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Surname

Other names

Pearson Edexcel
International GCSE

Centre Number

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Candidate Number

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Further Pure Mathematics

Level 2
Paper 2



Sample assessment material for first teaching September 2017

Time: 2 hours

Paper Reference

4PM1/02**Calculators may be used.**

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r} \quad |r| < 1$

Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 The n th term of a geometric series is $3e^{(1-2n)}$

Find the sum to infinity of this series.

Give your answer in the form $\frac{ae}{e^b - 1}$ where a and b are integers to be found.

(5)

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(Total for Question 1 is 5 marks)

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2 Find the set of values of x for which

(a) $3 + x < 2x - 1$

(1)

(b) $x(x - 1) > 6$

(3)

(c) **both** $3 + x < 2x - 1$ **and** $x(x - 1) > 6$

(1)

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(Total for Question 2 is 5 marks)

3 O , A and B are fixed points such that

$$\vec{OA} = 4\mathbf{i} + 3\mathbf{j} \quad \vec{OB} = 8\mathbf{i} + p\mathbf{j} \quad \text{and} \quad |\vec{AB}| = 2\sqrt{13}$$

(a) Find the possible values of p .

(3)

Given that $p > 0$

(b) find a unit vector parallel to \vec{AB}

(2)

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(Total for Question 3 is 5 marks)

4

$$f(x) = 2x^3 + px^2 + qx + 12 \quad p, q \in \mathbb{Z}$$

Given that $(x + 3)$ is a factor of $f(x)$ and that when $f'(x)$ is divided by $(x + 3)$ the remainder is 37

(a) show that $p = 1$ and find the value of q (6)

(b) hence factorise $f(x)$ completely (2)

(c) show that the equation $f(x) = 0$ has only one real root. (2)

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Question 4 continued

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(Total for Question 4 is 10 marks)

5 (a) Show that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$ (2)

(b) Hence express $2 \sin 5x \sin 3x$ in the form $\cos mx - \cos nx$ where m and n are integers, giving the value of m and the value of n , (1)

(c) (i) Find $\int 4 \sin 5\theta \sin 3\theta \, d\theta$

(ii) Hence evaluate $\int_0^{\frac{\pi}{6}} 4 \sin 5\theta \sin 3\theta \, d\theta$, giving your answer in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers. (4)

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Question 5 continued

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(Total for Question 5 is 7 marks)

Question 6 continued

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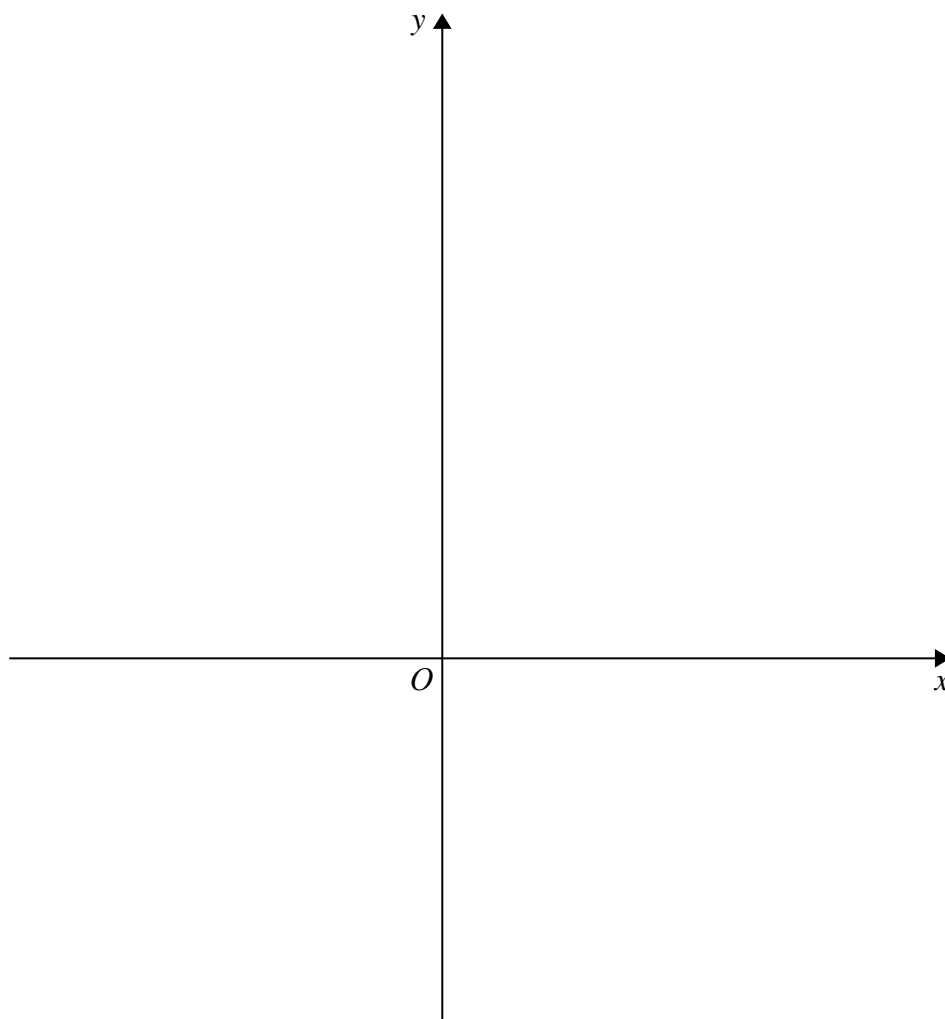
(Total for Question 6 is 7 marks)

