

Please check the examination details below before entering your candidate information


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Centre Number					Candidate Number				
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**Pearson Edexcel International GCSE**

**Thursday 8 June 2023**

Morning (Time: 2 hours) Paper reference **4PM1/02**

**Further Pure Mathematics**  
**PAPER 2**



**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain **NO** credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

**Mensuration**

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Curved surface area of cone} = \pi r \times \text{slant height}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

**Series****Arithmetic series**

$$\text{Sum to } n \text{ terms, } S_n = \frac{n}{2}[2a + (n-1)d]$$

**Geometric series**

$$\text{Sum to } n \text{ terms, } S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\text{Sum to infinity, } S_\infty = \frac{a}{1-r} \quad |r| < 1$$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

**Calculus****Quotient rule (differentiation)**

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

**Trigonometry****Cosine rule**

$$\text{In triangle } ABC: a^2 = b^2 + c^2 - 2bc \cos A$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

**Logarithms**

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Given that  $\frac{a + 2\sqrt{5}}{3 - \sqrt{5}} = \frac{11 + b\sqrt{5}}{2}$  where  $a$  is an integer and  $b$  is prime,

find the value of  $a$  and the value of  $b$

Show your working clearly.

(5)

(Total for Question 1 is 5 marks)



P 7 2 8 6 6 A 0 3 3 6

2 The  $n$ th term of a convergent geometric series is  $8^{(1-2n)}$

Find the sum to infinity of the series.

Give your answer in the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers to be found.

(6)

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**Question 2 continued**

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**(Total for Question 2 is 6 marks)**



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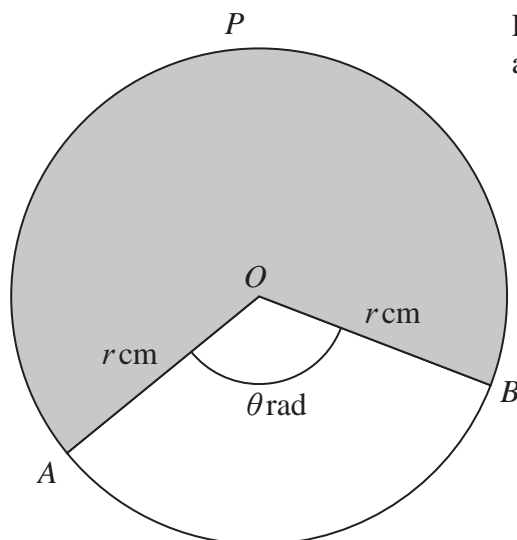
Diagram NOT  
accurately drawn

Figure 1

Figure 1 shows a circle, centre  $O$ , with radius  $r \text{ cm}$ .

The points  $A$ ,  $P$  and  $B$  lie on the circle.

The obtuse angle  $AOB = \theta$  radians.

The area of the sector  $APBO$ , shown shaded, is  $372.4 \text{ cm}^2$  and the length of the arc  $APB$  is  $53.2 \text{ cm}$ .

Find, to 3 significant figures where appropriate, the value of

- (i)  $r$
- (ii)  $\theta$

(6)

6



**Question 3 continued**

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**(Total for Question 3 is 6 marks)**



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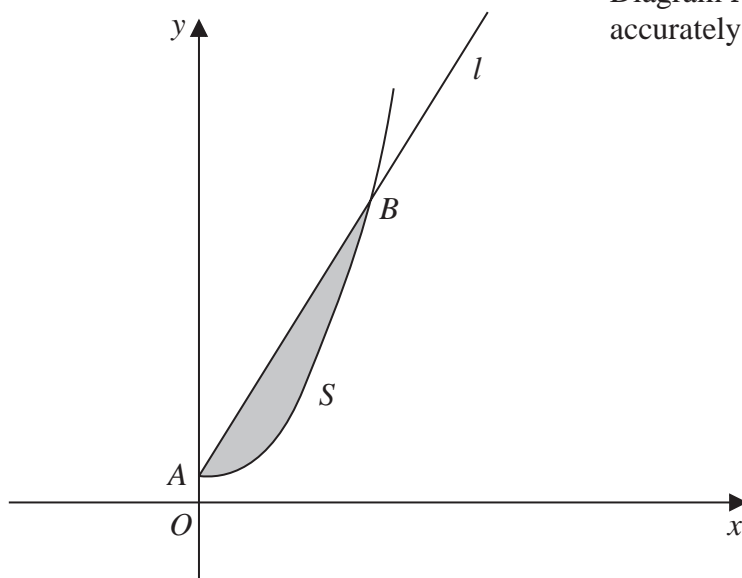
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Figure 2

The curve  $S$  with equation  $y = \frac{x^2}{4} + 2$  where  $x \geq 0$  and the line  $l$  with equation  $2y - x - 4 = 0$  where  $x \geq 0$  intersect at the points  $A$  and  $B$ , as shown in Figure 2.

