AQA Model Solutions

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

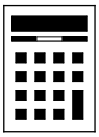
H

Higher Tier Paper 2 Calculator

Thursday 7 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.



Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

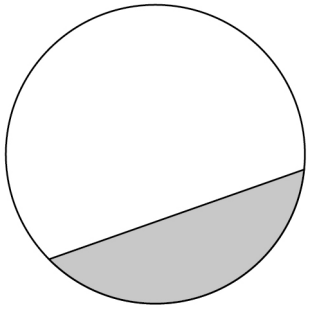
Advice
- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
28–29	
TOTAL	

Answer **all** questions in the spaces provided

Do not write outside the box

1 Here is a circle.



Circle the word that describes the shaded part.

[1 mark]

segment chord sector arc

2 Circle the number that is in standard form.

[1 mark]

0.25×10^4 6×10^7 38×10^{-3} $4 \times 10^{\frac{1}{2}}$

whole number

$1 \leq x < 10$

Do not write outside the box

3 y is $1\frac{1}{2}$ times x.

Circle the ratio that is equivalent to $y : x$

[1 mark]

$y : x$

$1\frac{1}{2} : 1$

$\frac{3}{2} : 1$

$\times 2$ ($3 : 2$) $\times 2$

2 : 5 | 5 : 2 | (3 : 2) | 2 : 3

4 Work out 40 as a percentage of 10

Circle your answer.

[1 mark]

$\frac{40}{10} \times 100\%$.

$= 400\%$.

4% | 25% | 300% | (400%)