7 The curve C with equation

$$y = \frac{ax - 5}{x - b}$$

where a and b are integers, crosses the x -axis at the point $(2.5, 0)$. The asymptote to C which is parallel to the y -axis has equation $x = 1$

- (a) (i) Show that $a = 2$ (3)
- (ii) Find the value of b . (1)
- (b) Find the coordinates of the point where C crosses the y -axis. (1)
- (c) Find the equation of the asymptote to C which is parallel to the x -axis. (1)
- (d) Using the axes below, sketch the curve C showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes. (3)



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Question 7 continued

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(Total for Question 7 is 8 marks)

- 8 (a) Expand $\frac{3}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^3 and simplifying each term as far as possible. (4)
- (b) Write down the range of values of x for which this expansion is valid. (1)
- (c) Show that $\frac{3}{\sqrt{0.9}} = \sqrt{10}$ (1)
- (d) Express $\frac{1}{\sqrt{10}-3}$ in the form $a\sqrt{10} + b$, where a and b are integers. (2)
- (e) Hence, using your expansion with a suitable value for x , obtain an approximation to 5 decimal places of $\frac{1}{\sqrt{10}-3}$ (3)

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Question 8 continued

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(Total for Question 8 is 11 marks)

9

$$f(x) = 7 + 4x - 2x^2$$

Given that $f(x)$ can be written in the form $P(x + Q)^2 + R$ where P , Q and R are constants,

(a) find the value of P , the value of Q and the value of R . (3)

(b) hence write down

- (i) the maximum value of $f(x)$,
- (ii) the value of x for which this maximum occurs. (2)

The curve C has equation $y = 7 + 4x - 2x^2$

The line l with equation $y = 4 - x$ intersects C at two points.

(c) Find the x coordinates of these two points. (3)

The finite region bounded by the curve C and the line l is rotated 360° about the x -axis.

(d) Use algebraic integration to find, to 3 significant figures, the volume of the solid generated. (5)

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Question 9 continued

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Question 9 continued

Area with horizontal dotted lines for writing answers.

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Question 9 continued

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(Total for Question 9 is 13 marks)

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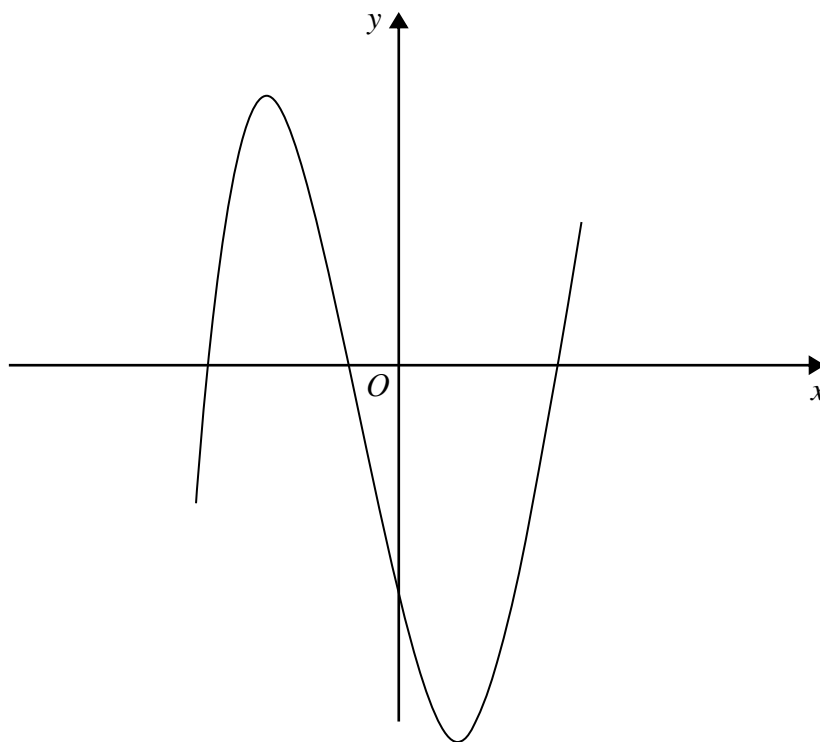


Figure 1

Figure 1 shows the curve M with equation $y = x^3 - 13x - 12$

The point P , with x coordinate -2 , lies on M and line l_1 is the tangent to M at the point P .

- (a) Find an equation for l_1 (5)

The point Q lies on M and the line l_2 is the tangent to M at the point Q .

Given that l_1 and l_2 are parallel,

- (b) find an equation for l_2 (4)

The normal to M at P meets l_2 at the point R .

- (c) Find the coordinates of R . (4)

- (d) Find the exact length of the line PR . (2)

The tangent and normal at P and the tangent and normal at Q form a rectangle.

- (e) Find the exact area of this rectangle. (3)

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Question 10 continued

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Question 10 continued

A large rectangular area containing numerous horizontal dotted lines for writing.

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Question 10 continued

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(Total for Question 10 is 18 marks)

11

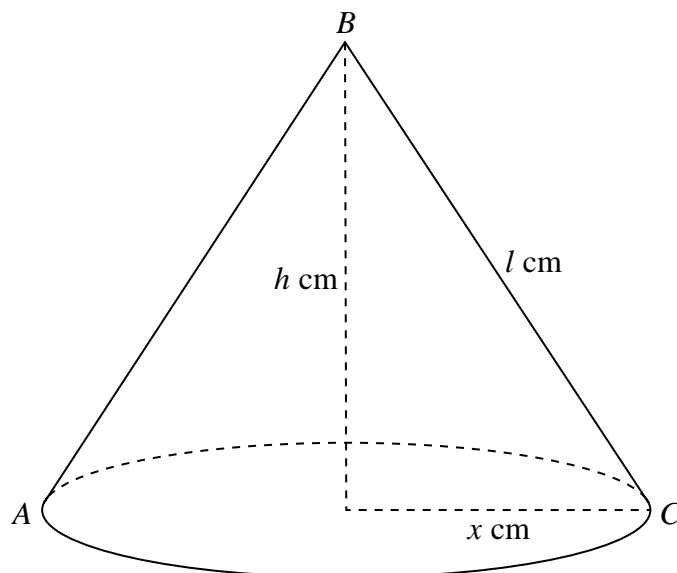
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows a right circular cone with a base radius of x cm. The slant height of the cone is l cm and the height of the cone is h cm. The vertex of the cone is B and the points A and C , on the base of the cone, are such that AC is a diameter of the base.

The cone is increasing in size in such a way that the size of the angle ABC is constant at 60° and the **total** surface area of the cone is increasing at a constant rate of $10 \text{ cm}^2/\text{s}$.

Find the exact rate of increase of the volume of the cone when $x = 6$

(11)

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Question 11 continued

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Question 11 continued

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Question 11 continued

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(Total for Question 11 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

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