(a) (i) Show that the coordinates of point  $A$  are  $(0, 2)$

(ii) Find the coordinates of the point  $B$

(4)

The finite region bounded by  $S$  and  $l$ , shown shaded in Figure 2, is rotated through  $2\pi$  radians about the  $y$ -axis.

(b) Use algebraic integration to find the volume of the solid generated.  
Give your answer in terms of  $\pi$

(4)

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**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

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**(Total for Question 4 is 8 marks)**



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5 (a) On the grid opposite draw the line with equation

(i)  $y = 2x + 5$

(ii)  $4y = x - 8$

(iii)  $5y + 3x = 30$

(3)

(b) Show, by shading, the region  $R$  defined by the inequalities

$y \leq 2x + 5$

$4y \geq x - 8$

$5y + 3x \leq 30$

(1)

For all points in  $R$  with coordinates  $(x, y)$

$P = 2x - 5y$

(c) Using your graph, find the least value of  $P$

(3)

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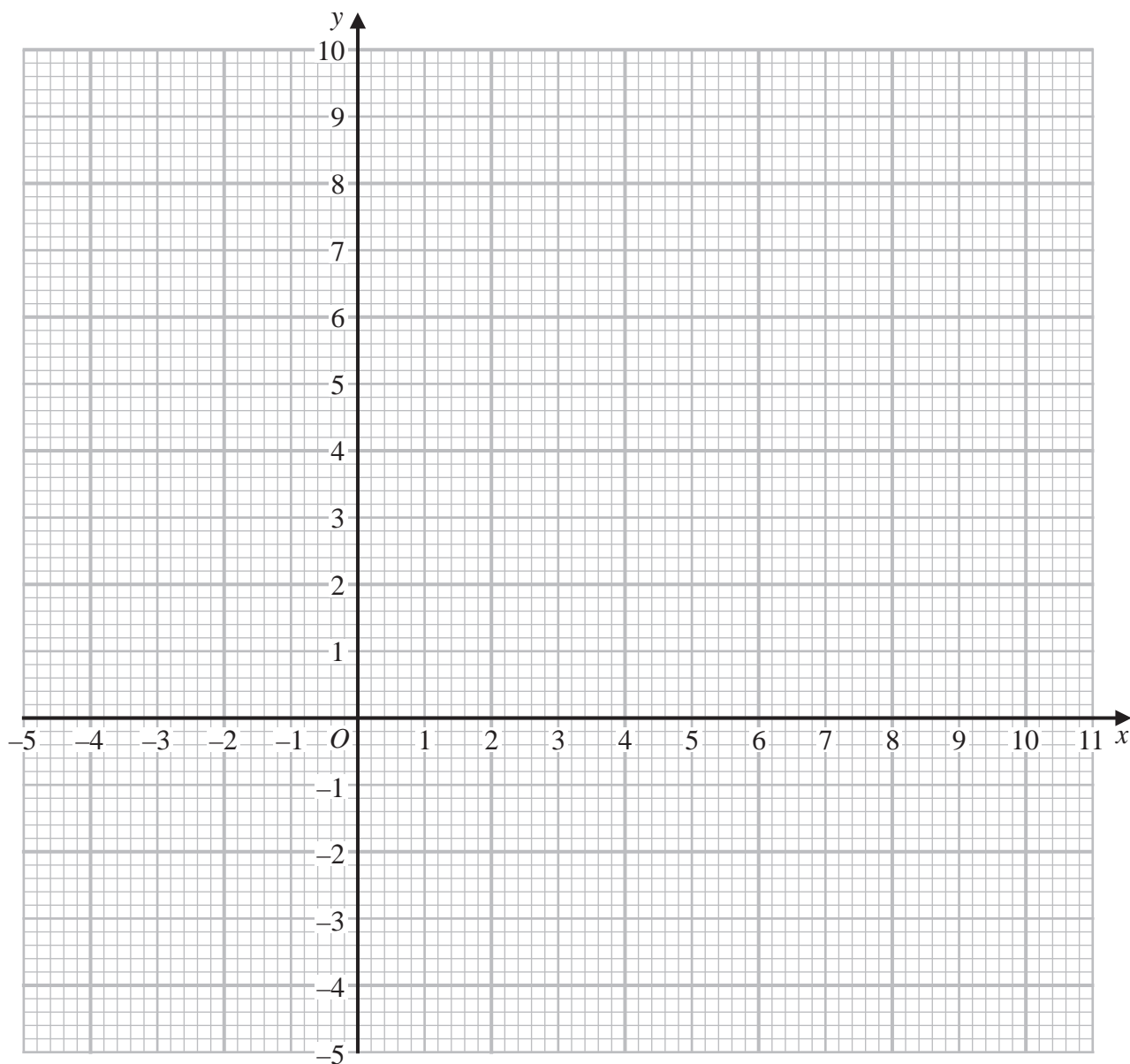
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Question 5 continued



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Turn over for a spare grid if you need to redraw your graph.



**Question 5 continued**

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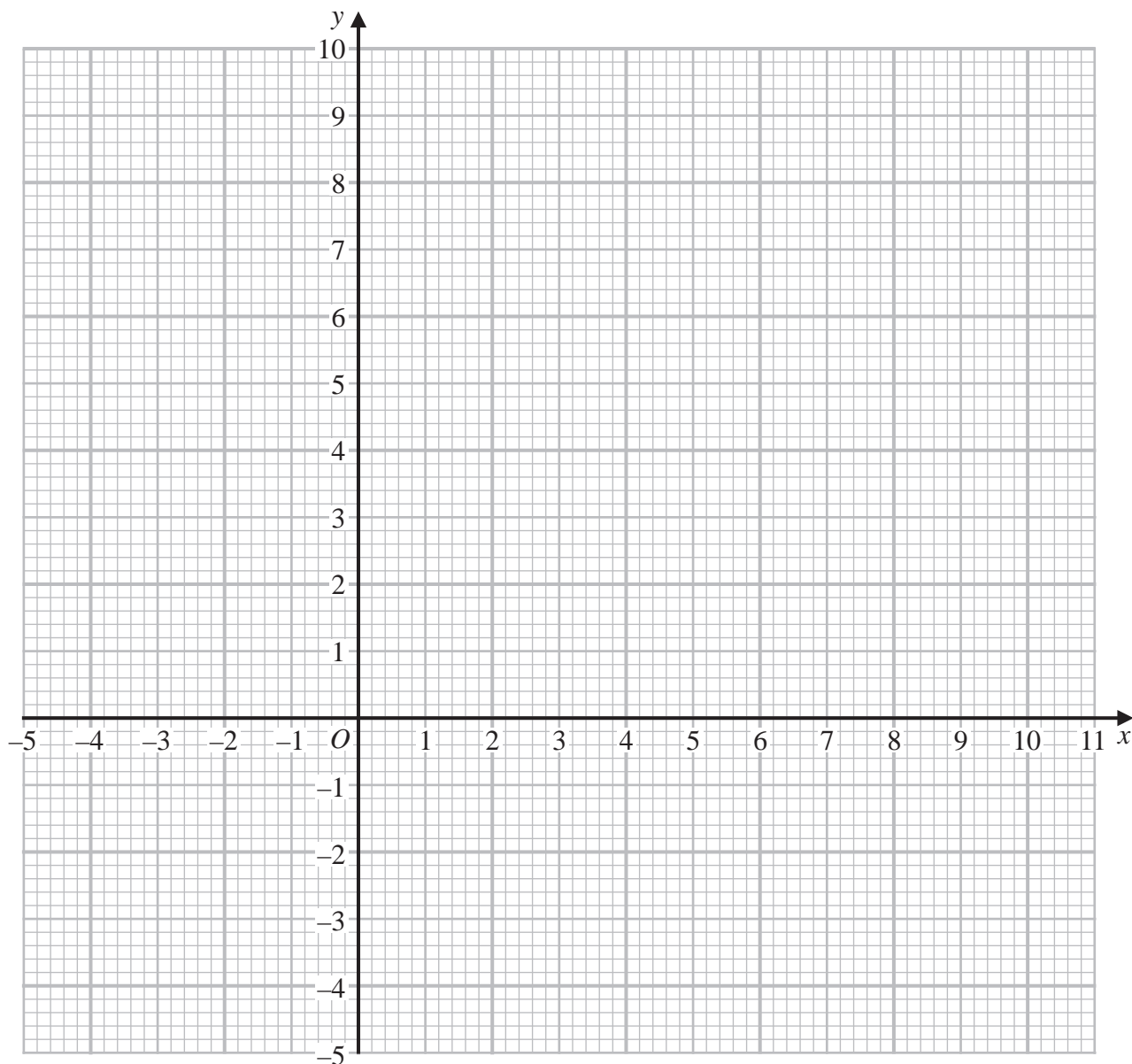
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Question 5 continued

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(Total for Question 5 is 7 marks)



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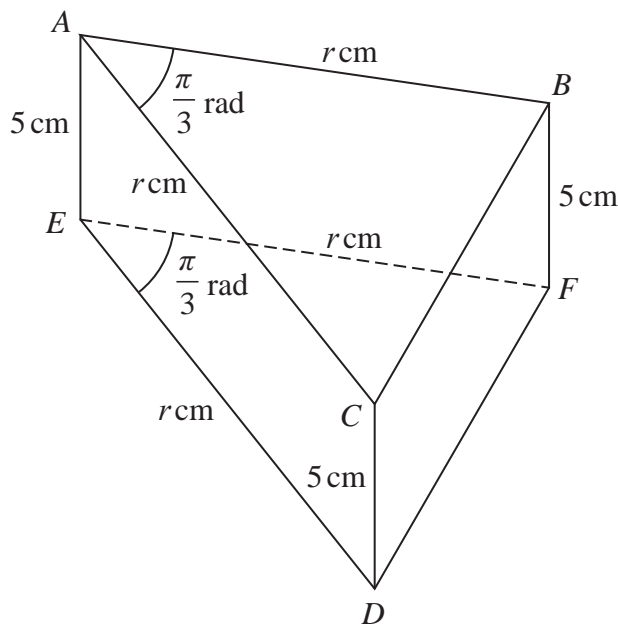


Diagram NOT accurately drawn

Figure 3

Figure 3 shows a right triangular prism  $ABCDEF$ . A cross section  $ABC$  of the prism is a triangle in which  $AB = AC = r$  cm and  $\angle CAB = \frac{\pi}{3}$  radians.

In the prism

$$AE = BF = CD = 5 \text{ cm} \quad ED = EF = r \text{ cm and } \angle DEF = \frac{\pi}{3} \text{ radians}$$

- (a) Show that the volume of the prism is  $\frac{5\sqrt{3}}{4} r^2 \text{ cm}^3$  (1)

The volume of the prism is increasing in such a way that the size of  $\angle CAB$  and the size of  $\angle DEF$  remain constant and the length of  $AE$ , the length of  $BF$  and the length of  $CD$  remain constant.

The lengths of  $AB$ ,  $AC$ ,  $ED$  and  $EF$  are each increasing at a constant rate of  $0.2 \text{ cm/s}$

- (b) Find the exact rate of increase, in  $\text{cm}^3/\text{s}$ , of the volume of the prism when the area of the rectangular face  $BCDF$  is  $60 \text{ cm}^2$  (5)

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

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**(Total for Question 6 is 6 marks)**



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7 (a) Expand  $\left(1 + \frac{x}{3}\right)^{-3}$  in ascending powers of  $x$  up to and including the term in  $x^3$   
Where appropriate express each coefficient as an exact fraction in its lowest terms. (3)

(b) Write down the range of values of  $x$  for which your expression is valid. (1)

(c) Express  $(3 + x)^{-3}$  in the form  $P(1 + Qx)^{-3}$  where  $P$  and  $Q$  are rational numbers whose values should be stated. (2)

$$f(x) = \frac{(1 + 4x)}{(3 + x)^3}$$

(d) Obtain a series expansion for  $f(x)$  in ascending powers of  $x$  up to and including the term in  $x^2$  (2)

(e) Hence, using algebraic integration, obtain an estimate of  $\int_0^{0.2} f(x) dx$   
Give your answer to 5 significant figures. (3)

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**(Total for Question 7 is 11 marks)**



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8 The points  $A$  and  $B$  have coordinates  $(-6, 8)$  and  $(12, 2)$  respectively.

(a) Find an equation of the straight line passing through  $A$  and  $B$  in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers to be found.

(3)

(b) Find the exact length of  $AB$

(2)

The point  $X$  with coordinates  $(m, n)$  lies on  $AB$  such that  $AX:XB = 1:2$

(c) Find the value of  $m$  and the value of  $n$

(2)

The line  $L$  passes through the point  $X$  and is perpendicular to  $AB$

The point  $C$  with coordinates  $(p, q)$  lies on  $L$  where  $p > 0$  and  $q > 0$

Given that  $AB$  is a diameter of a circle and  $C$  also lies on the circumference of the circle,

(d) find

(i) the exact value of  $p$

(ii) the exact value of  $q$

(7)

(e) Find the exact area of triangle  $ABC$

(3)

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

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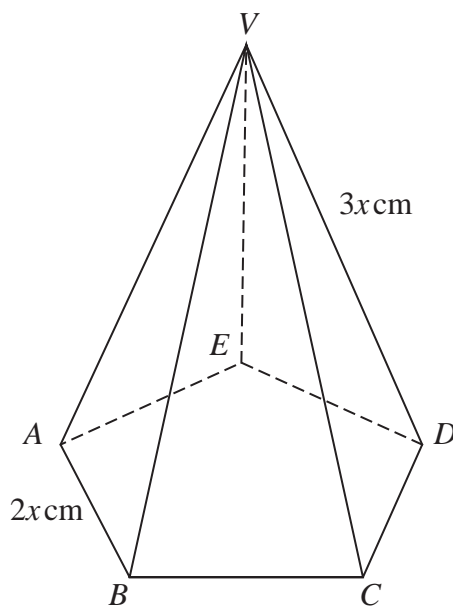
**(Total for Question 8 is 17 marks)**



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9

Diagram **NOT** accurately drawn



**Figure 4**

Figure 4 shows a right pyramid with vertex  $V$  and base  $ABCDE$  which is a regular pentagon.

$$AB = BC = CD = DE = EA = 2x \text{ cm}$$

$$VA = VB = VC = VD = VE = 3x \text{ cm}$$

Find, in degrees to one decimal place, the size of the angle between the plane  $VBC$  and the base  $ABCDE$

(6)

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**Question 9 continued**

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**(Total for Question 9 is 6 marks)**



10 The curve  $C$  with equation  $y = \frac{6 - 3x}{x - 4}$  where  $x \neq 4$ , crosses the  $x$ -axis at the point  $P$  and the  $y$ -axis at the point  $Q$

(a) Find the coordinates of

- (i)  $P$
- (ii)  $Q$

(2)

(b) Write down an equation of the asymptote to  $C$  which is

- (i) parallel to the  $y$ -axis
- (ii) parallel to the  $x$ -axis

(2)

(c) Sketch  $C$  showing clearly the asymptotes and the coordinates of the points  $P$  and  $Q$

(3)

The line  $L$  is the normal to  $C$  at the point on  $C$  where  $x = 2$

(d) Find an equation of  $L$

(6)

The line  $L$  intersects  $C$  again at the point  $R$

(e) Find the  $x$  coordinate of  $R$

(3)

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**Question 10 continued**

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**Question 10 continued**

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**(Total for Question 10 is 16 marks)**



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11 The roots of a quadratic equation  $E$  are  $\alpha$  and  $\beta$  where  $\alpha > \beta > 0$

Given that  $\alpha - \beta = 2\sqrt{6}$  and  $\alpha^2 + \beta^2 = 30$

(a) show that

(i)  $\alpha\beta = 3$  (4)

(ii)  $\alpha + \beta = 6$  (2)

(b) Without solving  $E$

(i) find the value of  $\alpha^4 + \beta^4$  (2)

(ii) find the exact value of  $\alpha^4 - \beta^4$  (2)

Given that  $\alpha^4 = P + Q\sqrt{6}$  where  $P$  and  $Q$  are positive integers,

(c) find the value of  $P$  and the value of  $Q$  (2)

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**Question 11 continued**

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Question 11 continued

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(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