Turn over for the next question

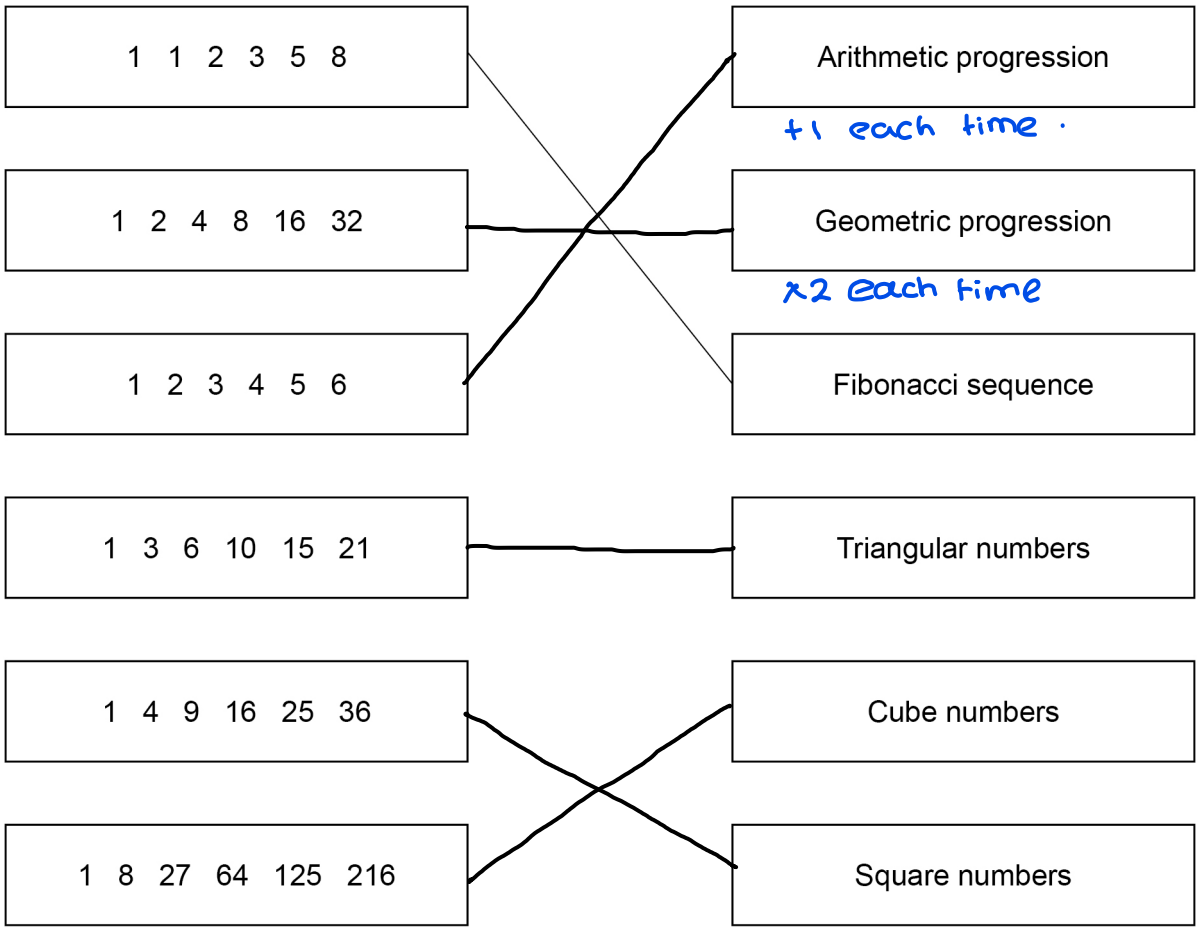
4

Do not write outside the box

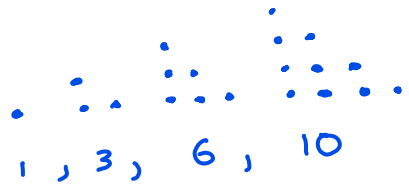
5 Match each sequence to its description.

One has been done for you.

[4 marks]



Triangular numbers:



Do not write outside the box

6 The table shows information about the population of a city.

Population in 2001	Population in 2011
420 000	480 000

Liam claims,

"From 2011 to 2021 the population of the city will increase by the same percentage as from 2001 to 2011"

He works out,

population increase from 2001 to 2011 = 480 000 – 420 000
= 60 000

population in 2021 = 480 000 + 60 000 ← Increase between 2011 and 2021
= 540 000

Does the population of 540 000 match his claim?

You **must** show your working.

[3 marks]

$\frac{60000}{420000} \times 100 = 14.3\%$.

There is a 14.3% increase in population between 2001 and 2011.

$\frac{60000}{480000} \times 100 = 12.5\%$.

There is only a 12.5% increase between 2011 and 2021.

Answer No, the population does not increase by the same amount.

Turn over for the next question

7

Do not write outside the box

7 On three days, Ali throws darts at a target.

Here are his results.

	Number of throws	Number of hits	Number of misses
Monday	20	15	5
Tuesday	30	22	8
Wednesday	40	17	23
Total	90	54	36

7 (a) Work out **two** different estimates for the probability of Ali hitting the target.

[2 marks]

Probability of hitting the target $= \frac{\text{no. of hits}}{\text{no. of throws}}$.

Answer $\frac{15}{20}$ and $\frac{22}{30}$

7 (b) Which of your two answers is the better estimate for the probability of Ali hitting the target?

Give a reason for your answer.

[1 mark]

Answer $\frac{22}{30}$

Reason Because this probability was calculated from a greater number of throws.

Do not write outside the box

8 Theo starts with savings of £18
James starts with no savings.

Each week from now,

Theo will save £4.50 and James will save £4

In how many weeks will Theo and James have savings in the ratio 15 : 8 ?

[3 marks]

x is the number of weeks.

$$\frac{18+(4.5)x}{15} \quad \frac{(4)x}{8} \qquad \text{Theo : James} \quad 15 : 8$$

Cross multiply

$$8(18 + 4.5x) = 15(4x)$$

$$144 + 36x = 60x$$

$-36x$ $\qquad$ $-36x$

$$144 = 24x$$

$$x = \frac{144}{24} = 6 \text{ weeks}$$

Answer 6 weeks

Alternatively:

week	Theo : James
0	18 : 0
1	22.5 : 4
2	27 : 8
3	31.5 : 12
4	36 : 16
5	40.5 : 20
(6)	45 : 24
7	49.5 : 28

45 : 24 $\div 3$ → 15 : 8 ← required ratio

6

Do not write outside the box

9 The length of each side of a regular pentagon is 8.4 cm to 1 decimal place.

9 (a) Complete the error interval for the length of one side.

[2 marks]

Would round to 8.4 at 1dp.

8.35 cm ⩽ length < 8.45 cm

9 (b) Complete the error interval for the perimeter.

[1 mark]

$8.35 \times 5 = 41.75$ $8.45 \times 5 = 42.25$

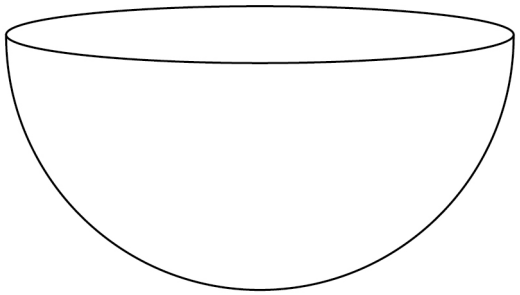
41.75 cm ⩽ perimeter < 42.25 cm

Do not write outside the box

10

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

A container is a hemisphere of radius 30 cm



Sand fills the container at a rate of 4000 cm^3 per minute.

Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

[3 marks]

hemisphere

Volume of container $= \frac{1}{2} \times \frac{4}{3} \times \pi \times (30)^3$

$= \frac{2}{3} \times \pi \times 27000$

$= 18000\pi \text{ cm}^3$

$\frac{18000\pi \text{ cm}^3}{4000 \text{ cm}^3/\text{min}} = \frac{9}{2}\pi \text{ min}$

rate

$\frac{9}{2}\pi = 14.1 \text{ min} < \frac{1}{4} \text{ hr} = \frac{1}{4} \times 60 = 15 \text{ min}$

Answer Yes, as $14.1 < 15$

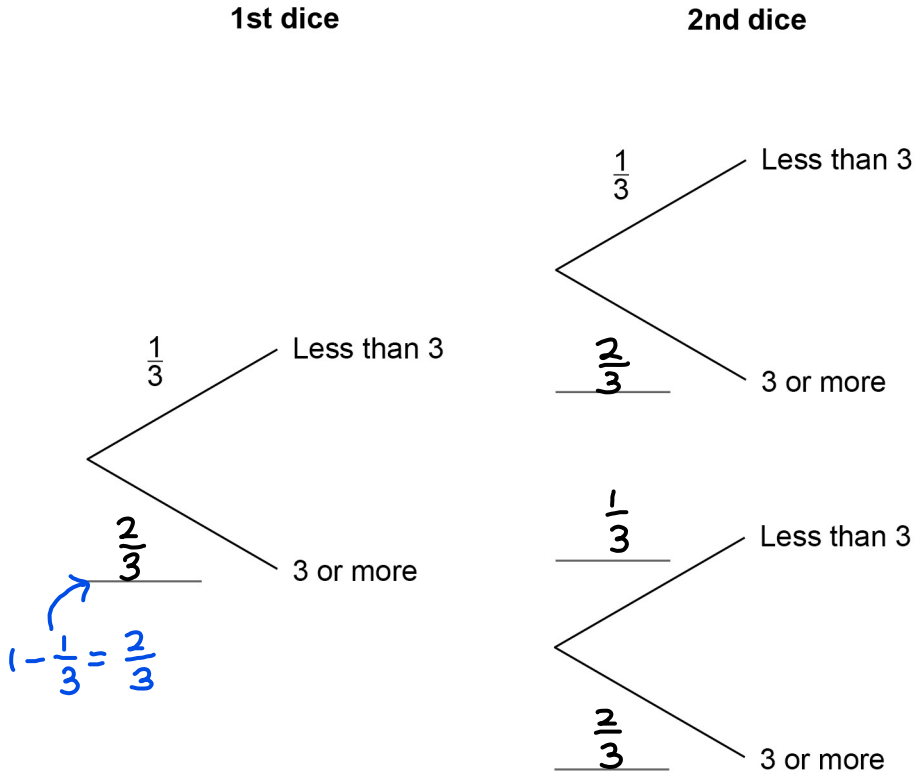
6

Do not write outside the box

11 Two ordinary fair dice are rolled.

11 (a) Complete the tree diagram.

[1 mark]



11 (b) Work out the probability that **both** dice land on a number less than 3

[1 mark]

$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Answer $\frac{1}{9}$

Do not write outside the box

11 (c) Work out the probability that **exactly one** of the dice lands on a number less than 3

[2 marks]

[(less than 3) AND (3 or more)] OR [(3 or more) AND (less than 3)]

$\left(\frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{1}{3}\right) = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

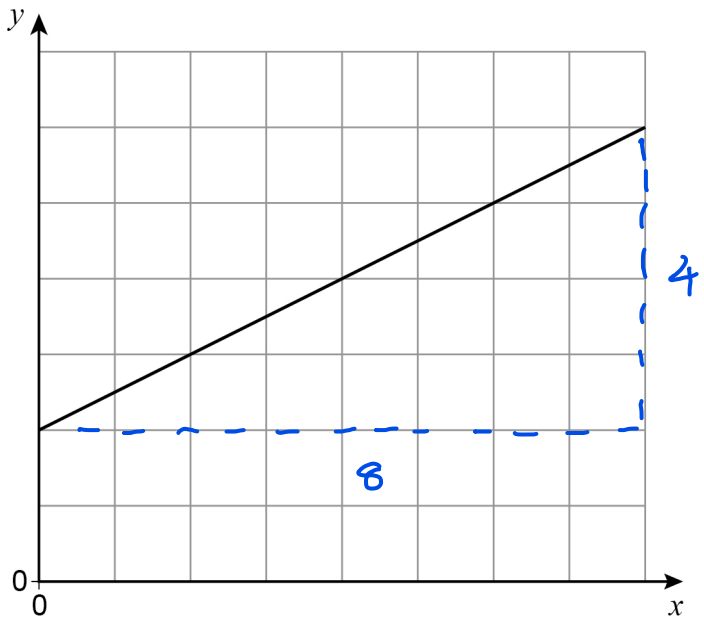
Answer 4/9

Turn over for the next question

4

Do not write outside the box

12 A straight line is drawn on the centimetre grid.



Fay assumes that the scale is 1 cm represents 1 unit.

12 (a) Use her assumption to work out the gradient of the line.

[1 mark]

gradient = $\frac{4}{8} = \frac{1}{2}$

$\frac{\text{change in } y}{\text{change in } x}$

Answer $\frac{1}{2}$

Do not write outside the box

12 (b) In fact, the scale is 1 cm represents 2 units.

Which statement is correct?

Tick **one** box.

[1 mark]

- ☐ The answer to part (a) is too big
- ☑ The answer to part (a) stays the same
- ☐ The answer to part (a) is too small

1 unit = 0.5 cm

gradient = $\frac{4 \times 0.5}{8 \times 0.5} = \frac{1}{2}$

Turn over for the next question

2

Do not write outside the box

13 Show that, for $x \neq -1$

$$\frac{8x^2 - 8}{4x + 4}$$ simplifies to the form $ax + b$ where a and b are integers.

[3 marks]

$\frac{8(x^2-1)}{4(x+1)} = \frac{8(x+1)(x-1)}{4(x+1)} = 2(x-1) = 2x-2$

* difference of 2 squares.

Do not write outside the box

14 The scale drawing represents a garden.

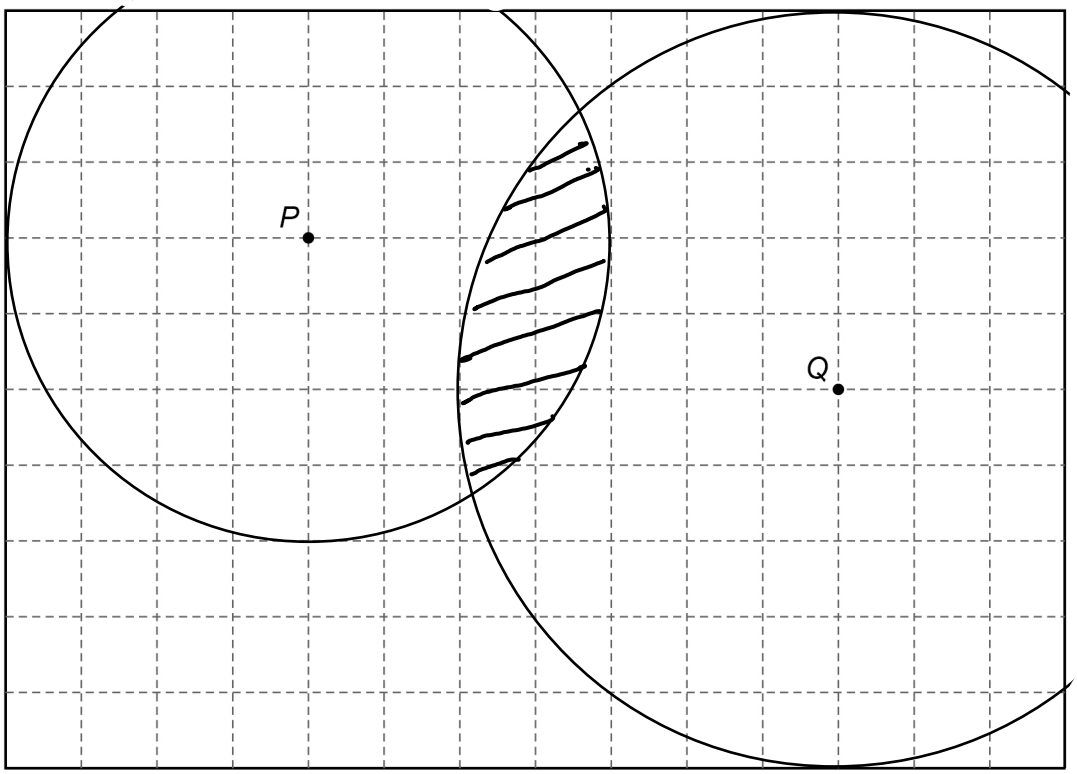
Water from a sprinkler at *P* reaches up to 20 metres from *P*.

Water from a sprinkler at *Q* reaches up to 25 metres from *Q*.

P : 20m : 4cm ← Draw an arc radius 4cm from P, etc.

Q : 25m : 5cm

Scale: 1 cm represents 5 m



Using a pair of compasses,

show the region that water from **both** sprinklers reaches.

[2 marks]

Turn over for the next question

5

Do not write outside the box

15 100 men and 100 women took a test.

Scores

	Median	Interquartile range	Range
Men	28	7.5	31
Women	30	9	37

Using this data, which statement **must** be true?

Tick **one** box.

[1 mark]

- ☐ Men had a higher average score than women

IQR is lower for men, so data is less spread out.

- ☑ Men had more consistent scores than women
- ☐ A woman had the highest score
- ☐ A man had the lowest score

Do not write outside the box

16 Some concrete has volume 3.8 m^3

16 (a) The density of the concrete is 2400 kg/m^3

Work out the mass of the concrete.

[2 marks]

density = $\frac{\text{mass}}{\text{volume}}$; mass = density × volume

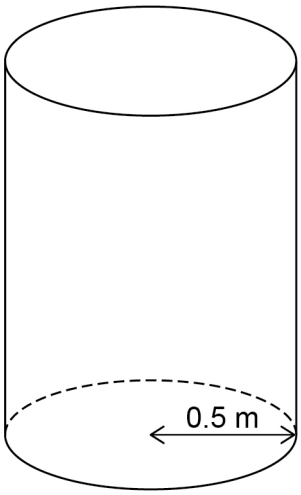
= 2400 × 3.8

= 9120 kg

Answer 9120 kg

16 (b) The 3.8 m^3 of concrete is made into the shape of a cylinder.

The base has radius 0.5 metres.



Work out the height of the cylinder.

[2 marks]

Volume = 3.8 m^3 (volume of cylinder = $\pi r^2 h$)

$3.8 = \pi \times (0.5)^2 \times h$

$h = \frac{3.8}{0.5^2 \times \pi} = 4.8m$

Answer 4.8 m

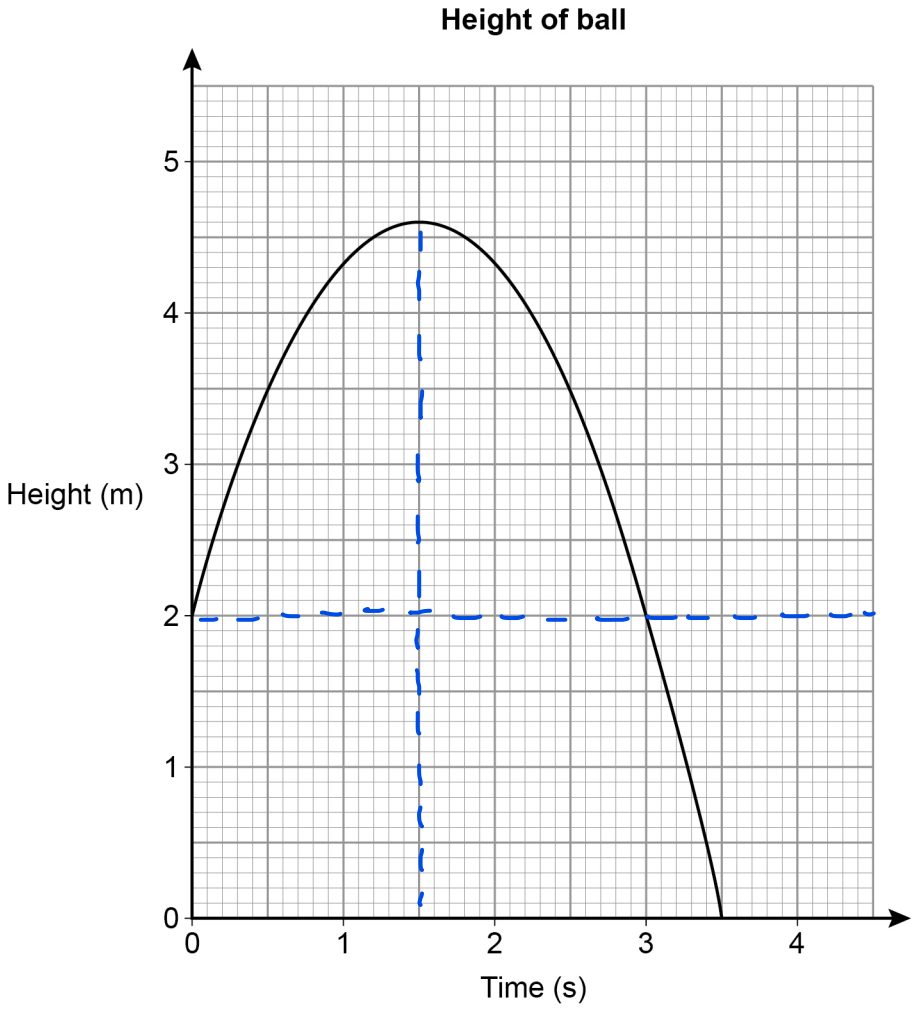
5

Turn over ►

Do not write outside the box

17 A ball is thrown vertically upwards.

The graph shows the height of the ball above the ground after it is thrown.



17 (a) For how many seconds is the ball at a height of **more than** 2 metres?

[1 mark]

Answer 3 s

17 (b) After how many seconds is the ball at instantaneous rest when it is in the air?

[1 mark]

Answer 1·5 s

(Instantaneous rest at turning point because gradient = $\frac{H}{T}$ = 0, So Speed = 0 m/s)

Do not write outside the box

17 (c) Work out the average speed of the ball when it is moving downwards.

[2 marks]

$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{4.6 - 0}{3.5 - 1.5} = \frac{4.6}{2} = 2.3$

Answer 2.3 m/s

18 The solution of $3^x = 300$ lies between two consecutive integers.

Work out the two integers.

[1 mark]

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

$243 < 300 < 729$

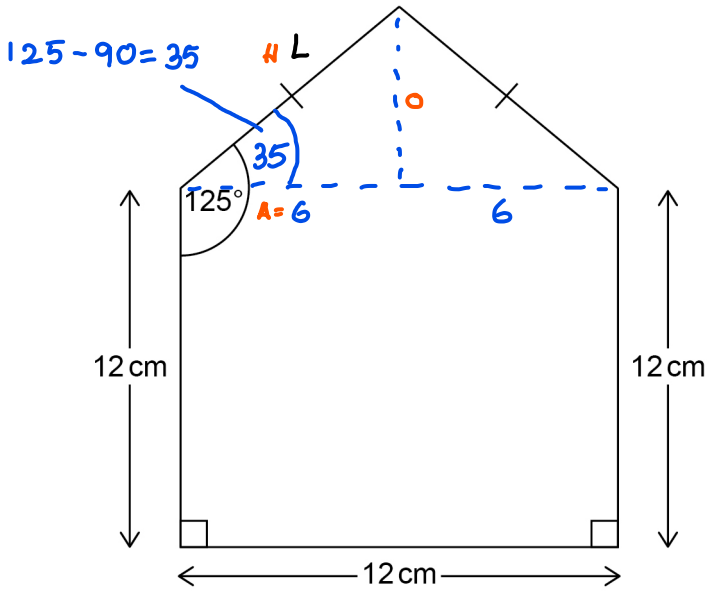
Answer 5 and 6

Turn over for the next question

5

Do not write outside the box

19 A pentagon is made from a square and an isosceles triangle.



Not drawn accurately

SOH CAH TOA

$\cos x = \frac{A}{H}$

Work out the perimeter of the pentagon.

[4 marks]

work out the length of a side of the triangles : $\cos 35 = \frac{6}{L}$

$L = \frac{6}{\cos 35} = 7.3\text{ cm}$

Perimeter $= (3 \times 12) + (2 \times 7.3)$

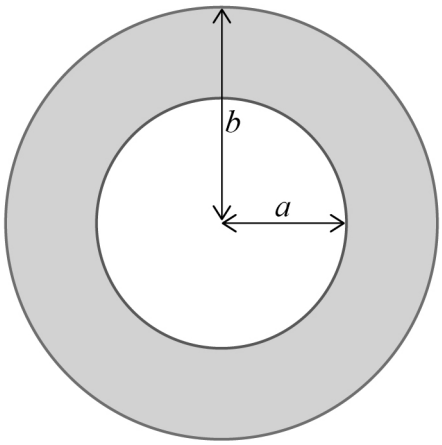
$= 36 + 14.6$

$= 50.6\text{ cm}$

Answer 50.6 cm

Do not write outside the box

20 Here is an inflated swimming ring with dimensions in centimetres.



The volume of the ring, V cm^3, is given by

$$V = 0.25\pi^2(b - a)^2(b + a)$$

Work out the volume when $a = 20$ and $b = 30$

Give your answer to 3 significant figures.

[3 marks]

$b - a = 30 - 20 = 10$ $\quad b + a = 30 + 20 = 50$

$V = 0.25\pi^2(10)^2(50)$

$= 0.25\pi^2(100)(50)$

$= 1250\pi^2 = 12300 \text{ cm}^3$ (3 sf)

Answer 12300 cm^3

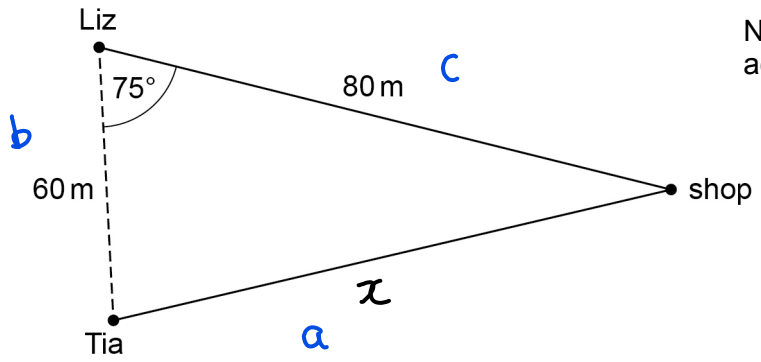
Turn over for the next question

7

Do not write outside the box

21 Liz and Tia are walking towards a shop along different straight paths.

The diagram shows their positions at 2 pm



Not drawn accurately

21 (a) Assume they walk at the same speed.

Who will arrive at the shop first?

You **must** show your working.

[3 marks]

Distance Tia walks: $x^2 = 80^2 + 60^2 - 2(80)(60)\cos 75$

Cosine rule $\Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 6400 + 3600 - 9600\cos 75$

$x^2 = 7515$

$x = 86.7\text{ m} > 80\text{m}$

They are walking at the same speed so Liz arrives first.

Answer Liz

21 (b) In fact, Liz walks at a faster speed than Tia.

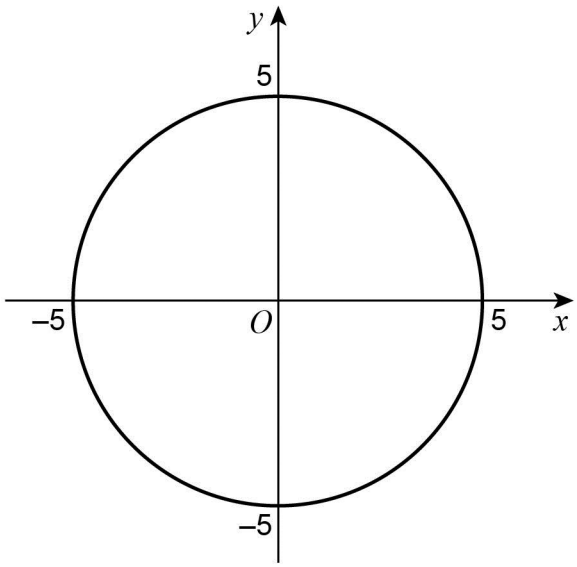
How does this affect the answer to part (a)?

[1 mark]

Liz will still arrive first.

Do not write outside the box

22 A circle, centre O, passes through (5, 0).



What is the equation of the circle?

Circle your answer.

radius = 5
center = 0,0

[1 mark]

$x^2 + y^2 = 25$ $\quad$ $x^2 + y^2 = 5$ $\quad$ $x^2 + y^2 = 10$ $\quad$ $x^2 + y^2 = 100$

$r^2 = 25$

Turn over for the next question

5

Do not write outside the box

23 Solids X and Y are similar.

X has volume 64 cm^3

Y has volume 343 cm^3

The surface area of X is 176 cm^2

Work out the surface area of Y.

[3 marks]

$X = \sqrt[3]{64}$ $Y = \sqrt[3]{343}$

length ratio $= 4$ length ratio $= 7$

Area ratio $\Rightarrow 4^2 = 16$ Area ratio $\Rightarrow 7^2 = 49$

$16 \times 11 = 176$ $\therefore$ $49 \times 11 = 539\ cm^3$

Answer 539 cm^2

Do not write outside the box

24 A tank is a cuboid measuring 50 cm by 35 cm by 20 cm

All lengths are to the **nearest centimetre**.

A container has a capacity of **exactly** 34 litres.

1 litre = 1000 cm^3

Which has the greater capacity?

Tick **one** box.

☐ Tank ☐ Container ☑ Cannot tell

Show working to support your answer.

[4 marks]

$49.5 \leqslant 50\text{cm} < 50.5$

$34.5 \leqslant 35\text{cm} < 35.5$

$19.5 \leqslant 20\text{cm} < 20.5$

Smallest capacity of tank = $49.5 \times 34.5 \times 19.5 = 33301.125\ cm^3$

largest capacity of tank = $50.5 \times 35.5 \times 20.5 = 36751.375\ cm^3$

So capacity of tank could be smaller or larger than $34000\ cm^3$.

container = 34×1000

$= 34000\ cm^3$

Turn over for the next question

7

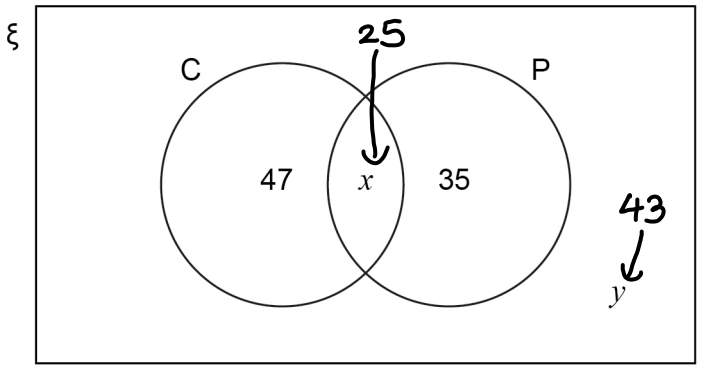
Do not write outside the box

25 The Venn diagram shows some information about 150 students.

ξ = 150 students

C = students who study Chemistry

P = students who study Physics



The probability that a Physics student, chosen at random, also studies Chemistry is $\frac{5}{12}$

One of the 150 students is chosen at random.

Work out the probability that the student does **not** study either Chemistry or Physics.

[4 marks]

Probability of studying chemistry given studies physics. $= \frac{x}{x+35} = \frac{5}{12}$

CROSS MULTIPLY $12x = 5(x+35)$

$12x = 5x + 175$

$7x = 175$

$x = \frac{175}{7} = 25$

$150 = 47 + x + 35 + y \Rightarrow 150 - 47 - 25 - 35 = 43$

Answer $\frac{43}{150}$

Do not write outside the box

26 A curve has equation $y = 4x^2 + 5x + 3$

A line has equation $y = x + 2$

Show that the curve and the line have **exactly** one point of intersection.

Do **not** use a graphical method.

[4 marks]

Equate the two equations:

$-(x+2)$ ($4x^2 + 5x + 3 = x + 2$) $-(x+2)$

$4x^2 + 4x + 1 = 0$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(1)}}{2(4)} \qquad \sqrt{16-16} = \sqrt{0} = 0$$

$$= \frac{-4 \pm \sqrt{16-16}}{8} = \frac{-4 \pm 0}{8} = -\frac{4}{8} = -\frac{1}{2}$$

There is only one solution, so there is only one point of intersection.

Turn over for the next question

8

Do not write outside the box

27 Prove algebraically that $2.7\dot{5}$ converts to the fraction $\frac{124}{45}$ **[3 marks]**

$x = 2.7555\ldots$

$-\quad 10x = 27.5555\ldots$ } Subtract upward.

$9x = 27.5 - 2.7 = 24.8$

$9x = 24.8 \Rightarrow x = \frac{24.8}{9} = \frac{124}{45}$ (×5 above, ×5 below)

Do not write outside the box

28 $f(x) = 5 - x$ and $g(x) = 3x + 7$

28 (a) Simplify $f(2x) + g(x - 1)$

[3 marks]

$f(2x) = 5 - (2x) = 5 - 2x$

$g(x-1) = 3(x-1) + 7$

$= 3x - 3 + 7$

$= 3x + 4$

So, $f(2x) + g(x-1) = (5 - 2x) + (3x + 4) = x + 9$

Answer $x + 9$

28 (b) Solve $g^{-1}(x) = 2x$

[3 marks]

Find $g^{-1}(x)$:

$y = 3x + 7$

$y - 7 = 3x$ ↓ Re arrange to find x

$\frac{y-7}{3} = x$

↓ replace y with x.

So $g^{-1}(x) = \frac{x-7}{3}$

$\frac{x-7}{3} = 2x$

$x - 7 = 3(2x)$

$x - 7 = 6x$

$-7 = 5x$

$\div 5$ ↓ ↓ $\div 5$

$x = -7/5$

END OF QUESTIONS

9

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED