NEW SPECIMEN PAPERS
PUBLISHED JUNE 2015

AQA

GCSE
Mathematics
Specification (8300/2H)

Paper 2 Higher tier

H

Date	Morning	1 hour 30 minutes

Materials

For this paper you must have:
• a calculator
• mathematical instruments.

Model Solutions

Instructions

• Use black ink or black ball-point pen. Draw diagrams in pencil.
• Fill in the boxes at the bottom of this page.
• Answer **all** questions.
• You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
• Do all rough work in this book.
• In all calculations, show clearly how you work out your answer.

Information

• The marks for questions are shown in brackets.
• The maximum mark for this paper is 80.
• You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Please write clearly, in block capitals, to allow character computer recognition.

Centre number ☐☐☐☐☐   Candidate number ☐☐☐☐

Surname ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

Forename(s) ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

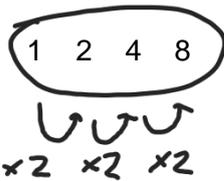Candidate signature _____

8300/2H

Answer **all** questions in the spaces provided.

1 Which sequence is a geometric progression?

Circle your answer.

[1 mark]

1   2   3   4   1   2   4   7

1   2   4   8   1   2   3   5

×2 ×2 ×2

2 Which of these is **not** used to prove that triangles are congruent?

Circle your answer.

[1 mark]

SSS   SAS   AAA   RHS

3 Circle the expression that is equivalent to $2a + 5a \times 4a - a$

[1 mark]

$a + 20a^2$ $21a^2$ $28a^2 - a$ $2a + 15a^2$

$2a + (5a \times 4a) - a$

$2a + 20a^2 - a$

$a + 20a^2$

4 Circle the equation of a line that is parallel to $y = 5x - 2$

[1 mark]

$y = 2x - 5$ $y = 5x + 2$ $y = 3x - 2$ $y = -\dfrac{1}{5}x - 2$

Parallel lines have the same gradients.

5 In a sale, the original price of a bag was reduced by $\dfrac{1}{5}$

The sale price of the bag is £29.40

Work out the original price.

$1/5 = 20.1.$

[3 marks]

Original $\times 0.8 = £29.40$

Original $= \dfrac{£29.40}{0.8} = £36.75$

Answer £   36.75

Turn over for the next question

6   ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

S = square numbers

E = even numbers

6   (a)   Complete the Venn diagram.

[3 marks]

ξ

S   E

1

9   4   12
                                2
                      6

3 5 7   8 10

11

6   (b)   One of the numbers is chosen at random.

Write down P (S ∩ E)

[1 mark]

$\frac{1}{12}$ → only one is square and even
      → total numbers

Answer   1/12

7 A coin is rolled onto a grid of squares.

It lands randomly on the grid.

To win, the coin must land completely within one of the squares.

Meera and John each roll the coin a number of times and record their results.

	Number of wins	Number of losses
Meera	6	44
John	28	72

7 (a) Work out **two** different estimates for the probability of winning.

[2 marks]

$$\text{Meera} \rightarrow \frac{6}{6+44} = \frac{6}{50}$$

$$\text{John} \rightarrow \frac{28}{28+72} = \frac{28}{100}$$

Answer $\frac{6}{50}$ and $\frac{28}{100}$

7 (b) Which of your estimates is the better estimate for the probability of winning?

Give a reason for your answer.

[1 mark]

Answer _Johns $\frac{28}{100}$_

Reason _More trials were involved in calculating the probability_

8   Here is the graph of $4x - 3y = 12$ for values of x from 0 to 4
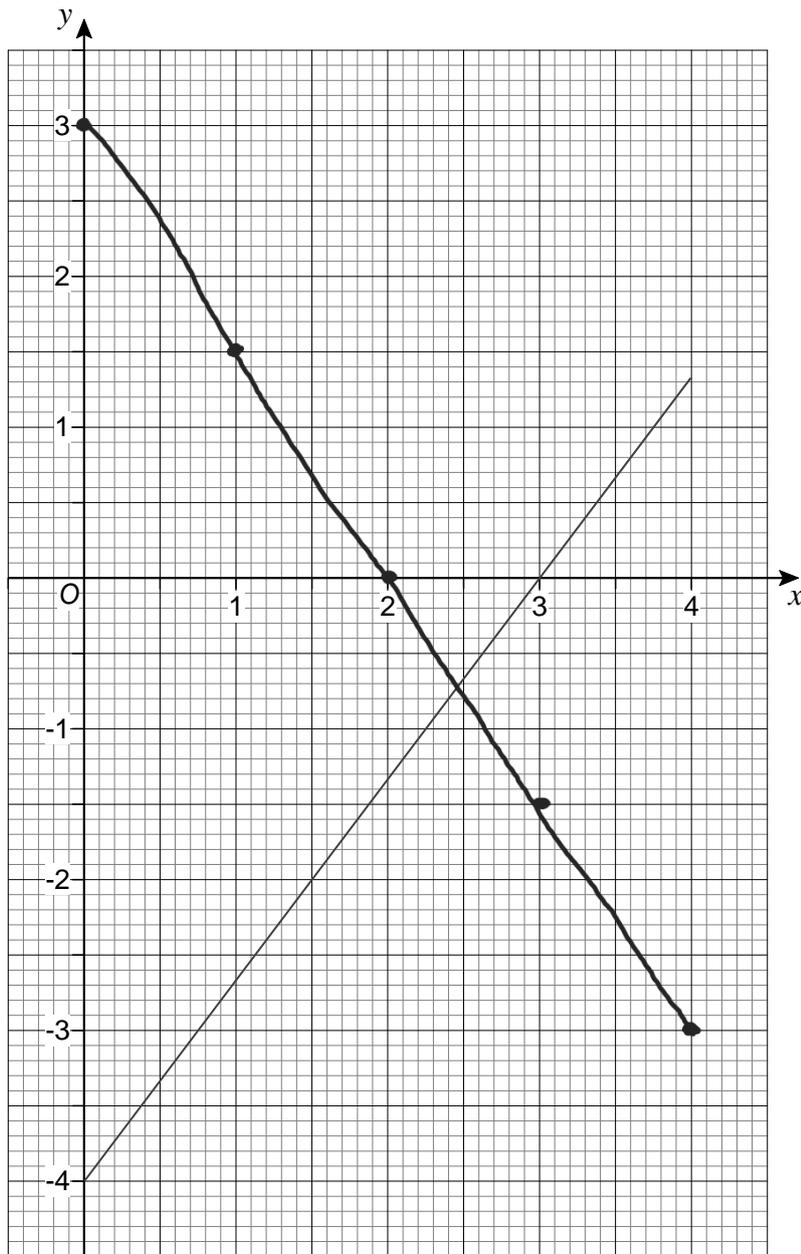
By drawing a second graph on the grid,

work out an approximate solution to the simultaneous equations

$$4x - 3y = 12 \quad \text{and} \quad 3x + 2y = 6$$

Solution is where graphs intrsect

[3 marks]

Answer ___(2.45, -0.75) → x=2.45 y=-0.75___

3x+2y=6

2y=6-3x

y=3-1.5x →

x	0	1	2	3	4
y	3	1.5	0	-1.5	-3

9   Written as the product of its prime factors

$$672 = 2^5 \times 3 \times 7$$

9 (a)   Write 252 as the product of its prime factors.

[2 marks]

252

2   126

2   63

7   9

3   3
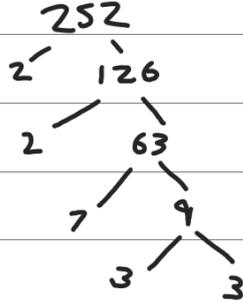
$$252 = 2 \times 2 \times 3 \times 3 \times 7 = 2^2 \times 3^2 \times 7$$

Answer $2^2 \times 3^2 \times 7$

9 (b)   Work out the value of the highest common factor of 672 and 252

[1 mark]

Use factors that are common to both → $2^2 \times 3 \times 7 = 84$

Answer   84

Turn over for the next question

Turn over ▶

10   At a school

number of boys : number of girls = 9 : 7

There are 116 **more** boys than girls.

Work out the total number of students at the school.

[3 marks]

Boys have 2 more parts in the ratio → 9−7=2

That gives them 116 more, so 116÷2 = 58 per part

Total parts 9+7=16

16×58 = 928 total students

Answer _____**928**_____

11   Circle the equation with roots 4 and −8

[1 mark]

$$4x(x - 8) = 0 \qquad (x - 4)(x + 8) = 0$$

$(x-4)=0 \qquad (x+8)=0$
$x=4 \qquad\quad x=-8$

$$x^2 - 32 = 0 \qquad (x + 4)(x - 8) = 0$$

12 $R = \dfrac{x^2}{y}$

$x = 3.6 \times 10^5$

$y = 7.5 \times 10^4$

Work out the value of R.

Give your answer in standard form to an appropriate degree of accuracy.

[3 marks]

$R = \dfrac{x^2}{y} \rightarrow \dfrac{(3 \cdot 6 \times 10^5)^2}{7 \cdot 5 \times 10^4} = 1728000$

$= 1 \cdot 728 \times 10^6$

$= 1 \cdot 7 \times 10^6$

Answer $1 \cdot 7 \times 10^6$

13 Two spheres have radii in the ratio 5 : 3

Circle the ratio of their volumes.

[1 mark]

5 : 3   15 : 9   25 : 9   125 : 27

Radii $\rightarrow$ 5:3

Volume $\rightarrow$ 125 : 27   cube

Turn over for the next question

14 (a) A pattern is made from two **similar** trapeziums.

Not drawn
accurately
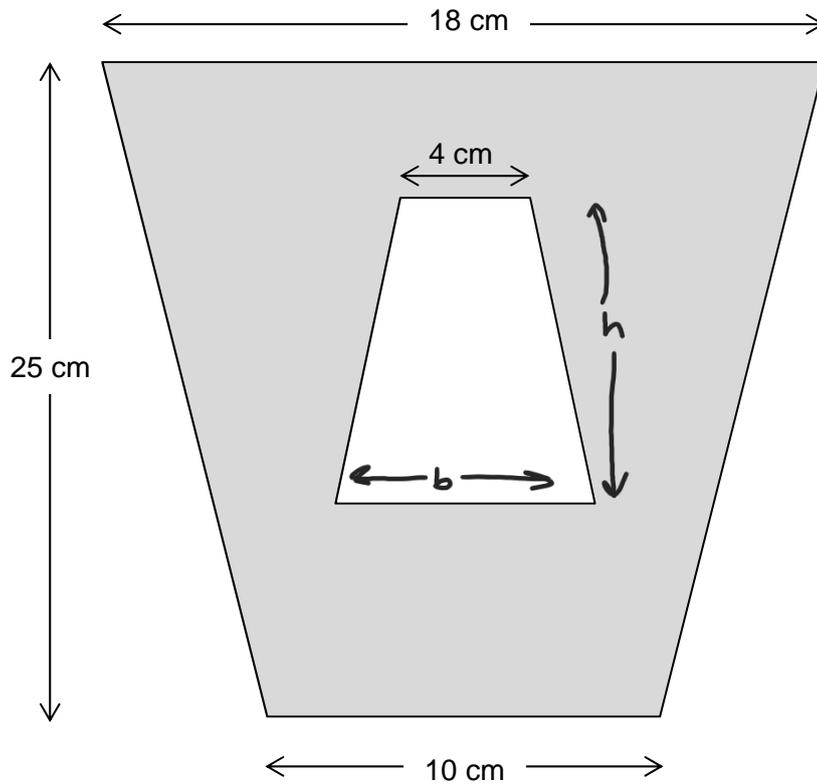
Show that the shaded area is 294 cm^2

[4 marks]

Big trapezium — small trapezium = Shaded area

Big trapezium → $\frac{18+10}{2} \times 25 = 14 \times 25 = \underline{350 \ cm^2}$

$\frac{4}{10} = \frac{b}{18}$ because trapeziums are similar. $\frac{h}{25} = \frac{4}{10}$

$4/10 \times 18 = b = 7.2$ $4/10 \times 25 = h = 10$

Small trapezium → $\frac{4+7.2}{2} \times 10 = \underline{56 cm^2}$

Shaded → $350 - 56 = \underline{\underline{294 \ cm^2}}$

14 (b) The pattern has one line of symmetry.

Not drawn
accurately

←——————— 18 cm ———————→

x

25 cm

←—— 10 cm ——→
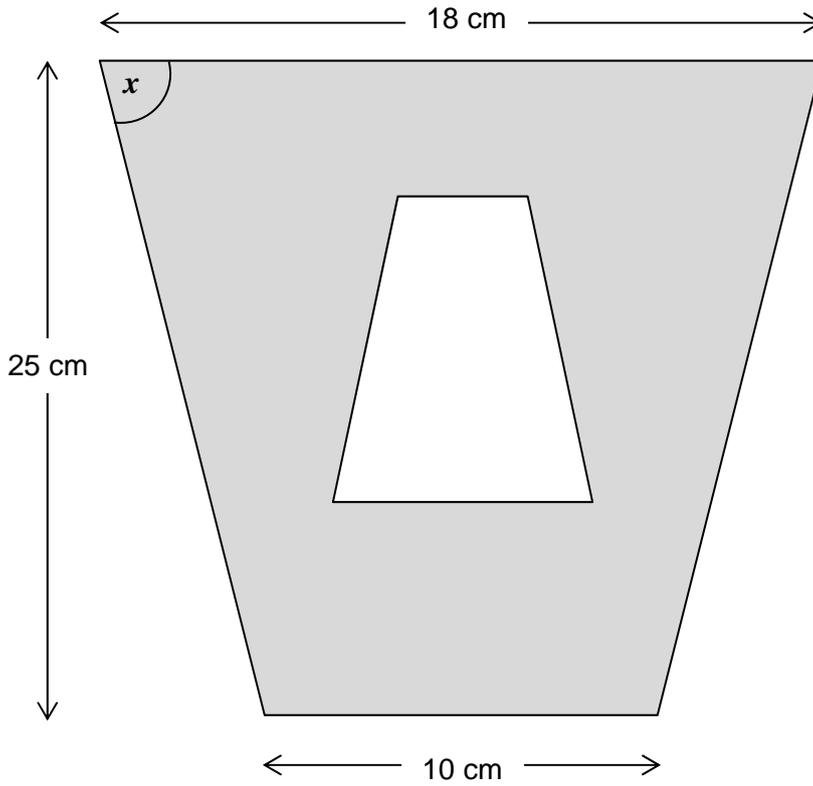
Work out the size of angle *x*.

[3 marks]

$\frac{18-10}{2} = 4$

4

x

25

$\tan x = \frac{25}{4}$

$x = \tan^{-1}\left(\frac{25}{4}\right)$

$= 80.9°$

Answer _____ 80.9° _____ degrees

Turn over ▶

15   Ann picks a 4-digit number.

The first digit is **not** zero.

The 4-digit number is a multiple of 5

How many different 4-digit numbers could she pick?

[3 marks]

___ ___ ___ ___
↓ can be 1-9 (9 possabilitis)   ↓ can be 0-9 (10 possibilitis)   ↓ can be 0 or S (2 possibilitis)

$9 \times 10 \times 10 \times 2 = \underline{1800}$

Answer   1800

16 c is a positive integer.

Prove that $\dfrac{6c^3 + 30c}{3c^2 + 15}$ is an even number.

[3 marks]

$\dfrac{6c^3 + 30c}{3c^2 + 15} \rightarrow \dfrac{6c(c^2 + 5)}{3(c^2 + 5)} \rightarrow \dfrac{6c}{3} = \underline{\underline{2c}}$

$\underline{2c \text{ is a multiple of } 2, \text{ so its even.}}$

17   The distance from the Earth to the Sun is 93 million miles.

Assume

it takes 365 days for the Earth to travel once around the Sun

the Earth travels in a circle with the Sun at the centre.

17 (a)   Work out the average speed of the Earth in miles per hour.

[4 marks]

Speed = $\dfrac{distance}{time}$

distance ⟶ circumference of path

93 000 000 × 2π = 186 000 000 π miles.

Time taken for 1 orbit ⟶ 365 days

365 × 24 = 8760 hours

Speed ⟶ $\dfrac{186\,000\,000}{8760}$ = 66,700 mph

Answer _____66,700_____ miles per hour

17 (b)   It actually takes $365\frac{1}{4}$ days for the Earth to travel once around the Sun.

How does this affect your answer to part (a)?

[1 mark]

This mean the time will increase so speed should be lower.
Makes my answer an overestimate.

18   In the formula $T = (n - 6)^2 + 1$ n is a positive integer.

18 (a)   Kim says,

> "The value of T is always greater than 1
> because $(n - 6)^2$ is always greater than 0"

Comment on her statement.

[1 mark]

If $n = 6 \rightarrow (6-6)^2 = (0)^2 = 0$

So her statement is wrong as $(n-6)^2$ is not <u>always</u> greater than 0.

18 (b)   What is the only value of T that is a square number?

[1 mark]

$\underline{1}$

All other values of T are 1 more than a square number.

Answer   1

19 $f(x) = 3x$

Circle the expression for $f^{-1}(x)$

[1 mark]

$-3x$ $\boxed{\dfrac{3}{x}}$ $\dfrac{1}{3x}$ $\dfrac{x}{3}$

inverse of $3x$

20 y is directly proportional to $\sqrt{x}$

x	36	a
y	2	5

Work out the value of a.

[4 marks]

$y \propto \sqrt{x} \rightarrow y = k\sqrt{x}$

$2 = k\sqrt{36} \rightarrow k = \dfrac{2}{6} = \frac{1}{3}$

$5 = \frac{1}{3}\sqrt{a}$

$15 = \sqrt{a} \rightarrow 15^2 = a = \underline{\underline{225}}$

Answer   __225__

21 A company makes boxes of cereal.

A box usually contains 450 grams of cereal.

Here are two options for a special offer.

Option A	Option B
20% more cereal	Usual amount of cereal
Price remains the same	15% off the price

Which option is the better value for the customer?

You **must** show your working.

[3 marks]

Set price at €10 for normal 450g box.

A → 450g × 1.2 = 540g for €10
    1g for €0.0185

B → €10 × 0.85 = €8.5 for 450g
    €0.0188 for 1g

So option A is better value as it is cheaper per gram

Answer _____ A _____

22 The histogram shows the ages, in years, of members of a chess club.

Frequency density / Age (years)
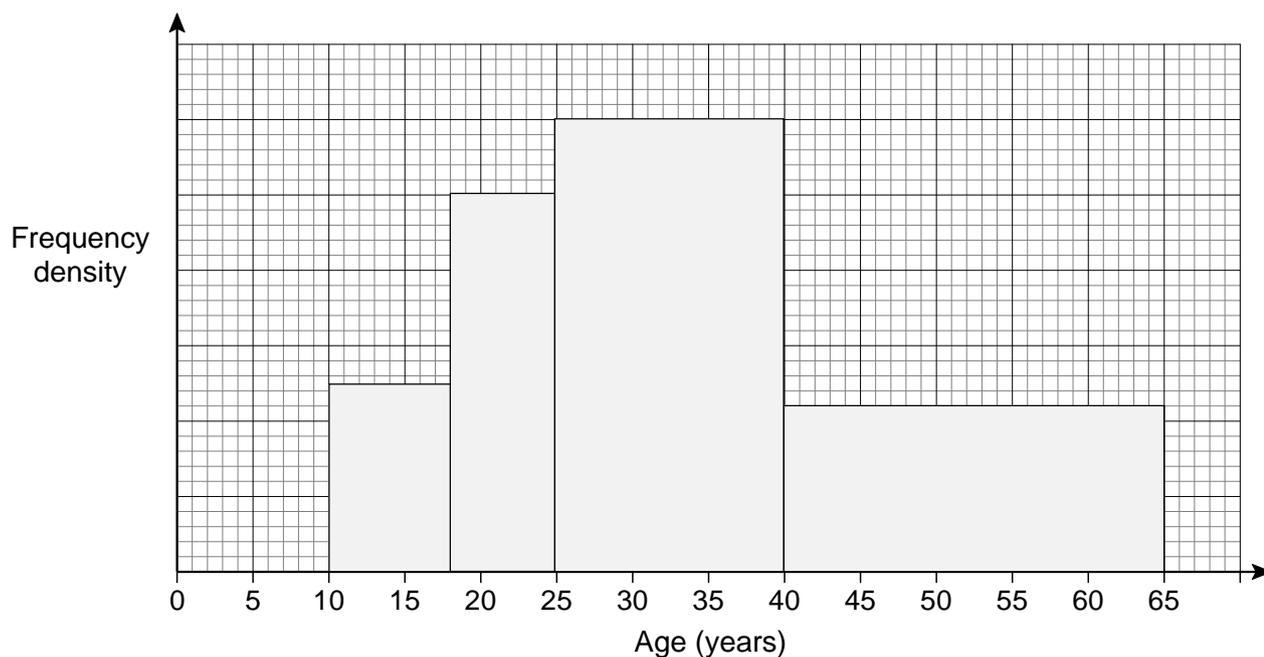
There are 22 members with ages in the range $40 \leqslant \text{age} < 65$

Work out the number of members with ages in the range $25 \leqslant \text{age} < 40$

[4 marks]

$40 \leq \text{age} < 65 \rightarrow \text{width} = 25$

$\text{height} = 11$

$\text{area} = 25 \times 11 = 275$

This 275 represents 22 people $\rightarrow \frac{275}{22} = 12.5$ per person

$25 \leq \text{age} < 40 \rightarrow \frac{15 \times 30}{12.5} = 36$ people

Answer   **36**

23   A bowl is a hemisphere with radius 6 cm

Water fills two-fifths of the volume of the bowl.

←——— 6 cm ———→
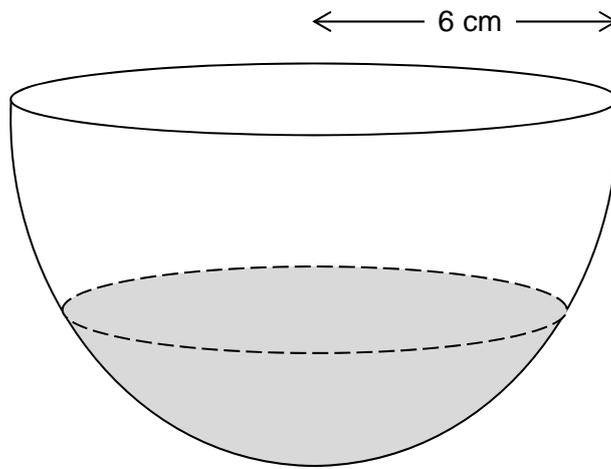
The water is poured into a hollow cone.

The depth of the water in the cone is 12 cm

12 cm
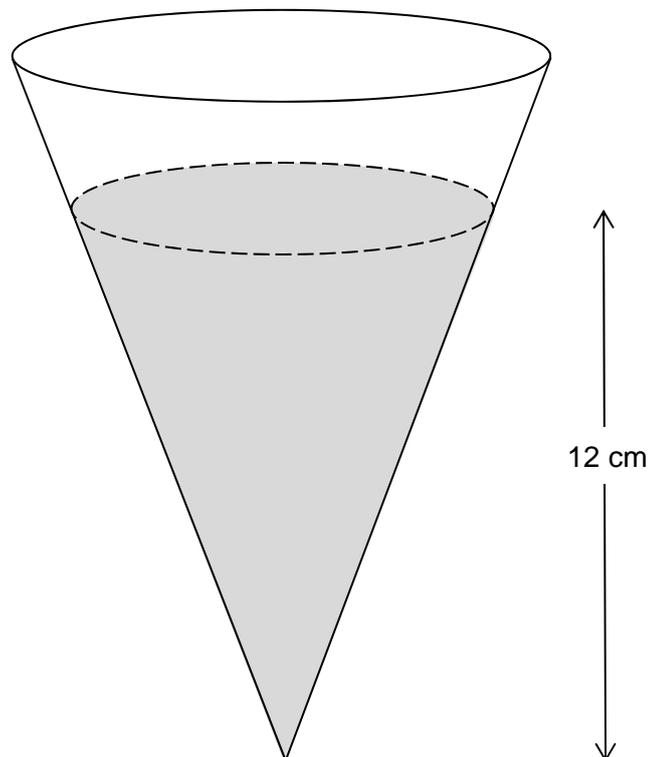
> Volume of a sphere $= \dfrac{4}{3}\pi r^3$ where r is the radius.
>
> Volume of a cone $= \dfrac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height

Work out the radius of the surface of the water in the cone.

[4 marks]

Volume of water in bowl $= \dfrac{2}{5} \times \dfrac{1}{2} \times \dfrac{4}{3} \times \pi \times 6^3 = 57.6\pi$

Same volume of water in cone $57.6\pi = \dfrac{1}{3}\pi \times r^2 \times 12$

$$r^2 = \frac{57.6\pi}{4\pi} = 14.4$$

$$r = \sqrt{14.4} = 3.79\,cm$$

Answer _____ 3.79 _____ cm

Turn over ▶

24 A Big Wheel is modelled as a circle with centre *O* and radius 15 metres.

The wheel turns in an anticlockwise direction.

The lowest point on the wheel is always 2 metres above horizontal ground.

Not drawn
accurately

O

x° 15 m

A ⊐ - - - - - - ✱*C*

h m

2 m

B
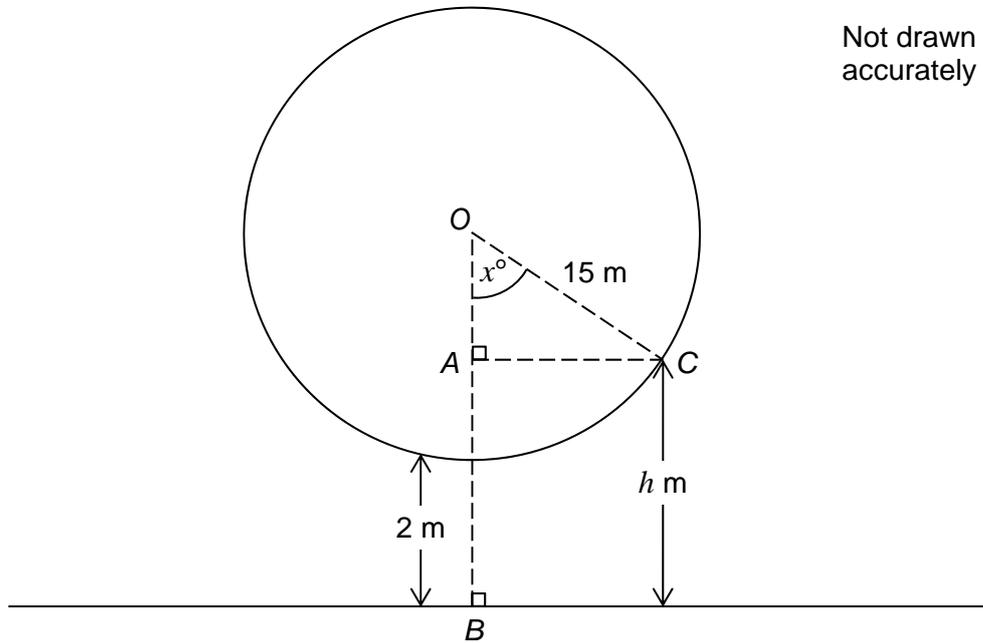
24 (a) *C* is a point on the wheel, *h* metres above horizontal ground.

Angle *COB* = *x*°

Show that $h = 17 - 15 \cos x°$

[2 marks]

Vertical height OA → $\cos x = \frac{OA}{15}$ → OA = 15 cos x

Height OB = 15 + 2 = 17

h = OB − OA = 17 − 15 cos x

24 (b) *D* is a point on the wheel.

Angle *DOB* = 120°

15 m

30⁰

O

120°

D

Not drawn
accurately

B
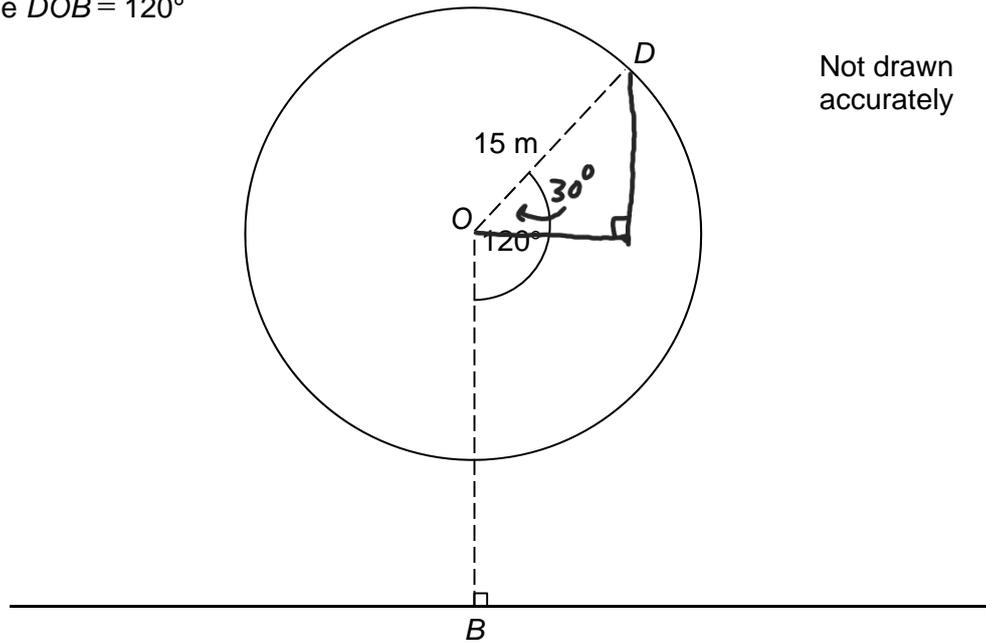
Work out the height of *D* above horizontal ground.

[2 marks]

$\sin 30 = \dfrac{y}{15} \rightarrow y = 15 \sin 30 = 7.5m$

height above ground $= 17 + 7.5 = 24.5 \ m$

Answer _____ **24.5** metres

24 (c) Here is a sketch of the graph $h = 17 - 15 \cos x°$ for one **complete** turn of the wheel.

P is the highest point on the graph.

h

P

2

0

0

x
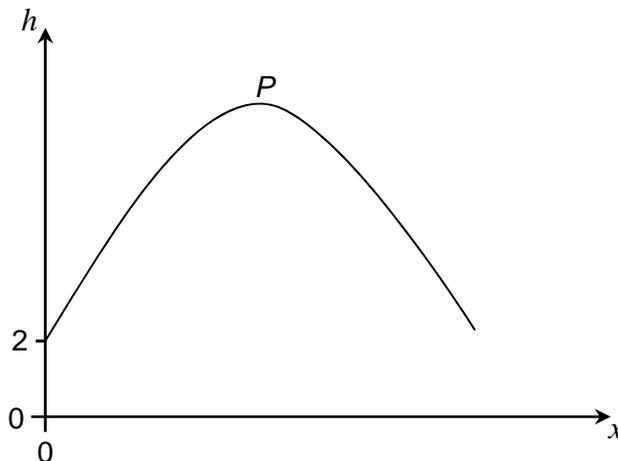
Work out the coordinates of *P*.

[2 marks]

highest point is $15 + 15 + 2 = 32m = y$ value

so p is $(180, 32)$

Answer (**180** , **32**)

25 $2x^2 - 6x + 5$ can be written in the form $a(x - b)^2 + c$

where a, b and c are positive numbers.

25 (a)   Work out the values of a, b and c.

[3 marks]

$$2\left(x^2 - 3x + \frac{5}{2}\right)$$

$$2\left[(x - 3/2)^2 + 5/2 - 9/4\right]$$

$$2\left[(x - 3/2)^2 + 1/4\right]$$

$$2(x - 3/2)^2 + 1/2$$

$a = 2$

$b = 1.5$

$c = 0.5$

$a = $ _____ 2 _____

$b = $ _____ 1.5 _____

$c = $ _____ 0.5 _____

25 (b) Using your answer to part (a), or otherwise, solve $2x^2 - 6x + 5 = 8.5$

[3 marks]

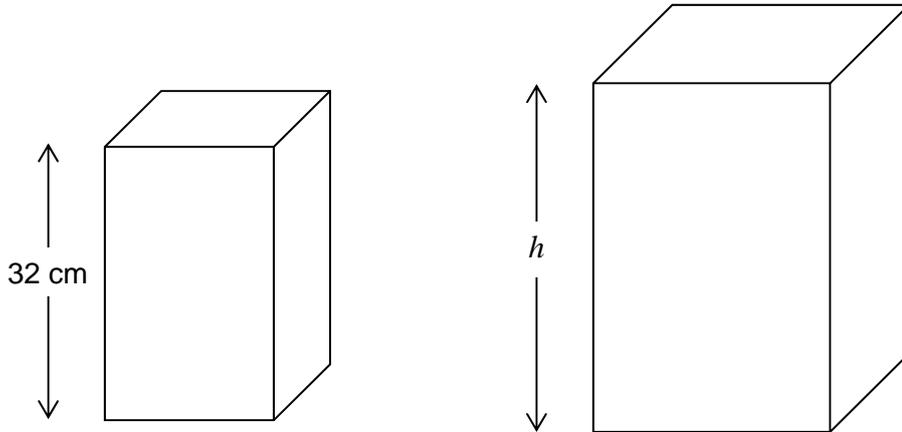$$2(x - 3/2)^2 + 1/2 = 8.5$$

$$2(x - 3/2)^2 = 8$$

$$(x - 3/2)^2 = 4$$

$$x - 3/2 = \pm 2$$

$$x = \frac{7}{2} \quad \text{or} \quad x = \frac{-1}{2}$$

Answer $\quad x = 7/2 \quad \text{or} \quad -1/2$

Turn over for the next question

Turn over ▶

26 Two boxes are made with card.

The boxes are similar cuboids.

The smaller box has height 32 cm

32 cm

h

It takes 44% more card to make the larger box.

Work out the height, h, of the larger box.

[4 marks]

Surface area → small : large

1 : 1.44

Length → 1 : $\sqrt{1.44}$

1 : 1.2

32 : 38.4

So height of larger box = 38.4 cm

Answer _____38.4_____ cm

END OF QUESTIONS