Model Solutions

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Tuesday 21 May 2019

Morning (Time: 2 hours) | Paper Reference **4MA1/1HR**

Mathematics A
Level 1/2
Paper 1HR
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

P60260A

1/1/1/

P 6 0 2 6 0 A 0 1 2 4

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

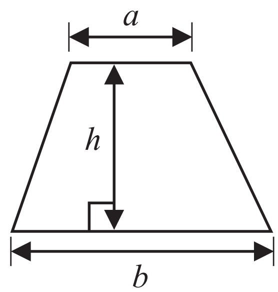
Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

The quadratic equation

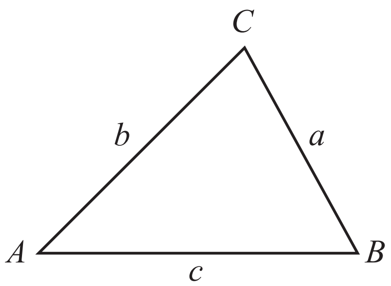
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle *ABC*

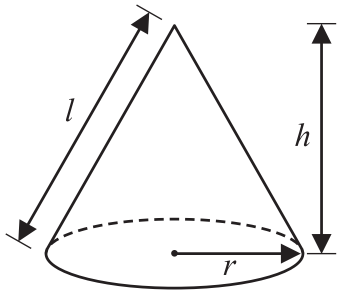
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

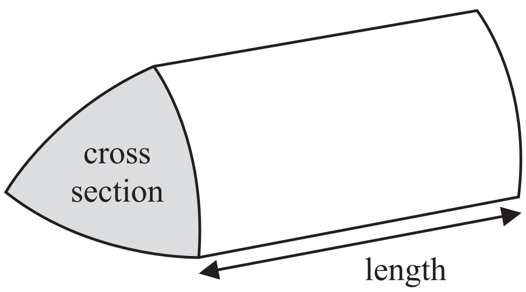
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

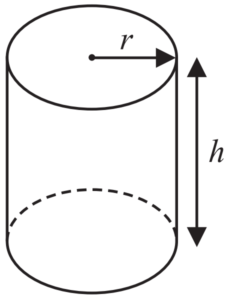
Curved surface area of cone $= \pi rl$



Volume of prism
= area of cross section × length

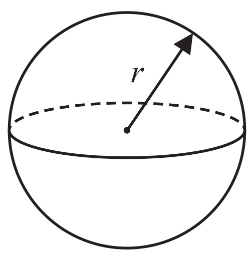


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi rh$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Answer ALL TWENTY FIVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The diagram shows a cylinder.

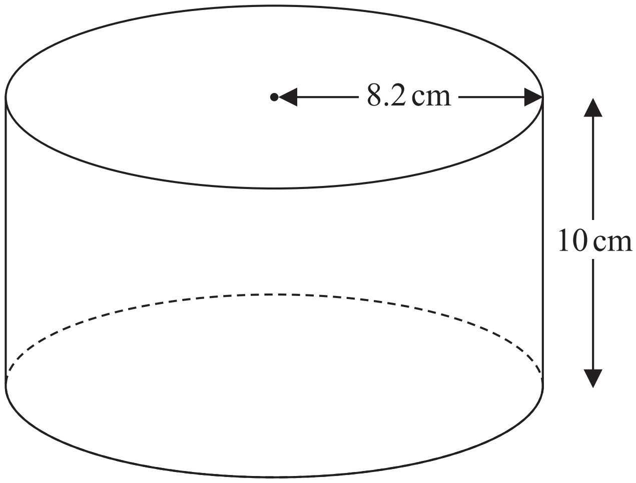


Diagram **NOT** accurately drawn

The cylinder has radius 8.2 cm and height 10 cm.
The cylinder is empty.

Pam pours 1.5 litres of water into the cylinder.

$1\,ml = 1\,cm^3$

Work out the depth of the water in the cylinder.
Give your answer correct to 1 decimal place.

$1.5\,l \xrightarrow{\times 1000} 1500\,ml = 1500\,cm^3$

Area of circle base : $\pi \times 8.2^2 = 211.24\ldots cm^2$

Depth : $\underbrace{1500\,cm^3}_{\text{Volume}} \div \underbrace{211.24\ldots\,cm^2}_{\text{Base Area}}$

$= 7.1009\ldots\,cm$

round down

7.1 cm

(Total for Question 1 is 3 marks)

2 Each interior angle of a regular polygon is 162°

Work out the number of sides the polygon has.

Interior + Exterior = 180

$\text{Exterior} = \frac{360}{n}$ no. of sides

162 + Exterior = 180 (Angles in a straight line add up to 180)

−162

Exterior Angle = 18°

$18 = \frac{360}{n}$

×n

$18n = 360$

÷18

$n = 20$

20 sides

(Total for Question 2 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

3 $\mathscr{E} = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
$A = \{\text{even numbers}\}$
$B = \{\text{multiples of } 3\}$

List the members of the set

(i) $A \cap B$ ← in List A and B
↳ even multiple of 3

12, 18

(ii) $A \cup B$ ← in A or B
↳ even or multiples of 3

12, 14, 15, 16, 18, 20

(iii) A' - not in A
↳ odd numbers

11, 13, 15, 17, 19

(Total for Question 3 is 3 marks)

---

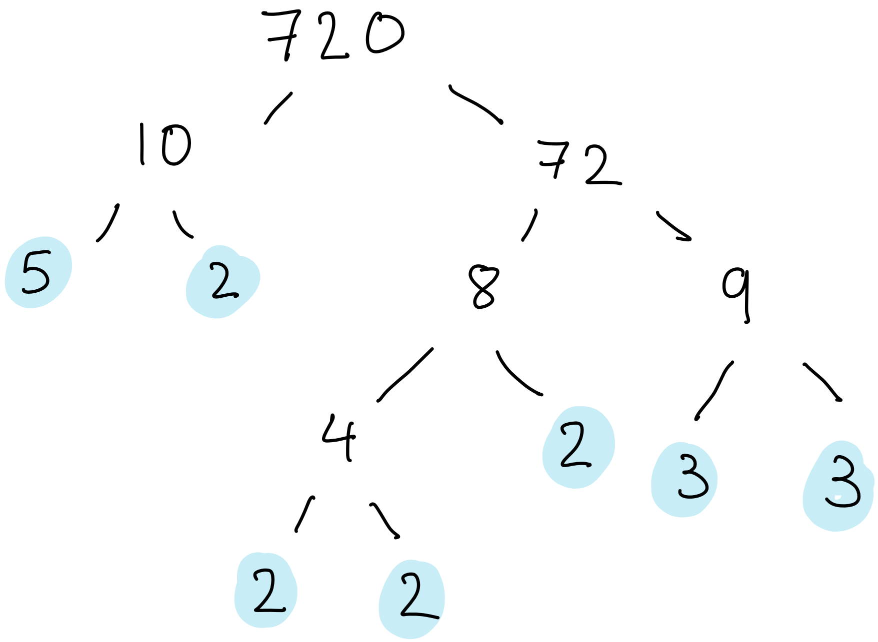
4 Solve $4x - 13 = 17 + 8x$

$4x - 13 = 17 + 8x$
$-4x$
$-13 = 17 + 4x$
-17
$-30 = 4x$
$\div 4$
$-7.5 = x$

$x =$ -7.5

(Total for Question 4 is 2 marks)

5 (a) Write 720 as a product of its prime factors.
Show your working clearly.



$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$2^4 \times 3^2 \times 5$

(3)

(b) Find the smallest whole number that 720 can be multiplied by to give a square number.

To be square, all prime factor products must be square.

$2^4 = 16$ - square

$3^2 = 9$ - square

$5 = 5$ - not square

↳ 5×5 = square

To make 720 a square, x by 5

5

(1)

(Total for Question 5 is 4 marks)

DO NOT WRITE IN THIS AREA

6 Lorenzo increases all the prices on his restaurant menu by 8%

Before the increase, the price of a dessert was \$4.25

(a) Work out the price of the dessert after the increase.

Multiplier: $100 + 8 = 108\% \overset{\div 100}{=} 1.08$

$\underbrace{4.25}_{\text{before}} \times \underbrace{1.08}_{\text{multiplier}} = 4.59$

\$ 4.59

(3)

After the increase, the price of lasagne is \$9.45

(b) Work out the price of lasagne before the increase.

$x =$ price before

multiplier $1.08x = 9.45$

$\div 1.08$

$x = 8.75$

\$ 8.75

(3)

(Total for Question 6 is 6 marks)

7 The diagram shows isosceles triangle *ABC*.

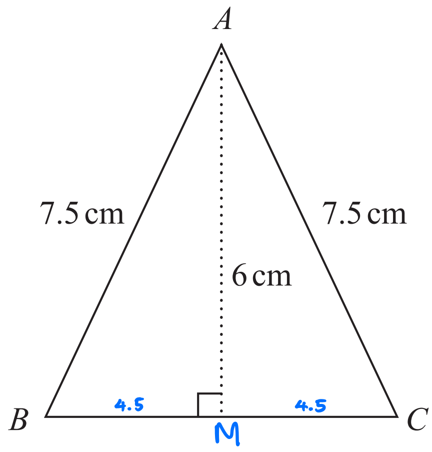


Diagram **NOT** accurately drawn

$AB = AC = 7.5$ cm.

The height of the triangle is 6 cm.

Pythagoras theorem : $a^2 + b^2 = c^2$

Calculate the area of the triangle.

$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

Base: $BM^2 + AM^2 = AB^2$

$- AM^2$

$BM^2 = AB^2 - AM^2$

$= 7.5^2 - 6^2$

$= 20.25$

$BM = 4.5$

$BC = 4.5 \times 2 = 9$ cm

$\text{Area} = \frac{1}{2} \times 9 \times 6 = 27$

27 cm^2

(Total for Question 7 is 4 marks)

DO NOT WRITE IN THIS AREA

8 There are 10 people in a lift.
These 10 people have a mean weight of 79.2 kg. ①

Mean = $\frac{\text{Total Sum}}{\text{frequency}}$

3 of these people get out of the lift.
These 3 people have a mean weight of 68 kg. ②

Work out the mean weight of the 7 people left in the lift.

① $79.2 = \frac{\text{Total}}{10}$ ×10

Total weight of 10 people = 792 kg

② $68 = \frac{\text{Total}}{3}$ ×3

Total weight of 3 people = 204 kg

Total weight of 7 people: 792 − 204 = 588

mean: $\frac{588}{7}$ =

84 kg

(Total for Question 8 is 3 marks)

9 (a) Simplify $t^9 \div t^3$

t^{9-3} $\quad a^b \div a^c = a^{b-c}$

t^6

(1)

(b) Simplify $w^5 \times w^7$

w^{5+7} $\quad a^b \times a^c = a^{b+c}$

w^{12}

(1)

(c) Simplify $(5xy^2)^3$

$= 5^3 \times x^3 \times y^{2\times 3}$

$(a^b)^c = a^{b\times c}$

$125x^3y^6$

(2)

(Total for Question 9 is 4 marks)

10 Change 22 metres per second to a speed in kilometres per hour.
Show your working clearly.

$$\frac{22\,m}{1\,sec} \times \frac{1km}{1000m} \times \frac{1\,sec}{\frac{1}{3600}h}$$

ie.

22m → 1Sec

79200 m → 3600 Sec ×3600

1hr = 60 × 60
min sec

$$1\,sec \overset{\div 60}{=} \frac{1}{60}\,min \overset{\div 60}{=} \frac{1}{3600}\,h$$

$$79200\,m = \frac{79200}{1000}\,km$$

$$= 79.2\,km$$

$$\frac{22\cancel{m}}{1\,\cancel{sec}} \times \frac{1km}{1000\cancel{m}} \times \frac{3600\,\cancel{sec}}{1\,h}$$

$$= \frac{79200\,km}{1000\quad h}$$

79.2 km/h

(Total for Question 10 is 3 marks)

---

11 3 years ago, the ratio of Tom's age to Clemmie's age was 2 : 7
Tom is now 15 years old and Clemmie is now x years old.

Find the value of x.

3 years ago: Tom was 15 − 3 = 12

Ratio: T : C
2 : 7
×6 12 : 42 ×6

3 years ago Clemmie was 42.

Now: 42 + 3

x = 45

(Total for Question 11 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

12

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

A box, in the shape of a cuboid, is going to be put on a table.

The whole of one face of the box will be in contact with the table.
The force exerted by the box on the table is always 105 newtons.

The box is 5 m by 4 m by 3 m.

The greatest pressure exerted by the box on the table is P newtons/m^2
The least pressure exerted by the box on the table is Q newtons/m^2

Work out the value of $P - Q$

Areas:
$5 \times 4 = 20\text{m}^2$
$5 \times 3 = 15\text{m}^2$
$4 \times 3 = 12\text{m}^2$

Greatest Pressure: $\dfrac{105}{\text{smallest area}} = \dfrac{105}{12}$

$= 8.75 = P$

Least Pressure: $\dfrac{105}{\text{largest area}} = \dfrac{105}{20}$

$= 5.25 = Q$

$P - Q = 8.75 - 5.25$

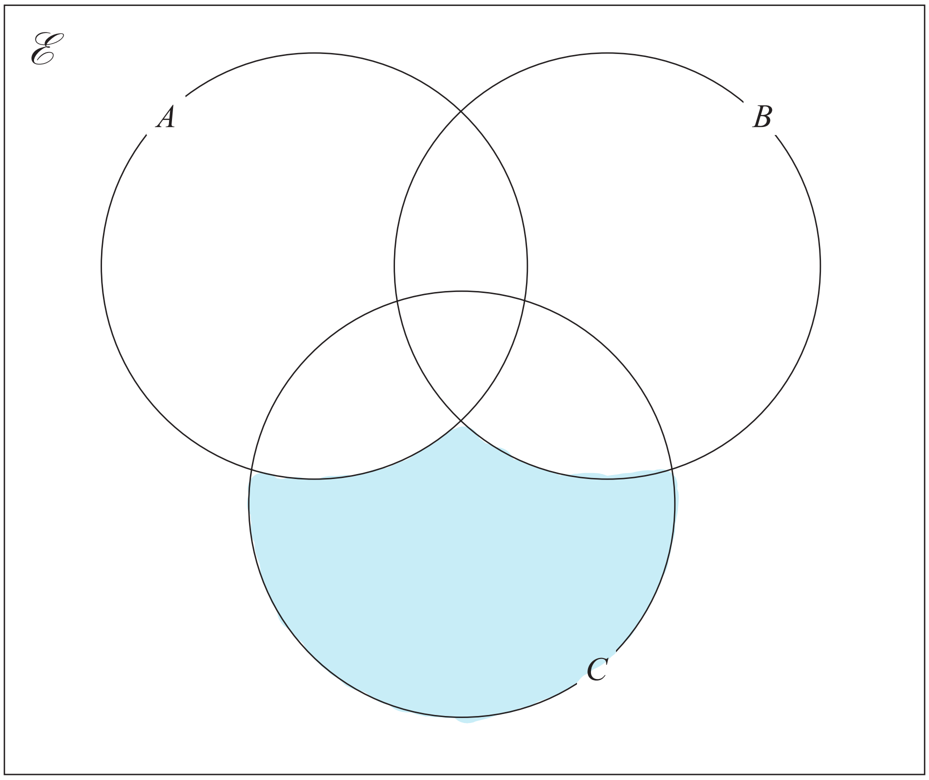
$= 3.5$

3.5

(Total for Question 12 is 3 marks)

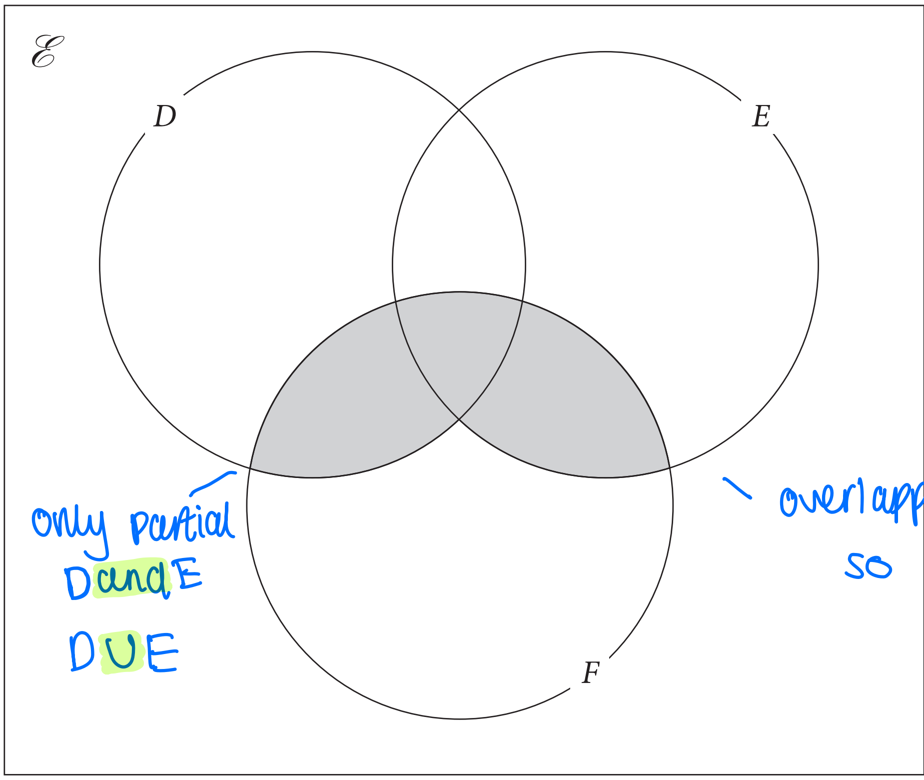
Not (A or B) and C

13 (a) On the Venn diagram, shade the set $(A \cup B)' \cap C$



(1)

(b) Use set notation to describe the shaded region in the Venn diagram below.



$(D \cup E) \cap F$

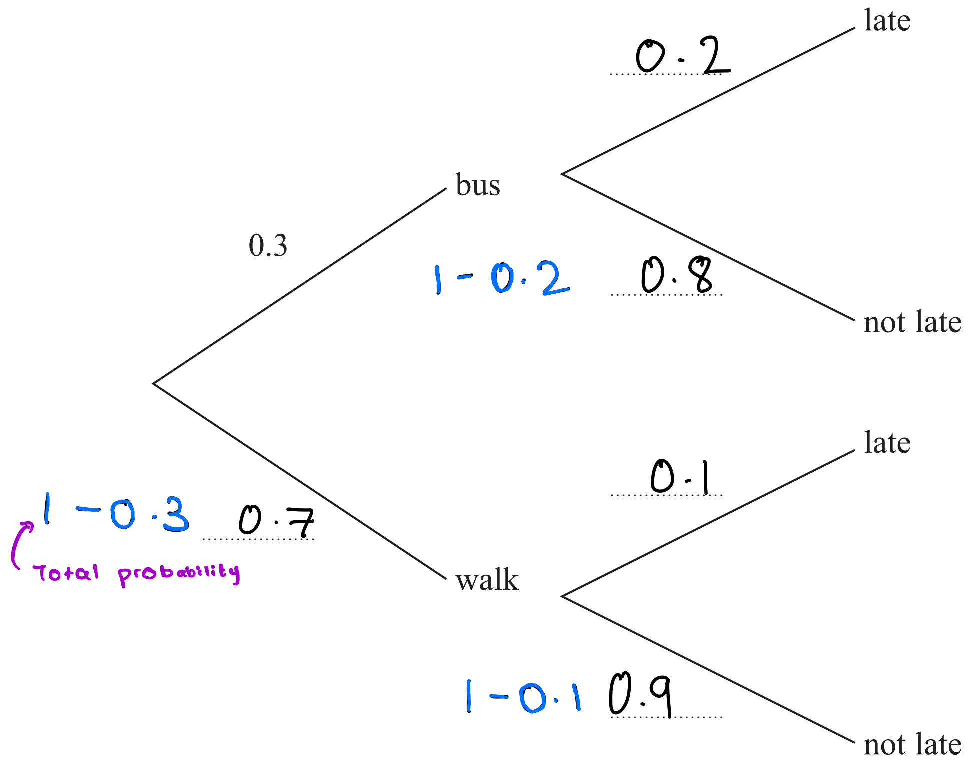
(1)

(Total for Question 13 is 2 marks)

14 Each day that Barney goes to college, he either goes by bus or he walks.
The probability that Barney will go to college by bus on any day is 0.3

When Barney goes to college by bus, the probability that he will be late is 0.2
When Barney walks to college, the probability that he will be late is 0.1

(a) Complete the probability tree diagram.



(2)

Barney will go to college on 200 days next year.

(b) Work out an estimate for the number of days Barney will be late for college next year.

$P(\text{Late}) = P(\text{Bus} \cap \text{Late}) + P(\text{Walk} \cap \text{Late})$

'and'

$= 0.3 \times 0.2 + 0.7 \times 0.1$

$= 0.06 + 0.07 = 0.13$

Number of days: $0.13 \times 200 =$

26

(4)

(Total for Question 14 is 6 marks)

15 The straight line $\mathbf{L}_1$ has equation $2y = 6x - 5$

The straight line $\mathbf{L}_2$ is perpendicular to $\mathbf{L}_1$ and passes through the point $(9, -1)$

Find an equation for $\mathbf{L}_2$

Give your answer in the form $ay + bx = c$

L_1 gradient : $2y = 6x - 5$

$y = 3x - \frac{5}{2}$ $\quad m_1 = 3$

$m_1 m_2 = -1$

$\hookrightarrow m_2 = -1 \div m_1$

$m_2 = \frac{-1}{3}$

Lines are perpendicular when the gradients are negative reciprocals.

L_2 : $y = mx + c$

substitute values

$-1 = \frac{-1}{3}(9) + c$

$-1 = -3 + c$

$+3$

$2 = c$

$y = \frac{-1}{3}x + 2$

$\times 3$

$3y = -x + 6$

$+x$

$x + 3y = 6$

$3y + x = 6$

(Total for Question 15 is 4 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 6 0 2 6 0 A 0 1 4 2 4

DO NOT WRITE IN THIS AREA

16 A particle P is moving along a straight line.
The fixed point O lies on this line.

At time t seconds, the displacement, s metres, of P from O is given by

$$s = 4t^3 - 6t^2 + 5t$$

At time t seconds, the velocity of P is v m/s.

(a) Find an expression for v in terms of t.

$v = \frac{ds}{dt} = 3 \times 4t^2 - 2 \times 6t + 5$

$v =$ $12t^2 - 12t + 5$
(2)

(b) Find the time at which the acceleration of the particle is 6 m/s^2

$acc = \frac{dv}{dt} = 2 \times 12t - 12$

$= 24t - 12$

$24t - 12 = 6$

$+12$

$24t = 18$

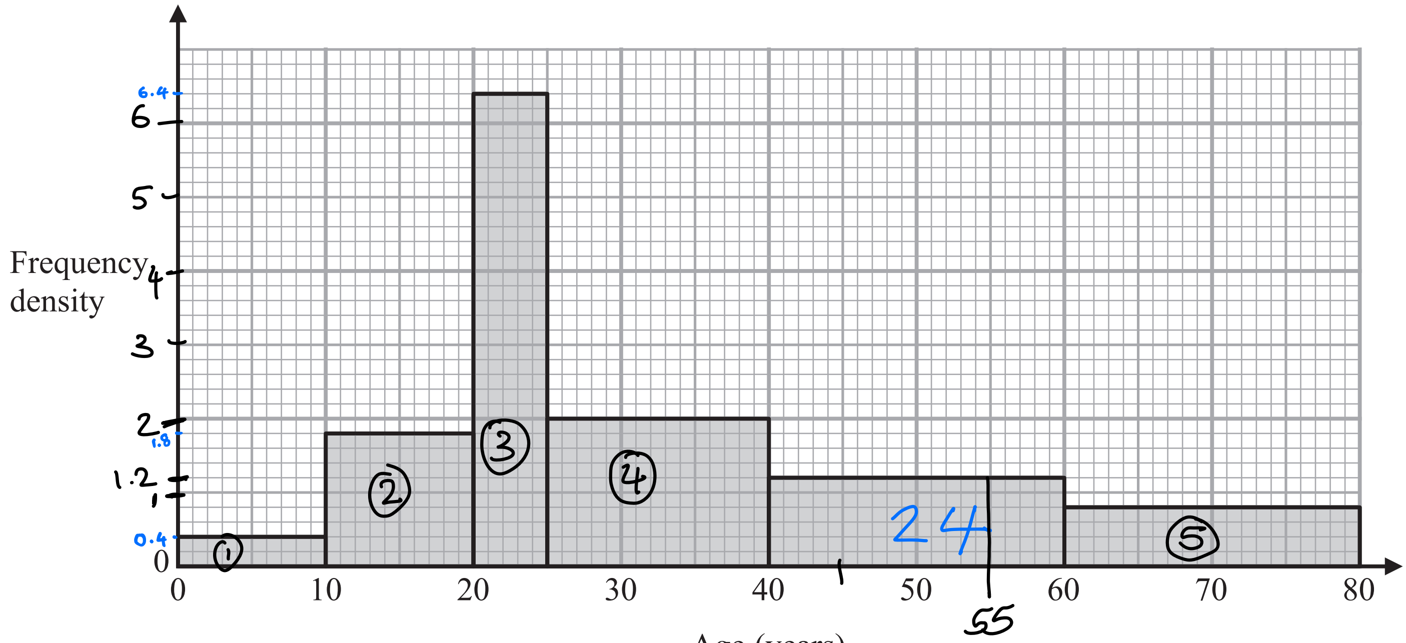
$\div 24$

$t = 3/4$

$\frac{3}{4}$ seconds
(3)

(Total for Question 16 is 5 marks)

17 The histogram shows information about the ages of all the passengers travelling on a plane.
No one on the plane is older than 80 years.



24 passengers on the plane are aged between 40 years and 60 years.

(a) Work out the total number of passengers on the plane.

Frequency = Freq density × class width

$24 = FD \times 20$

$\div 20$

$FD = 1.2$

Now draw the scale accordingly

① $F = 0.4 \times 10 = 4$

② $1.8 \times 10 = 18$

③ $6.4 \times 5 = 32$

④ $2 \times 15 = 30$

⑤ $0.8 \times 20 = 16$

Total:

$4 + 18 + 32 + 30 + 24 + 16$

$= 124$

(3)

A passenger on the plane is picked at random.

(b) Work out an estimate for the probability that this person is older than 55 years.

$> 55 \quad = 16 + \frac{1}{4} \times 24$

$= 16 + 6 = 22$

$P = \frac{22}{124}$ ← Total passengers

$= \frac{11}{62}$

(2)

(Total for Question 17 is 5 marks)

DO NOT WRITE IN THIS AREA

18 (a) Expand and simplify $(x + 2)(2x + 3)(x - 7)$
Show your working clearly.

$$2x^2 + 3x + 4x + 6 \quad (x - 7)$$

$$2x^2 + 7x + 6\,(x - 7)$$

$$2x^3 - 14x^2 + 7x^2 - 49x + 6x - 42$$

$$2x^3 - 7x^2 - 43x - 42$$

(3)

(b) Make m the subject of $p^2 = \dfrac{x + m}{2m - y}$

isolate m

$\times(2m - y)$

$$2mp^2 - p^2y = x + m$$

$+p^2y$
$-m$

move all m terms to one side

$$2mp^2 - m = x + p^2y$$

– all non-m terms

factorise out m

$$m(2p^2 - 1) = x + p^2y$$

$\div(2p^2 - 1)$

$$m = \frac{x + p^2y}{2p^2 - 1}$$

(3)

(Total for Question 18 is 6 marks)

19 The 25th term of an arithmetic series is 44.5 ①
The sum of the first 30 terms of this arithmetic series is 765 ②

Find the 16th term of the arithmetic series.
Show your working clearly.

$S_n = \frac{n}{2}(2a + (n-1)d)$ $a_n = a + (n-1)d$

① $a + 24d = 44.5$

② $\frac{30}{2}(2a + (30-1) \times d) = 765$

$15(2a + 29d) = 765$ $\div 15$

② $2a + 29d = 51$

① $\times 2$ $2a + 48d = 89$ −

$-19d = -38$ $\div -19$

$d = 2$

16^{th} term $= 25^{th} - 9d$

$= 44.5 - 9(2)$ 18

$= 26.5$

26.5

(Total for Question 19 is 5 marks)

DO NOT WRITE IN THIS AREA

20 $a = 25 \times 10^{14n}$ where n is an integer.

Find an expression, in terms of n, for $a^{\frac{3}{2}}$

Give your answer in standard form.

$(a^b)^c = a^{bc}$

$$a^{\frac{3}{2}} = (25 \times 10^{14n})^{\frac{3}{2}}$$

$$= (\sqrt{25})^3 \times 10^{14n \times 3/2}$$

5^3

$$= 125 \times 10^{21n}$$

$14 \times \frac{3}{2}\ n$

$$= 1.25 \times 10^2 \times 10^{21n}$$

standard form

add powers

$1.25 \times 10^{21n+2}$

(Total for Question 20 is 3 marks)

21 A curve has equation $y = f(x)$

There is only one maximum point on the curve.
The coordinates of this maximum point are (4, 3)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 5)$

5 to the right

$(x, y) \Rightarrow ((x+5), y)$

(.......9......., ...3.......)

(ii) $y = 3f(x)$

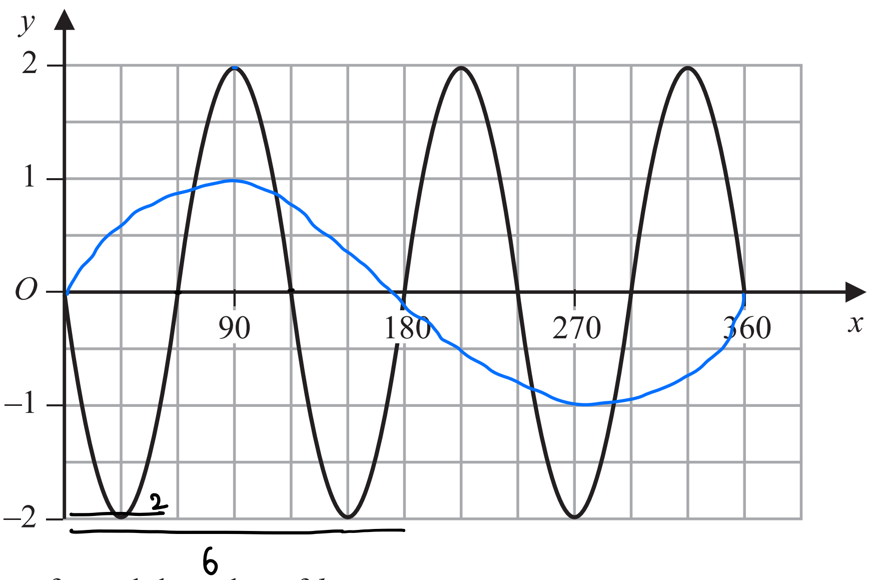
stretch y by 3

$(x, y) \Rightarrow (x, 3y)$

(.......4......., ...9.......)

(2)

Here is the graph of $y = a\sin(bx)°$ for $0 \leqslant x \leqslant 360$



(b) Find the value of a and the value of b.

$f(x) = \sin x$

Looking at difference:

y values by stretched by a factor 2 and have been reflected.

x values have squished by 3

$a =$ ....-2........

$b =$ ....3........

(2)

(Total for Question 21 is 4 marks)

P 6 0 2 6 0 A 0 2 0 2 4

22 Solve the simultaneous equations

$$2x^2 + 3y^2 = 5 \quad (1)$$

$$y = 2x + 1 \quad (2)$$

Show clear algebraic working.

Substitute (2) into (1)

$$2x^2 + 3(2x+1)(2x+1) = 5$$

$$2x^2 + 3(4x^2 + 2x + 2x + 1) = 5$$

$$2x^2 + 12x^2 + 12x + 3 = 5$$

-5

$$14x^2 + 12x - 2 = 0$$

$\div 2$

$$7x^2 + 6x - 1 = 0$$

By inspection:

$$(7x - 1)(x + 1) = 0$$

$7x - 1 = 0$
$7x = 1$
$x = \frac{1}{7}$

$x = \frac{1}{7}$ $\quad$ $x = -1$

$x + 1 = 0$
$x = -1$

Subs x into $y = 2x + 1$ to find y

$y = 2\left(\frac{1}{7}\right) + 1$ $\quad$ $y = 2(-1) + 1$

$= \frac{9}{7}$ $\quad$ $y = -1$

$x = -1, y = -1$ and $x = \frac{1}{7}, y = \frac{9}{7}$

(Total for Question 22 is 5 marks)

23

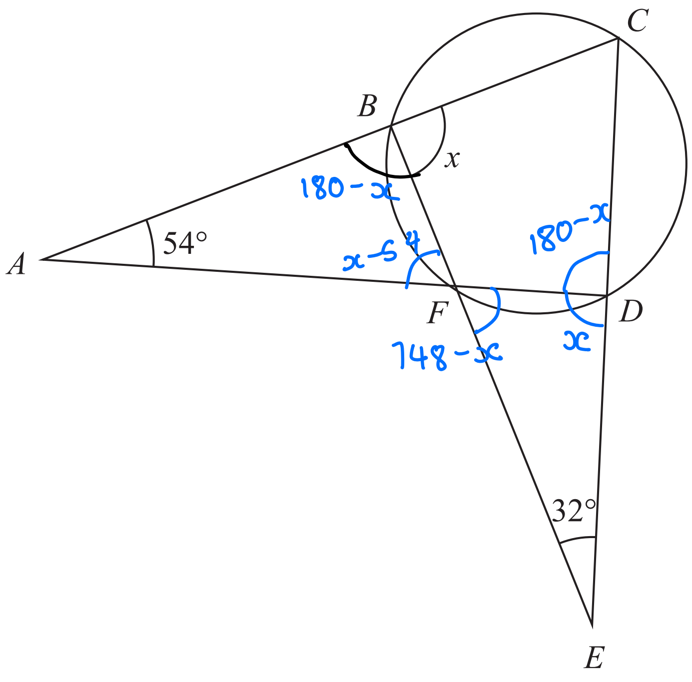


Diagram **NOT** accurately drawn

B, *C*, *D* and *F* are points on a circle.
ABC, *AFD*, *BFE* and *CDE* are straight lines.

Work out the size of angle *x*.
Show your working clearly.

$\angle ABF = 180 - x$ Angles on a straight line add to 180.

$\angle CDF = 180 - x$ Opposite angles in a cyclic quadrilateral add to 180

$\angle EDF = 180 - (180 - x)$
$= x$ Angles on a straight line add to 180.

$\angle DFE = 180 - x - 32$
$= 148 - x$ Angles in a triangle add to 180°.

$\angle AFB = 180 - 54 - (180 - x)$
$= x - 54$ Angles in triangle = 180°

$x - 54 = 148 - x$ Vertically opposite angles are equal
$+x$
$+54$
$2x = 202$
$\div 2$
$x = 101$

$x =$ 101 °

(Total for Question 23 is 4 marks)

24

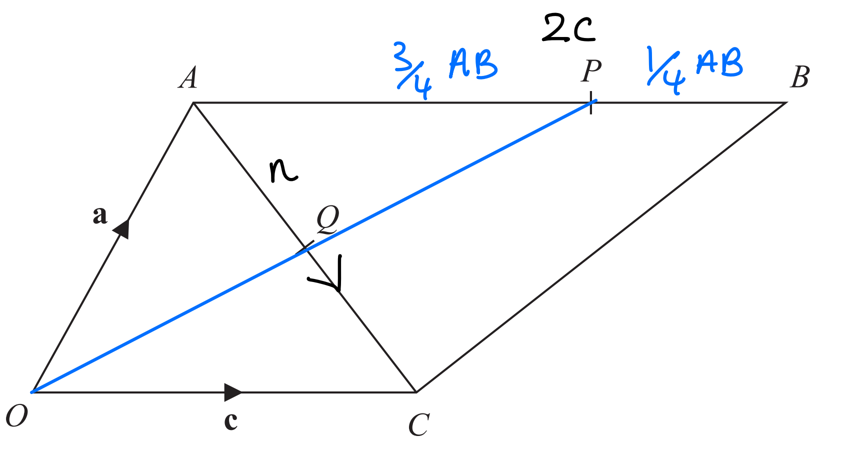


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{a}$ $\quad \overrightarrow{OC} = \mathbf{c}$ $\quad \overrightarrow{AB} = 2\mathbf{c}$

P is the point on AB such that $AP : PB = 3 : 1$
Q is the point on AC such that OQP is a straight line.

$OP = \overrightarrow{OQ} + \overrightarrow{QP}$

Use a vector method to find $AQ : QC$
Show your working clearly.

$AC = -a + c$

$AP = \frac{3}{4}(\overset{\overrightarrow{AB}}{2c}) = \frac{3}{2}c$

$OQ = \overset{\overrightarrow{OA}}{a} + n(\overset{\overrightarrow{AC}}{c - a}) = (1-n)a + nc$

$OP = a + \frac{3}{2}c$

OQP is a straight line so OQP is multiple of OQ

$(1-n)ka + nkc = a + \frac{3}{2}c$

Equate a and c:

(a) $k - \cancel{nk} = 1$
(c) $\cancel{n}k = 3/2$ $\quad +$

$k = 5/2$

$n\left(\frac{5}{2}\right) = \frac{3}{2}$ $\quad \leftarrow nk = 3/2$
$\div 5/2$
$n = 3/5$

$\therefore AQ$ is $\frac{3}{5}$ of AC

$QC = \frac{2}{5}$

$AQ : QC =$ 3 : 2

(Total for Question 24 is 5 marks)

25 A boat sails from point X to point Y and then to point Z.

$a^2 = b^2 - 2bc\cos A$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

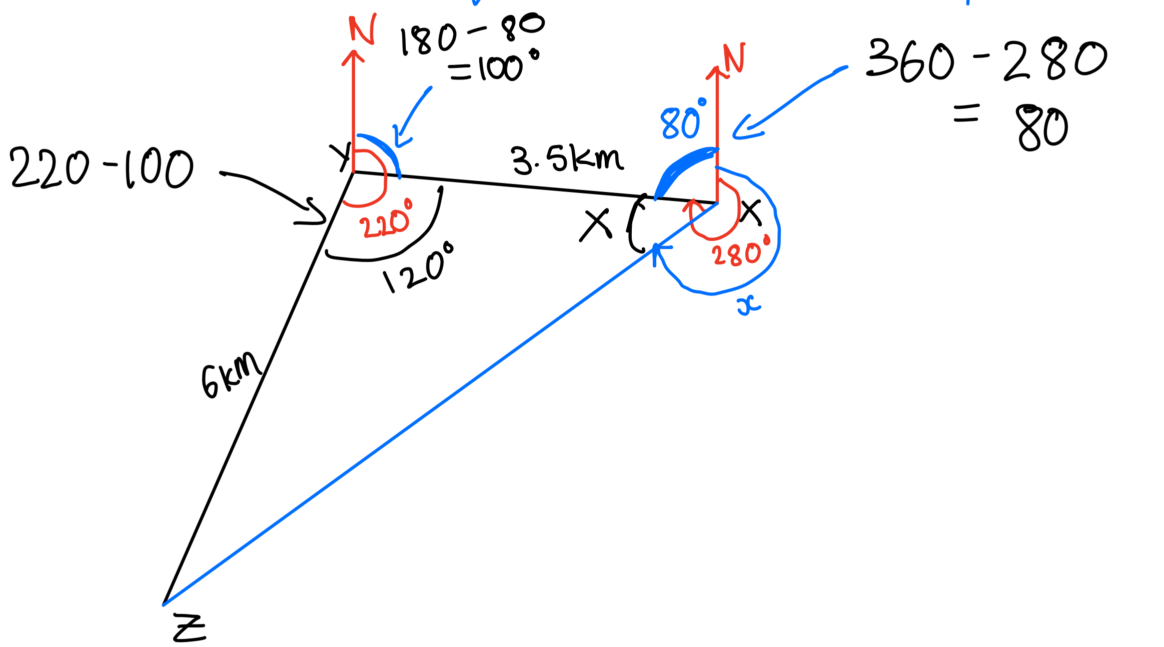
Y is on a bearing of 280° from X.
Z is on a bearing of 220° from Y.

The distance from X to Y is 3.5 km.
The distance from Y to Z is 6 km.

Work out the bearing of Z from X.
Give your answer correct to 1 decimal place.

Corresponding angles.

Angles around a point add up to 360°



$$XZ^2 = 6^2 + 3.5^2 - 2 \times 6 \times 3.5\cos 120$$

$$= 48.25 - (-21)$$

$$XZ^2 = \frac{277}{4}$$

$$\sqrt{}$$

$$XZ = \frac{\sqrt{277}}{2}$$

$$\frac{\sin X}{6} = \frac{\sin 120}{\frac{\sqrt{277}}{2}} \quad \times 6$$

$$\sin X = \frac{12\sin 120}{\sqrt{277}}$$

$$\sin^{-1}$$

$$X = 38.639...°$$

Bearing from X to Z:

$$280 - 38.639...°$$

$$= 241.36...°$$

round up

241.4 °

(Total for Question 25 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS