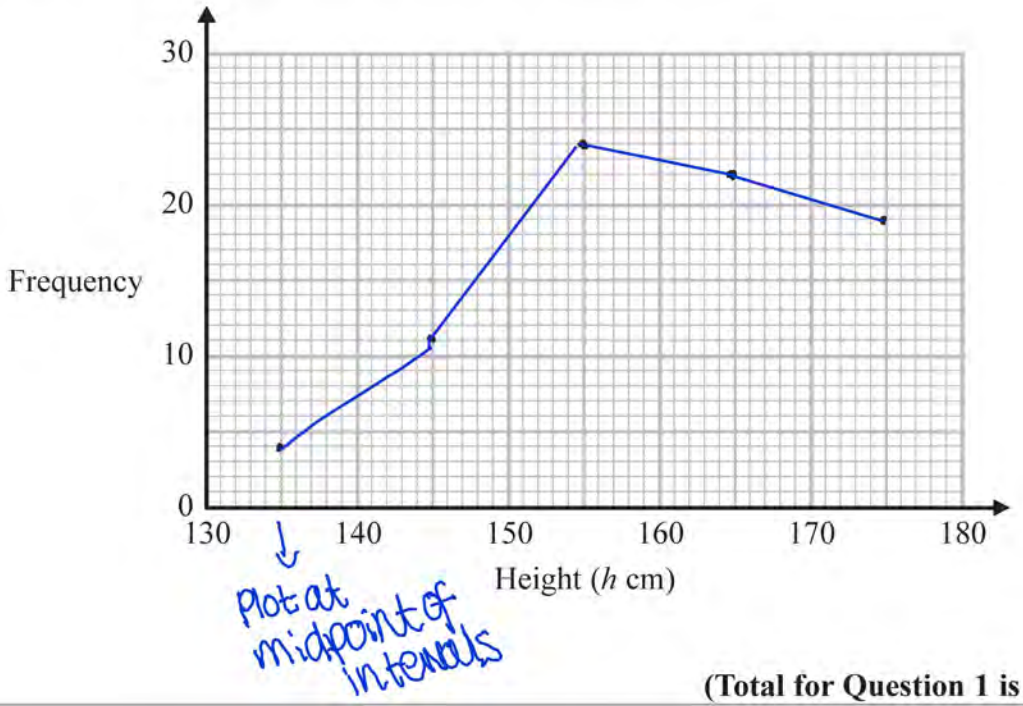
(a) Find the class interval that contains the median.

$\text{Median} = \frac{80+1}{2} = 40.5$

$160 < h \leq 170$

(1)

(b) Draw a frequency polygon for the information in the table.



Plot at midpoint of intervals

(2)

(Total for Question 1 is 3 marks)

2 In London, 1 litre of petrol costs 108.9p
In New York, 1 US gallon of petrol costs $2.83

1 US gallon = 3.785 litres
£1 = $1.46

In which city is petrol better value for money, London or New York?
You must show your working.

London: 1ℓ = £1.089 ⟶ x 3.785
x3.785 ⟶ 3.785 = £4.121865

x4.12... £1 = $1.46
£4.12... = $6.0179229 ⟵ x4.121865

In London, 1 US gallon costs $6.02, in New York this costs $2.83.
2.83 < 6.02
therefore it is cheaper in New York

(Total for Question 2 is 3 marks)

3 A gold bar has a mass of 12.5 kg. 12.5kg = 12500

The density of gold is 19.3 g/cm^3

Work out the volume of the gold bar.
Give your answer correct to 3 significant figures.

Density = Mass / Volume

Volume = 12500 / 19.3 = 647.688
↑ round up

648 ..... cm^3

(Total for Question 3 is 3 marks)

4 There are only blue pens, green pens and red pens in a box.

The ratio of the number of blue pens to the number of green pens is 2 : 5
The ratio of the number of green pens to the number of red pens is 4 : 1

There are less than 100 pens in the box.

What is the greatest possible number of red pens in the box?

Blue : Green Green : Red

×4 (2 : 5 ×4 → 8 : 20) ×5 (4 : 1 ×5 → 20 : 5)

Blue : Green : Red

8 : 20 : 5

8 + 20 + 5 = 33 parts

$\frac{100}{33} = 3 \text{ r } 1$ $3 \times 5 = 15$

To find largest possible total

15 red pens

(Total for Question 4 is 3 marks)

5 (a) Find the value of the reciprocal of 1.6
Give your answer as a decimal.

reciprocal = $\frac{1}{x}$ $\frac{1}{1.6} = 0.625$

0.625

(1)

Jess rounds a number, x, to one decimal place.
The result is 9.8

(b) Write down the error interval for x.

All Numbers in this interval round to 9.8

$9.75 \leqslant x < 9.85$

lower bound upper bound

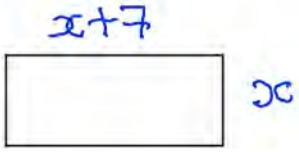
$9.75 \leqslant x < 9.85$

(2)

(Total for Question 5 is 3 marks)

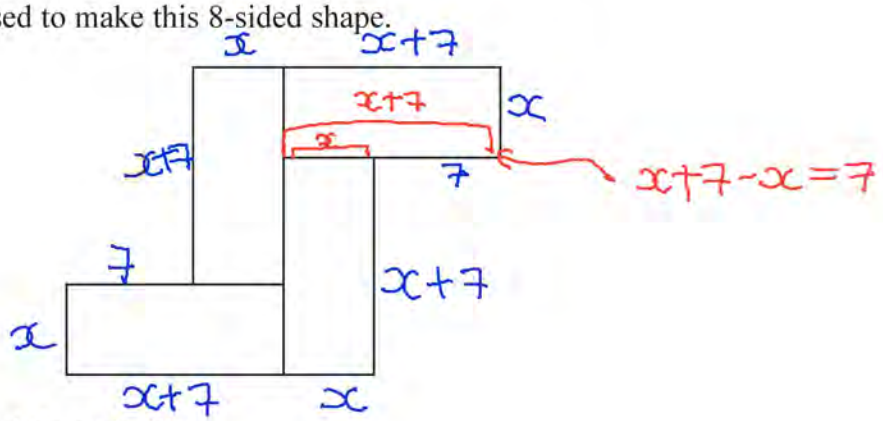
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

6 Here is a rectangle.



The length of the rectangle is 7 cm longer than the width of the rectangle.

4 of these rectangles are used to make this 8-sided shape.



The perimeter of the 8-sided shape is 70 cm.

Work out the area of the 8-sided shape.

Perimeter = $x+7+x+7+x+x+7+x+7+x+7+x+x+7$

$70cm = 8x + 42$

-42

$8x = 28cm$

$x = 3.5cm$ = width

$3.5+7 = 10.5cm$ = length

Area of 1 rectangle $= 3.5 \times 10.5 = 36.75cm^2$

A of 4 rect $= 36.75 \times 4 = 147cm^2$

147 ..... cm^2

(Total for Question 6 is 5 marks)

7 Work out $(13.8 \times 10^{7}) \times (5.4 \times 10^{-12})$
Give your answer as an ordinary number.

$13.8 \times 5.4 = 74.52$

$10^{7} \times 10^{-12} = 10^{-5}$

$74.52 \times 10^{-5} = 0.00007452$

0.00007452

(Total for Question 7 is 2 marks)

8 When a drawing pin is dropped it can land point down or point up.

Lucy, Mel and Tom each dropped the drawing pin a number of times.

The table shows the number of times the drawing pin landed point down and the number of times the drawing pin landed point up for each person.

	Lucy	**Mel**	**Tom**
point down	31	53	16
point up	14	27	9

Rachael is going to drop the drawing pin once.

(a) Whose results will give the best estimate for the probability that the drawing pin will land point up?
Give a reason for your answer.

Mel, because she carried out the most throws

(1)

Stuart is going to drop the drawing pin twice.

(b) Use all the results in the table to work out an estimate for the probability that the drawing pin will land point up the first time and point down the second time.

$31+53+16=100$ total number of landing ↓

$14+27+9=50$ total number of landing ↑

$100+50=150$ total number of throws

$\frac{100}{150}=\frac{2}{3}$ - P(lands ↓)

$\frac{50}{150}=\frac{1}{3}$ - P(land ↑)

$\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}$ $\frac{2}{9}$

(2)

(Total for Question 8 is 3 marks)

9 Jack bought a new boat for £12 500

The value, £V, of Jack's boat at the end of n years is given by the formula

$$V = 12\,500 \times (0.85)^n$$

(a) At the end of how many years was the value of Jack's boat first less than 50% of the value of the boat when it was new?

$12500 \times 0.85^n < \frac{12500}{2}$ ← less than 50% of its value

$0.85^n < \frac{1}{2}$

$0.85^3 = 0.614$ ×
$0.85^4 = 0.522$ ×
$0.85^5 = 0.444$ ✓

use trial and error to find n

$n = 5$

5 years

(2)

A savings account pays interest at a rate of R% per year.
Jack invests £5500 in the account for one year.

At the end of the year, Jack pays tax on the interest at a rate of 40%.
After paying tax, he gets £79.20

(b) Work out the value of R.

100 − 40%

Jack gets 60% of interest

60% = 79.20
10% = 13.20
100% = £132 = interest

$R\% = \frac{132}{5500} \times 100 = 2.4$

(interest / total money)

$R = 2.4\%$

2.4%

(3)

(Total for Question 9 is 5 marks)

10 There are only blue counters, yellow counters, green counters and red counters in a bag.
A counter is taken at random from the bag.

The table shows the probabilities of getting a blue counter or a yellow counter or a green counter.

Colour	blue	yellow	green	red
Probability	0.2	0.35	0.4	0.05

(a) Work out the probability of getting a red counter.

$1 - 0.2 - 0.35 - 0.4 = 0.05$

↳ probability adds to 1

0.05

(1)

(b) What is the least possible number of counters in the bag?
You must give a reason for your answer.

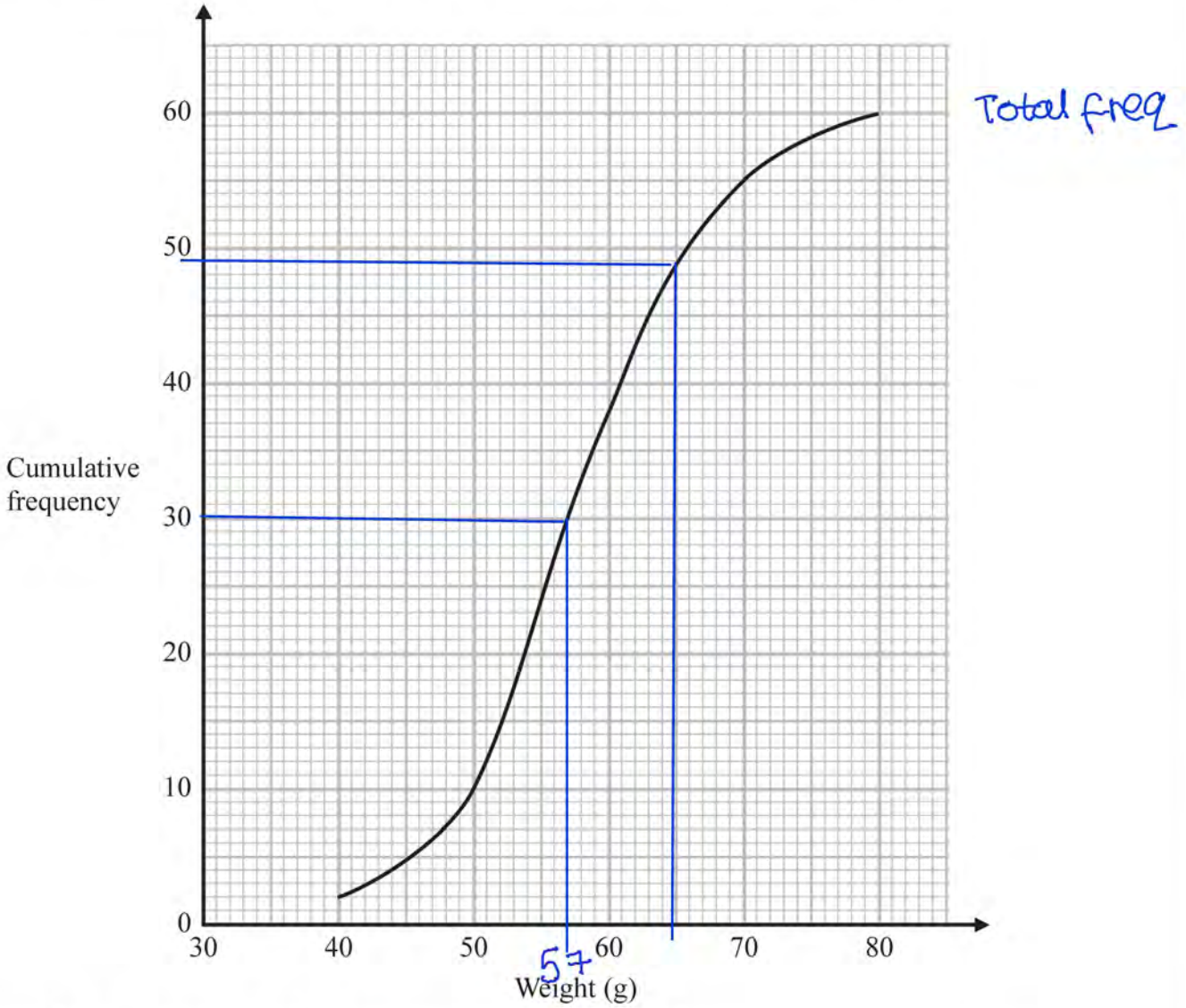
each colour must have a whole number
least possible number of counters when
0.05 = 1 counter
1/20 = 1 counter
×20 ×20

Total Number of counters = 20

(2)

(Total for Question 10 is 3 marks)

11 The cumulative frequency graph shows information about the weights of 60 potatoes.



(a) Use the graph to find an estimate for the median weight.

Total freq $= \frac{60}{2} = 30$

At 30, w = 57g

57 g

(1)

Jamil says,

"80 − 40 = 40 so the range of the weights is 40 g."

(b) Is Jamil correct?
You must give a reason for your answer.

No, because the min and max weights may not always be 40g and 80g

(1)

(c) Show that less than 25% of the potatoes have a weight greater than 65 g.

When weight = 65g, cumlative freq = 49

60 - 49 = 11 ← 11 potatoes have weight more than 65g

25% of 60 = 15

11 < 15 ∴ less than 25% of potatoes have a weight greater than 65g

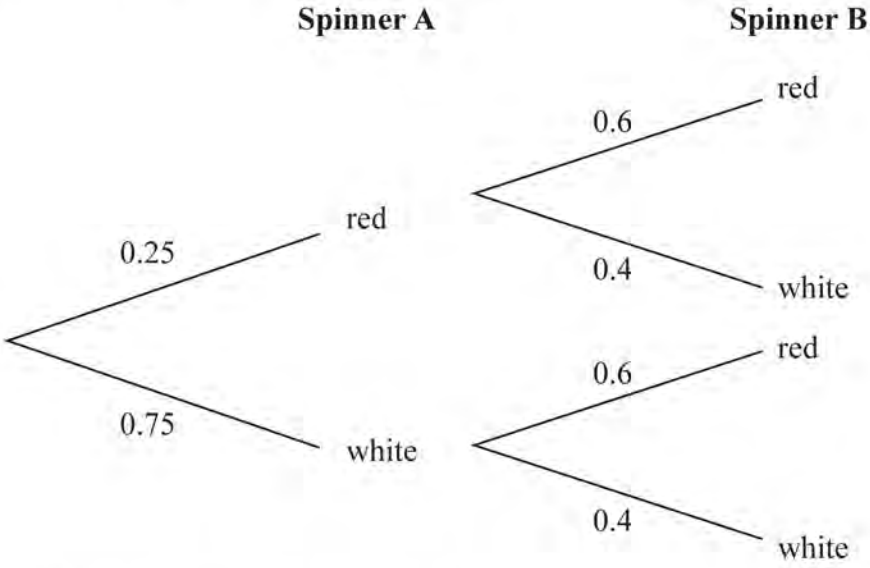
(2)

(Total for Question 11 is 4 marks)

12 Alan has two spinners, spinner **A** and spinner **B**.
Each spinner can land on only red or white.

The probability that spinner **A** will land on red is 0.25
The probability that spinner **B** will land on red is 0.6

The probability tree diagram shows this information.



Alan spins spinner **A** once and he spins spinner **B** once.
He does this a number of times.

The number of times **both** spinners land on red is 24

Work out an estimate for the number of times **both** spinners land on white.

and rule

Both red = Red and Red

$= 0.25 \times 0.6 = 0.15$

$0.15 = 24$ times

$\frac{24}{\text{total number of spins}} = 0.15$

$\frac{24}{0.15} = 160 = \text{Total number of spins}$

P(white and white)

$= 0.75 \times 0.4 = 0.3$

$160 \times 0.3 = 48$

↳ land on white both times

48

(Total for Question 12 is 3 marks)

13 Write $x^2 + 6x - 7$ in the form $(x + a)^2 + b$ where a and b are integers.

Divide by 2 $(x+3)^2 - 7 - 9 = (x+3)^2 - 16$

$(x+3)^2 = x^2+6x+9$

extra +9

$(x+3)^2 - 16$

(Total for Question 13 is 2 marks)

14 Cone **A** and cone **B** are mathematically similar.
The ratio of the volume of cone **A** to the volume of cone **B** is 27 : 8

The surface area of cone **A** is 297 cm^2

Show that the surface area of cone **B** is 132 cm^2

A B

$27 : 8$ volume sf

$\sqrt[3]{}$ $\sqrt[3]{}$

$3 : 2$ sf linear

square

$9 : 4$ area sf

x33 → 297 : 132 ← x33

∴ Surface Area of B is 132 cm^2

(Total for Question 14 is 3 marks)

$f(x) =$

15 (a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

when $x=0$, $f(0) = -5$

when $x=1$, $f(1) = 1+7-5 = 3$

There is a sign change between $x=0$ and $x=1$

so for $f(x)=0$, x must be between 0 and 1

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \frac{5}{x^2 + 7}$

$x^3 + 7x - 5 = 0$

$x^3 + 7x = 5$

$x(x^2 + 7) = 5$

$\therefore x = \frac{5}{x^2+7}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{5}{x_n^2 + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

$x_0 = 1$

$x_1 = \frac{5}{1^2+7} = \frac{5}{8}$

$x_2 = \frac{5}{(\frac{5}{8})^2+7} = 0.67653277$

$x_3 = \frac{5}{(0.6765..)^2+7} = 0.6704430$

$x = 0.6704$

(3)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$,
comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

$x = 0.6704$

$0.6704^3 + 7 \times 0.6704 - 5 = -0.00588$. This is close to 0, so it is an accurate estimate

(2)

(Total for Question 15 is 9 marks)

16 The petrol consumption of a car, in litres per 100 kilometres, is given by the formula

$$\text{Petrol consumption} = \frac{100 \times \text{Number of litres of petrol used}}{\text{Number of kilometres travelled}}$$

Nathan's car travelled 148 kilometres, correct to 3 significant figures.
The car used 11.8 litres of petrol, correct to 3 significant figures.

Nathan says,

"My car used less than 8 litres of petrol per 100 kilometres."

Could Nathan be wrong?
You must show how you get your answer.

$147.5 \leqslant km < 148.5$

$11.75 \leqslant \ell < 11.85$

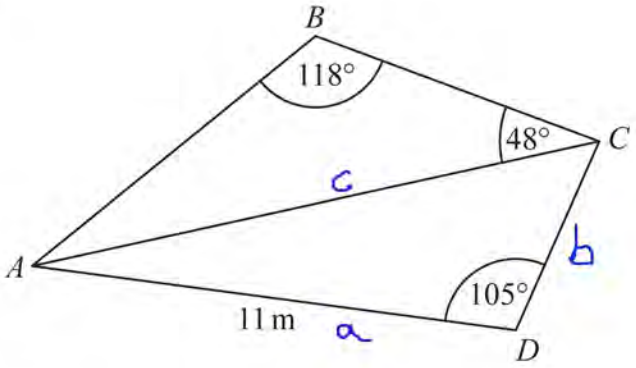
Max consumption $= \frac{100 \times 11.85}{147.5}$ ← upper bound / ← lower bound

$= 8.0339$ — so he could use more than 8L per 100km

Nathan could be wrong

(Total for Question 16 is 3 marks)

17 *ABC* and *ADC* are triangles.



The area of triangle *ADC* is 56 m²

Work out the length of *AB*.
Give your answer correct to 1 decimal place.

CD

Area of tri = $\frac{1}{2} \times a \times b \times \sin C$

$56 = \frac{1}{2} \times 11 \times b \times \sin 105$

$\frac{56}{\frac{11}{2} \times \sin 105} = b = 10.540..$ cm

$a^2 = b^2 + c^2 - 2bc\cos A$ ← cosine rule

$(AC)^2 = 11^2 + 10.540..^2 + 2 \times 11 \times 10.540 \times \cos 105$

$AC^2 = 292.1...$

$AC = 17.09$m

Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 48}{AB} = \frac{\sin 118}{17.09..}$

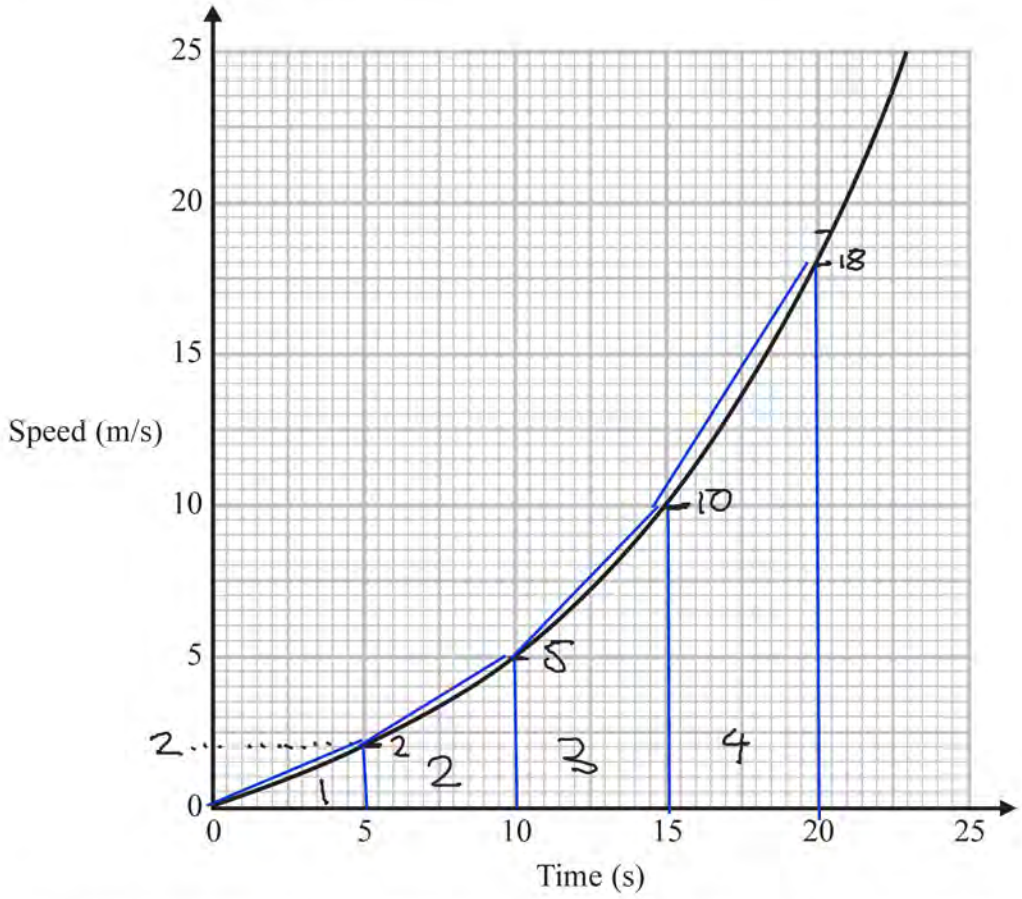
$AB = \frac{17.09 \sin 48}{\sin 118}$

$= 14.4$m

14.4 m

(Total for Question 17 is 5 marks)

18 Here is a speed-time graph for a train.



(a) Work out an estimate for the distance the train travelled in the first 20 seconds.
Use 4 strips of equal width.

$\frac{20}{4} = 5$ width = 5 sec

① $\frac{1}{2} \times 5 \times 2 = 5$

② $\frac{1}{2} \times 5 \times (2+5) = 17.5$ area of trapezium

③ $\frac{1}{2} \times 5 \times (5+10) = 37.5$

④ $\frac{1}{2} \times 5 \times (10+18) = 70$

$5 + 17.5 + 37.5 + 70$

$= 130$

130 m

(3)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance the train travelled?
Give a reason for your answer.

Overestimate because the area between the graph and the trapeziums has also been calculated

(1)

(Total for Question 18 is 4 marks)

19 Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$

$x - 2y = 10$

$x = 2y + 10$

$x^2 + y^2 = 20$

$(2y + 10)^2 + y^2 = 20$

$(2y + 10)(2y + 10)$

$4y^2 + 20y + 20y + 100$

$4y^2 + 40y + 100 + y^2 = 20$

-20

$5y^2 + 40y + 80 = 0$ $\div 5$

$y^2 + 8y + 16 = 0$

$(y + 4)(y + 4) = 0$

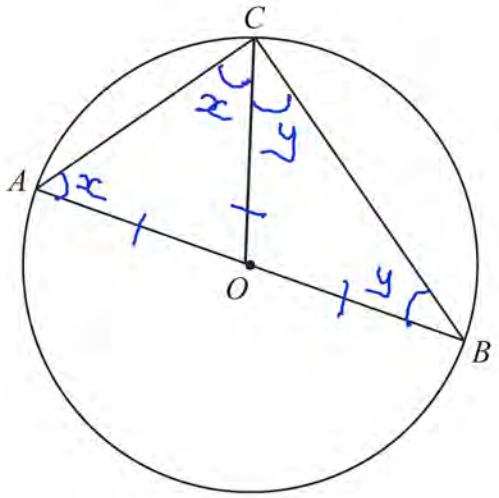
$y = -4$ $x = 10 + -8$

$= 2$

There is only one solution, therefore the curves only intersect once, meaning $x - 2y = 10$ is a tangent to $x^2 + y^2 = 20$ at $y = -4$ and $x = 2$

(Total for Question 19 is 5 marks)

20



A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* is 90°
You must **not** use any circle theorems in your proof.

$\angle OAC = \angle OCA$

$\angle OBC = \angle OCB$

radi are same length and base angles are =

$x + y + x + y = 180$

$2x + 2y = 180$

$x + y = 90$

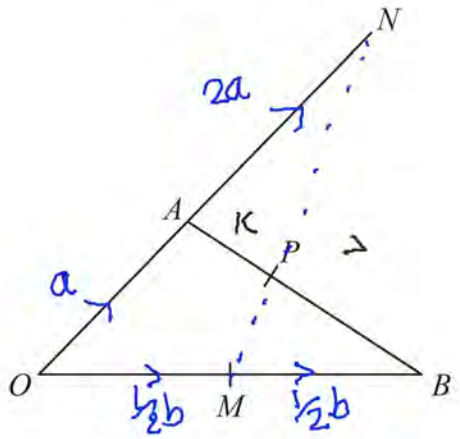
angles in tri ABC add up to 180

$\angle ACB = x + y$

$= 90°$

(Total for Question 20 is 4 marks)

21



OAN, *OMB* and *APB* are straight lines.
$AN = 2OA$.
M is the midpoint of *OB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where *k* is a scalar quantity.

Given that *MPN* is a straight line, find the value of *k*.

MPN is Astraight line therefore MN is a multiple Of PN and MP

$MN = x MP$

$AB = -a + b$ $MN = -\frac{1}{2}b + 3a$

$MP = MA + AP$

$= -\frac{1}{2}b + a + k(-a + b)$

$= (1-k)a + (k - \frac{1}{2})b$

$MN = x MP$

$3a - \frac{1}{2}b = x[(1-k)a + (k-\frac{1}{2})b]$

Equating a and b:

a: $3 = x(1-k)$

$x = \frac{3}{1-k}$

b: $-\frac{1}{2} = x(k - \frac{1}{2})$

$x = \frac{-1}{2(k-\frac{1}{2})}$

$x = x$

$\frac{3}{1-k} = \frac{-1}{2k-1}$

$6k - 3 = k - 1$

$+3 \quad -k$

$5k = 2$

$\therefore k = \frac{2}{5}$

$k = \frac{2}{5}$

(Total for Question 21 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS