Mark Scheme (Results)

Summer 2018

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM0) Paper 02

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2018
Publications Code 4PM0_02_1806_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part ofthe question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = ...$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = ...$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x = ...$

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x = ...$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

edexcel

June 18
4PM0 Paper 2
Mark Scheme

Question Number	Scheme	Marks
1	$\cos A = \frac{9^2 + 8^2 - 6^2}{2 \times 9 \times 8}$ or $6^2 = 9^2 + 8^2 - 2 \times 9 \times 8 \cos A$	M1
	$\cos A = \frac{109}{144}$	A1
	$A = 40.8°$	A1 [3]
M1	Use the cosine rule, either form. If not for angle *BAC* there must be a complete method shown for obtaining *BAC*	
A1	Correct numerical expression for cos *BAC* or for sin *BAC* if a longer method used. Need not be simplified.	
A1	Correct angle as shown 40.80^{o} scores A0 (Ignore any labelling)	
2(a)	$\frac{dy}{dx} = 3e^{3x}\cos 2x - 2e^{3x}\sin 2x$	M1B1A1 (3)
(b)	$\frac{dy}{dx} = \frac{2e^x\left(2x^2-1\right) - 2e^x \times 4x}{\left(2x^2-1\right)^2}$	M1A1A1 (3)
		[6]
ALT:	Use of product rule $y = 2e^x\left(2x^2-1\right)^{-1}$ $\frac{dy}{dx} = 2e^x\left(2x^2-1\right)^{-1} - 2e^x(4x)\left(2x^2-1\right)^{-2}$	M1A1A1
	NB: No simplification is required in either (a) or (b). isw any shown	
(a) **M1**	Use of product rule. If the rule is quoted it must be correct. 2 terms of form $ke^{3x}\cos 2x$, $k'e^{3x}\sin 2x$ added or subtracted (k, k' integers, inc 1)	
B1	NB: A mark on e-PEN. Either term in their attempt at the product rule correct	
A1	Fully correct	
(b) **M1**	Use of quotient rule. If the rule is quoted it must be correct. Numerator to be the difference of 2 terms (either way round) of form shown $ke^x\left(2x^2-1\right)$, $k'xe^x$ Denominator must be correct.	
A1	One numerator term correct	
A1	Fully correct numerator	
ALT: **M1**	Bring up denominator correctly and apply product rule. Difference of 2 terms (either way round) of form shown	
A1	Either term correct	
A1	Both terms correct	

Question Number	Scheme	Marks
3	$V=5h^3 \Rightarrow \frac{dV}{dh}=15h^2$ or $\frac{dh}{dV}=\frac{1}{15}\left(\frac{V}{5}\right)^{-\frac{2}{3}}$	M1A1
	$\frac{dV}{dt}=24$ or $\frac{dV}{dt}=-24$	B1
	$800=5h^3 \Rightarrow h^3=160,\ h=\sqrt[3]{160},\ h=4\sqrt{10},\ h=5.4288...$	B1
	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt},=\frac{24}{15\left(\sqrt[3]{160}\right)^2}\ \left(=\frac{24}{442.0...}\right)$	M1,A1ft
	$\frac{dh}{dt}=0.0543...$	
	(Rate of decrease =) 0.054 cm/s	A1cso [7]
	Intermediate decimal answers should be at least 3 sf.	
M1	Differentiate V wrt h or h wrt V	
A1	Correct expression for $\frac{dV}{dh}$ or $\frac{dh}{dV}$ These 2 marks can be given if $15h^2$ is seen used **correctly** in their chain rule.	
B1	$\frac{dV}{dt}=24$ or -24 seen explicitly or used.	
B1	Correct value for h^3 or h when $V = 800$, seen explicitly or used. Award for any of $h^3=160,\ h=\sqrt[3]{160},\ h=4\sqrt{10},\ h=5.42...$ min 3 sf OR if $\frac{dh}{dV}$ was found, use of $V = 800$	
M1	Quote a correct chain rule for solving the problem. Terms can be in any (correct) order	
A1ft	Correct numbers in the chain rule, follow through previous results.	
A1cso	Correct final answer must be positive.	

Question Number	Scheme	Marks
4(a)	$3x = \ln 8$ or $x = \frac{1}{3}\ln 8$ or $\log_e 8 = 3x$ or $e^x = 2$ or $e^x = \sqrt[3]{8}$ $e^x = 8^{\frac{1}{3}}$	M1
	$x = \ln 2$	A1 (2)
(b)	$2e^{3x} = \left(e^{3x} - 4\right)^2$ or $y = \left(\frac{y}{2} - 4\right)^2$	M1
	$0 = \left(e^{3x}\right)^2 - 10e^{3x} + 16$ $y^2 - 20y + 64 = 0$	A1
	$\left(e^{3x} - 8\right)\left(e^{3x} - 2\right) = 0$ $(y - 16)(y - 4) = 0$	
	$e^{3x} = 8$ $x = \frac{1}{3}\ln 8 = \ln 2$ $y = 16$	M1
	$e^{3x} = 2$ $x = \frac{1}{3}\ln 2$ $y = 4$	A1
	$(\ln 2, 16)$ $\left(\frac{1}{3}\ln 2, 4\right)$	A1 (5)
(c)	Length $PQ = \sqrt{\left(\ln 2 - \frac{1}{3}\ln 2\right)^2 + 12^2}$, $= 12.0088... = 12.009$	M1,A1 (2)
		[9]
(a)		
M1	For any one of the forms shown.	
A1	Correct *exact* value for *x*. If $x = \ln 2$ is seen ignore any decimal value that follows. (NB: This is the only form of the answer that fits the demand.) Correct answer without working scores M1A1	
(b)		
M1	Eliminate either variable to obtain an equation in one variable	
A1	A correct 3 term quadratic, terms in any order (any equivalent of those shown) If $\left(e^{3x}\right)^2$ has been expanded incorrectly but then the mistake reversed when factorising this mark should be awarded.	
M1	Factorise or use the formula for their 3TQ and solve to *x* = ... or *y* = ... Some candidates use a substitution here and sometimes it is $y = e^{3x}$ If they reverse their substitution they can achieve full marks; if they fail to reverse it the max mark available is M1A1M0A0A0	
A1	2 correct *exact* values for *x* or *y* (ie 2 *x* values or 2 *y* values or correct coordinates of 1 point) Values for *x* may be any equivalent, eg $\ln\sqrt[3]{2}$	
A1	Coordinates for both points correct – need not be written in coordinate brackets, but pairing must be clear. (Do not isw if incorrect pairing shown.)	
(c)		
M1	Use the correct formula for the length of a line with their coordinates found in (b)	
A1	Correct length of *PQ*, **must** be 3 dp	

Question Number	Scheme	Marks
5(a)	$a + ar^2 = 75$	M1
	$ar + ar^2 = 45$	A1
	$\frac{1+r^2}{r+r^2} = \frac{75}{45} \quad \left(= \frac{5}{3}\right)$	dM1
	$2r^2 + 5r - 3 = 0 \qquad (2r-1)(r+3) = 0$	
	$r = \frac{1}{2}$ or -3	M1 (NB A1 on e-PEN) A1 (5)
(b)	$a = \frac{75}{\left(1+\frac{1}{4}\right)} = 60$	B1
	$S = \frac{a}{1-r} = \frac{60}{\frac{1}{2}} = 120 \quad \left(\text{or } S = \frac{a(1-r^n)}{1-r} \quad \text{with } n = \infty\right)$	M1A1cao (3)
		[8]
(a) **M1** **A1** **dM1** **M1** **A1** **(b)** **B1** **M1** **A1cao**	Form an equation in *a* and *r* using either of the pieces of information given. Form a second equation **with both equations correct** Eliminate *a* from their equations using a correct method. Depends on the first M mark. Solve their 3 term quadratic to obtain at least one value for the common ratio. (The method used must be shown or correct answers from a correct equation seen) **Both** values correct (½ or 0.5) Correct answers from incorrect or no working – send to review. Obtain the correct value for *a* using $r = \frac{1}{2}$ Can be awarded if seen in (a) and **used in (b)** Use $S = \frac{a}{1-r}$ with their value of *a* and a value of *r* found in (a) for which $\lvert r \rvert < 1$ Correct answer only	

Question Number	Scheme	Marks
6(a)	$(V =)\ 5x \times 2x \times h = 1000$ or $10x^2h = 1000$	B1
	$(S =)\ 5x \times 2x + 2h(5x + 2x)$	M1
	$S = 10x^2 + \dfrac{1400}{x}$ *	M1A1cso (4)
(b)	$\dfrac{dS}{dx} = 20x - 1400x^{-2}$	M1
	$\dfrac{dS}{dx} = 0 \Rightarrow x^3 = 70$, or $x = \sqrt[3]{70}$ $(x = 4.121...)$	M1,A1
	$S_{\min} = 10\left(\sqrt[3]{70}\right)^2 + \dfrac{1400}{\sqrt[3]{70}} = 509.54... = 510$	M1A1 (5)
(c)	$\dfrac{d^2S}{dx^2} = 20 + 2800x^{-3}$	M1
	$x = \sqrt[3]{70} \Rightarrow \dfrac{d^2S}{dx^2} > 0\ \therefore \min$	A1ft (2)
		[11]
(a)		
B1	Obtain a correct equation connecting *x* and *h* (any equivalent allowed)	
M1	Obtain an expression for *S* in terms of *x* and *h*, correct or with top included. This is a "show that" question so we require adequate evidence for this expression, in particular areas of the separate sides must be identifiable. (14*xh* with no evidence scores M0)	
M1	Use the equation to eliminate *h* to give an expression for *S* in terms of *x* only.	
A1cso	Obtain the **given** expression for *S*. Must start *S* =... No errors in the working	
(b)		
M1	Differentiate the **given** expression, power of *x* to decrease in at least one term	
M1	Equate their derivative to zero and solve for x^3	
A1	Correct value of x^3 or *x*, seen explicitly or used. (Correct *x* implies correct method.)	
M1	Use their value of *x* to obtain the corresponding value of *S*	
A1	Correct value of *S*. **Must** be 3 sf.	
	NB: These last 2 marks may only be given for work seen in (b)	
(c)		
	Working for (c) must be seen or used in (c) to gain credit in (c). *If work not labelled (c) there must be no following work for marks to be awarded.*	
M1	Obtain the second derivative. (If signs of d*S*/d*x* on either side of their *x* are considered, numerical calculations must be shown.)	
A1ft	Establish that the minimum has been obtained and give a conclusion. No need to calculate the value of the second derivative. Follow through their *x* provided *x* > 0 **and the second derivative is algebraically correct.**	
NB:	Solutions for (b) and (c) by trial and improvement – send to Review.	

Question Number	Scheme	Marks
7(a)	$\left(1+\frac{2x}{5}\right)^{\frac{1}{2}}=1+\frac{1}{2}\left(\frac{2x}{5}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)}{2!}\left(\frac{2x}{5}\right)^2+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{2x}{5}\right)^3$	M1
	$=1+\frac{x}{5}-\frac{x^2}{50}+\frac{x^3}{250}...$	A1A1 (3)
(b)	$\left(1-\frac{2x}{5}\right)^{-\frac{1}{2}}=1-\frac{1}{2}\left(-\frac{2x}{5}\right)+\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{2!}\left(-\frac{2x}{5}\right)^2+\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(-\frac{2x}{5}\right)^3$	M1
	$=1+\frac{x}{5}+\frac{3x^2}{50}+\frac{x^3}{50}+...$	A1A1 (3)
(c)	$-\frac{5}{2}\le x\le\frac{5}{2}$ or $-\frac{5}{2}\le x<\frac{5}{2}$ or $-\frac{5}{2}<x\le\frac{5}{2}$ or $-\frac{5}{2}<x<\frac{5}{2}$ (Accept $\lvert x\rvert<\frac{5}{2}$ or $\lvert x\rvert\leqslant\frac{5}{2}$)	B1 (1)
(d)	$\left(\frac{5+2x}{5-2x}\right)^{\frac{1}{2}}=\left(\frac{1+\frac{2}{5}x}{1-\frac{2}{5}x}\right)^{\frac{1}{2}}=\left(1+\frac{2x}{5}\right)^{\frac{1}{2}}\times\left(1-\frac{2x}{5}\right)^{-\frac{1}{2}}$	M1
	$=\left(1+\frac{x}{5}-\frac{x^2}{50}...\right)\left(1+\frac{x}{5}+\frac{3x^2}{50}...\right)$	
	$=1+\frac{x}{5}+\frac{3x^2}{50}+\frac{x}{5}+\frac{x^2}{25}-\frac{x^2}{50}+...$	M1
	$=1+\frac{2x}{5}+\frac{2x^2}{25}+...$	A1 (3)
(e)	$\int_{0.1}^{0.3}\left(\frac{5+2x}{5-2x}\right)^{\frac{1}{2}}\mathrm{d}x\approx\int_{0.1}^{0.3}\left(1+\frac{2x}{5}+\frac{2x^2}{25}\right)\mathrm{d}x$	
	$=\left[x+\frac{x^2}{5}+\frac{2x^3}{75}\right]_{0.1}^{0.3}$	M1A1ft
	$=0.3+\frac{0.3^2}{5}+\frac{2\times0.3^3}{75}-\left(0.1+\frac{0.1^2}{5}+\frac{2\times0.1^3}{75}\right),=0.21669...=0.2167$	dM1,A1 cao (4)
		[14]

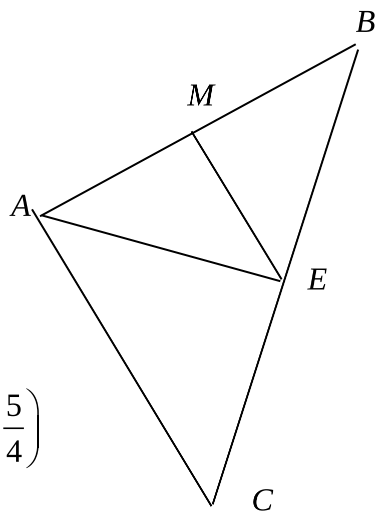
Question Number	Scheme	Marks
(a)		
M1	Attempt the binomial expansion. Must start with 1 and go up to at least x^3. $\left(\frac{2x}{5}\right)$ must be used in at least one term. Denominators 2 or 2!, 6 or 3!	
A1	2 correct algebraic terms; must be simplified, but fractions equivalent to those shown accepted for this mark.	
A1	Fully correct expansion as shown.	
(b)		
M1	Attempt the binomial expansion. Must start with 1 and go up to at least x^3. $\left(-\frac{2x}{5}\right)$ must be used in at least one term. Denominators 2 or 2!, 6 or 3!	
A1	2 correct algebraic terms, but fractions equivalent to those shown accepted for this mark.	
A1	Fully correct expansion as shown.	
(c)		
B1	Award for any of the ranges shown (5/2 or 2.5 accepted) (ie x between -5/2 and 5/2 with any inequality signs) Must be clear that the range applies to both expansions. Accept if just one range shown with no indication of expansion(s) it applies to or ranges for both expansions given and identical.	
(d)		
M1	Deal with the 5s to write the given expression in terms of the expressions in (a) and (b) - can be their expansions or $\left(1+\frac{2x}{5}\right)^{\frac{1}{2}} \times \left(1-\frac{2x}{5}\right)^{-\frac{1}{2}}$	
M1	Attempt the multiplication of their expansions from (a) and (b). Must have all terms needed up to x^2. Ignore higher powers. NB: This is not a dependent mark.	
A1	Simplify to the 3 terms shown.	
(e)		
M1	Attempt to integrate their expansion obtained in (d), min 2 terms. Must be a valid integration with powers of x increasing by 1 in at least 2 terms.	
A1ft	Correct integration of **their** expansion	
dM1	Use the given limits correctly in their integrated expression; ie attempt to substitute 0.3 and 0.1 in their terms and subtract the substitutions. Depends on the first M mark of (e).	
A1cao	Correct final answer. **Must** be 4 sf. **NB:** Correct answer w/o working scores 0/4 here as question states “use algebraic integration”.	

Question Number	Scheme	Marks
8(a)	$\cos 2\theta = \cos^2\theta - \sin^2\theta$	
(i)	$\cos 2\theta = 1 - \sin^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$ *	M1A1
(ii)	$\sin 2\theta = \sin\theta\cos\theta + \cos\theta\sin\theta = 2\sin\theta\cos\theta$ *	B1 (3)
(b)	$\cos 4\theta + 2\cos 2\theta = 1 - 2\sin^2 2\theta + 2\left(1 - 2\sin^2\theta\right)$	M1
	$= 1 - 2\left(2\sin\theta\cos\theta\right)^2 + 2 - 4\sin^2\theta$	M1
	$= 1 - 8\sin^2\theta\left(1 - \sin^2\theta\right) + 2 - 4\sin^2\theta$	M1
	$= 8\sin^4\theta - 12\sin^2\theta + 3$ *	A1cso (4)
ALT	**Working in reverse:** $8\sin^4\theta - 12\sin^2\theta + 3$	
	$= 8\left(\frac{1}{2}\left(1 - \cos 2\theta\right)\right)^2 - 12\left(\frac{1}{2}\left(1 - \cos 2\theta\right)\right) + 3$	M1
	$= 2\left(1 - 2\cos 2\theta + \cos^2 2\theta\right) - 6 + 6\cos 2\theta + 3$	M1
	$= 2 - 4\cos 2\theta + 2\left(\frac{1}{2}\left(1 + \cos 4\theta\right)\right) - 6 + \cos 2\theta + 3$	M1
	$= \cos 4\theta + 2\cos 2\theta$	A1cso
(c)	$4\sin^4 x° - 6\sin^2 x° - \cos 2x° = \frac{1}{2}\left(\cos 4x° + 2\cos 2x° - 3\right) - \cos 2x° + 1.2 = 0$	M1
	$\frac{1}{2}\cos 4x° = 0.3 \qquad \cos 4x° = 0.6$	A1
	$4x° = 53.13...°,\ 306.86...°$	M1
	$x = 13.28°..., 76.71°... \qquad x = 13.3°,\ 76.7°,$	A1 (4)
ALT	**Without using (b):** $4\sin^4 x - 6\sin^2 x - \cos 2x + 1.2 = 0 \quad 4\sin^4 x - 4\sin^2 x + 0.2 = 0$	
	$\sin^2 x = \frac{1}{2} \pm \frac{1}{\sqrt{5}} \quad \left(= 0.9472...\ 0.0527\right)$	M1A1
	$\sin x = \sqrt{0.9472...}\ \ x = 76.7°$ or $\sin x = \sqrt{0.0527}\ \ x = 13.3°$	M1A1

Question Number	Scheme	Marks
(d)(i)	$\int\left(2\sin^4\theta - 3\sin^2\theta\right)\mathrm{d}\theta = \frac{1}{4}\int\left(\cos 4\theta + 2\cos 2\theta - 3\right)\mathrm{d}\theta$	
	$= \frac{1}{4}\left(\frac{1}{4}\sin 4\theta + \sin 2\theta - 3\theta\right)\ (+c)$	M1A1
(ii)	$\frac{1}{4}\left[\frac{1}{4}\sin 4\theta + \sin 2\theta - 3\theta\right]_0^{\frac{\pi}{3}}$	
	$= \frac{1}{4}\left(\frac{1}{4}\sin\frac{4\pi}{3} + \sin\frac{2\pi}{3} - \pi\ (-0)\right)$	M1
	$= \frac{1}{4}\left(-\frac{1}{4}\times\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \pi\right), = \frac{3}{32}\sqrt{3} - \frac{1}{4}\pi$	A1,A1 (5)
		[16]

(a) (i)M1	Replace A and B with θ in $\cos(A+B) = \ldots$ and use $\cos^2\theta = 1 - \sin^2\theta$
A1cso	Obtain the **given** result with no errors seen
(ii)B1	Replace A and B with θ in $\sin(A+B) = \ldots$ and obtain the **given** result. $\sin\theta\cos\theta + \cos\theta\sin\theta$ must be seen.
(b)	There are many ways to do this part of the question. The following is for candidates who start from $\cos 4\theta + 2\cos 2\theta$ Some work separately on $\cos 4\theta$ and $(2)\cos 2\theta$ initially.
M1	Use (a) (i) or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ at least once anywhere in the work
M1	Use (a) (ii) and (a) (i) or $\cos 2\theta = \cos^2\theta - \sin^2\theta$ again to obtain an expression with no multiples of θ present. This mark **must only be awarded** when expressions for $\cos 4\theta$ and $2\cos 2\theta$ are combined. Candidates who never combine their separate expressions can gain M1M0M1A0 max.
M1	Use $\cos^2\theta = 1 - \sin^2\theta$ and the identity from (a)(i)to eliminate $\cos^2\theta$ and reach an expression or expressions for $\cos 4\theta$ and $2\cos 2\theta$ with powers of $\sin\theta$ only (May include a number but no terms with $\cos\theta$) Expressions for $\cos 4\theta$ and $2\cos 2\theta$ need not be combined.
A1cso	Obtain the **given** result with no errors seen.
ALT:	Working in reverse:
M1	Use $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ to replace the powers of $\sin\theta$
M1	Expand $\left(\frac{1}{2}(1 - \cos 2\theta)\right)^2$
M1	Use $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ to reach an expression in $\cos 4\theta$ and $\cos 2\theta$ with no other trig functions. There may be a number and the signs may be wrong
A1cso	Completely correct final expression obtained from correct working.

Question Number	Scheme	Marks
(c)		
M1	Use the result given in (b) to change the given equation to an equation in $\cos 4x$. No need to collect terms here. (M mark so need not be correct.)	
A1	Correct value for $\cos 4x$ obtained	
M1	For obtaining any correct value for $4x$. Need not be one of the 2 values shown. At least 3 sf must be shown.	
A1	For *both* values shown and no others within the range. Ignore extras outside the range.. **Must** be 3 sf.	
ALT	Without using (b)	
M1	Obtain a 3TQ in $\sin^2 x$ and solve to $\sin^2 x = ...$	
A1	Correct values for $\sin^2 x$ (exact or decimal)	
M1	Use their values of $\sin^2 x$ to solve for x	
A1	For *both* values shown and no others within the range. Ignore extras outside the range. **Must** be 3 sf.	
(d)		
(i)M1	Attempt to use the result given in (b) to change the given integrand into one which can be integrated and attempt the integration. (M mark so integrand need not be correct.) $\cos 4\theta \to \pm\frac{1}{4}\sin 4\theta$, $\cos 2\theta \to \pm\frac{1}{2}\sin 2\theta$	
A1	Fully correct after integration, constant not needed.	
(ii)M1	Substitute the given limits in their changed function, provided the result from (b) has been used in (i). (Candidates who use the equation from (c) cannot have this mark.)	
A1	Replace the trig functions with the *exact* values (not a follow through mark)	
A1cao	Correct final answer in the given form obtained.	

Question Number	Scheme	Marks
9	Grad $AB = \dfrac{6-4}{1-(-4)} = \dfrac{2}{5}$	M1
	Grad $AC = \dfrac{-1-4}{-2-(-4)} = -\dfrac{5}{2}$	A1
	$\dfrac{2}{5}\times\left(-\dfrac{5}{2}\right) = -1 \therefore AB$ is perpendicular to AC.	M1A1cso (4)
	See notes for 2 alt methods	
(b)	$\dfrac{y+1}{6+1} = \dfrac{x+2}{1+2}$ 	M1A1
	$7x-3y+11=0$	A1 (3)
(c)	Grad $l = -\dfrac{5}{2}$ (= grad AC)	
	Midpoint $AB = \left(-\dfrac{3}{2}, 5\right)$	B1B1
	Eqn. l: $y-5 = -\dfrac{5}{2}\left(x+\dfrac{3}{2}\right)$ $\left(y = \dfrac{-5}{2}x+\dfrac{5}{4}\right)$	M1A1 (4)
(d)	(E is midpoint of BC) E is $\left(-\dfrac{1}{2}, \dfrac{5}{2}\right)$ or decimal equivalents	B1, B1 (2)
(e)	AE perp to BC	M1
	$EC = \sqrt{(1.5^2+3.5^2)} = \sqrt{14.5}$	M1
	$AE = \sqrt{(3.5^2+1.5^2)} = \sqrt{14.5}$	A1
	Area $\Delta AEC = \dfrac{1}{2}AE\times EC = \dfrac{1}{2}\times 14.5 = 7.25$ oe	A1 (4)
		[17]
ALT 1	Area $\Delta AEC = \dfrac{1}{2}$ Area ΔABC	M1
	$AB = \sqrt{(5^2+2^2)} = \sqrt{29}$	M1
	$AC = \sqrt{2^2+5^2} = \sqrt{29}$	A1
	Area $\Delta AEC = \dfrac{1}{2}\times\dfrac{1}{2}\times AB\times AC = \dfrac{29}{4}$ oe $\left(7\dfrac{1}{4} \text{ or } 7.25\right)$	A1

Question Number	Scheme	Marks
ALT 2:	Use "determinant" method with coordinates of *A*, *E*, *C*	
	"Area ΔAEC" $= \frac{1}{2}\begin{vmatrix} -4 & -\frac{1}{2} & -2 & -4 \\ 4 & \frac{5}{2} & -1 & 4 \end{vmatrix}$	M1A1 (first M first A)
M1	$= \frac{1}{2}\left(-4\times\frac{5}{2}+-\frac{1}{2}\times -1+-2\times 4-\left(-4\times -1+-2\times\frac{5}{2}+-\frac{1}{2}\times 4\right)\right)$	
	$= -\frac{29}{4}$	M1 (second M)
A1	Area $\Delta AEC = \frac{29}{4}$	A1
(a)		
M1	Attempt gradient of either line. May find equation of either line and extract gradient from it.	
A1	Correct gradient of both lines	
M1	Attempt product of their gradients or state "negative reciprocals", provided the gradients are negative reciprocals, even if they are not correct. (no need for product)	
A1cso	Product $= -1$ or "negative reciprocals" and a conclusion (eg $\therefore$ perpendicular, shown, # or similar)	
ALT 1	Find lengths of *AB*, *AC* and *BC* and use Pythagoras	
M1	Attempt lengths of 2 of these lines	
A1	Correct lengths of all 3 lines ($\sqrt{29}, \sqrt{29}, \sqrt{58}$)	
M1	Use Pythagoras (sum of squares of the two shorter sides = square of longest)	
A1cso	Everything correct and a conclusion given (as above)	
ALT 2	Find an equation of the perpendicular to *AB* through *C*. Find the intersection of this line with *AB* and show it is *A*.	
M1	Attempt the gradient of *AB*	
A1	Correct equation of the perpendicular through *C* $\left(y+1=-\frac{5}{2}(x+2)\ \text{ oe}\right)$	
M1	Attempt an equation for *AB* **and** solve with their previous line	
A1	Correct intersection (- 4, 4) **and** a conclusion.	
(b)		
M1	Use any *complete* method for the equation of *BC*. (Use of $y = mx + c$ requires an attempt to find a numerical value for *c*.)	
A1	Correct numbers in their choice of method	
A1	Correct equation in the required form. All terms to be on one side of the = sign with 0 on the other. Can be an integer multiple of the one shown.	

Question Number	Scheme	Marks
(c)		
B1	Either coordinate of the midpoint of *AB*	
B1	Second coordinate of midpoint	
M1	Any *complete* method for the equation of the perpendicular bisector. Must include the gradient as the negative reciprocal of their gradient of *AB* or their gradient of *AC*. If (a) done by Pythagoras an appropriate gradient must be found for this M mark.	
A1	Correct equation of the perpendicular bisector, any equivalent form. Must have $y = ...$	
(d)		
B1	Either coordinate of *E*; fraction or decimal	
B1	Second coordinate of *E*; fraction or decimal	
(e)		
M1	For the statement shown. Give by implication if the following work implies use of this. No explanation needed.	
M1	Attempting the length of *EC* or *AE*	
A1	Both lengths correct.	
A1	Obtain the correct area of the triangle. (7.3 scores A0)	
ALT 1:		
M1	For the statement shown. Give by implication if the following work implies use of this. No explanation needed.	
M1	Attempting the length of *AB* or *AC*	
A1	Both lengths correct. Award marks if work seen in (a) and used here.	
A1	Obtain the correct area of the triangle. (7.3 scores A0)	
ALT 2:	By "determinant" method.	
M1	Area $\Delta AEC = \left(\frac{1}{2}\right)\begin{vmatrix} -4 & -\frac{1}{2} & -2 & -4 \\ 4 & \frac{5}{2} & -1 & 4 \end{vmatrix}$. $\frac{1}{2}$ not needed for this mark. Coords of *A*, *C* and their coords of *E* needed with first pair repeated at the end. Points in any order.	
A1	Correct numbers in the "determinant" (with or without the $\frac{1}{2}$ present)	
M1	Include the $\frac{1}{2}$ and attempt to multiply out their determinant.	
A1	Correct area, must be positive.	
NB	Enter marks in e-PEN order (M1M1A1A1) not in marking order (M1A1M1A1)	

1

Question Number	Scheme	Marks
10(a)	$4a^2 = 16a \quad a = 4$	M1A1 (2)
(b)	A is $(4,8)$ $x_B = 8$ (accept B is (8, 0))	M1A1 (2)
(c)	$(\text{Vol} = \pi)\int_0^4 y^2 \mathrm{d}x = (\pi)\int_0^4 16x\,\mathrm{d}x$	M1
	$= (\pi)\left[8x^2\right]_0^4$	dM1
	Vol of cone $= \frac{1}{3}\pi \times 8^2 \times 4 \left(= \frac{256\pi}{3}\right)$ or $\pi\int_4^8 (-2x+16)^2 \mathrm{d}x$	B1 NB A1 on e-PEN
	$128\pi + \frac{256\pi}{3} = 670$	ddM1A1cao (5) [9]
(a)		
M1	Use the coordinates of A and the equation of C to form an equation in a and solve to $a = ...$	
A1	$a = 4$	
(b)		
M1	Use their value of a and attempt to obtain the x coordinate of B. May find the equation of l or draw a diagram. Award by implication if the correct value is written down.	
A1	$x_B = 8$	
(c)		
M1	For $\int 16x\,\mathrm{d}x$ seen explicitly or implied by subsequent work. Limits and π not needed	
dM1	Attempt the integration. Limits and π not needed. Depends on the first M mark	
B1	NB A1 on e-PEN Correct volume of the cone, as a product from using the formula or in integral form **with correct limits**	
ddM1	Include π, substitute the limits 0 to their a in the volume of rev of the curve, evaluate the volume of the cone and add their two volumes. Depends on both the above M marks.	
A1cao	Correct complete volume, **must** be 3 sf.	
	Attempts at line – curve or curve – line:	
	$\int \left[16x - (-2x+16)\right]^2 \mathrm{d}x$ scores M0 (so 0/5)	
	$\int \left[16x - (-2x+16)^2\right] \mathrm{d}x$ scores M1	
	If $16x$ is integrated on its own award dM1 but no more marks are available.	
	If $\int \left[-4x^2 + 80x - 256\right] \mathrm{d}x$ is attempted award dM0	