AQA Model Solutions

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

H

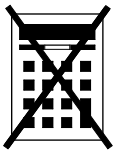
Higher Tier Paper 1 Non-Calculator

Thursday 24 May 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- mathematical instruments

You must **not** use a calculator.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided

1 Work out $\sqrt[3]{64 \times 1000}$

Circle your answer.

[1 mark]

40 80 400 4000

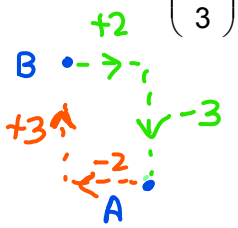
$\sqrt[3]{64} \times \sqrt[3]{1000} = 4 \times 10 = 40$ $(4 \times 4 \times 4 = 64)$

2 The vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ translates A to B.

Circle the vector that translates B to A.

[1 mark]

$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$



3 Circle the expression that is equivalent to $3a - a \times 4a + 2a$

[1 mark]

$8a^2 + 2a$ $12a^2$ $5a - 4a^2$ $3a - 6a^2$

BIDMAS $3a - (a \times 4a) + 2a$

$3a - 4a^2 + 2a$

$5a - 4a^2$

Do not write outside the box

4 Circle the number that is closest in value to $\frac{9.8}{0.0195}$

[1 mark]

5 50 (500) 5000

$\frac{9.8}{0.0195} \approx \frac{10}{0.02} = \frac{1000}{2} = 500$ (×100 on both numerator and denominator)

5 Solve $5(x + 3) < 60$

[2 marks]

$5x + 15 < 60$ (−15)

$5x < 45$ (÷5)

$x < 9$

Answer $x < 9$

Turn over for the next question

6

Do not write outside the box

6 The height of Zak is 1.86 metres.

The height of Fred is 1.6 metres.

Write the height of Zak as a fraction of the height of Fred.

Give your answer in its simplest form.

[3 marks]

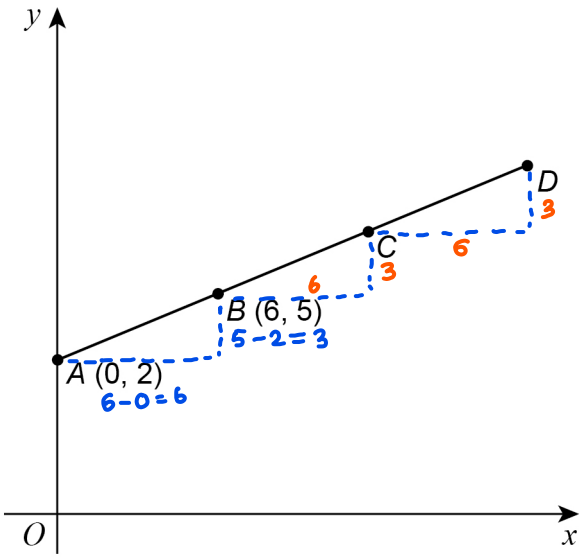
$$\frac{\text{Zak}}{\text{Fred}} = \frac{1.86}{1.6} = \frac{186}{160} = \frac{93}{80}$$

(×100, ÷2, ×100, ÷2)

Answer $\frac{93}{80}$

Do not write outside the box

7 *A* (0, 2) and *B* (6, 5) are points on the straight line *ABCD*.



Not drawn accurately

AB = *BC* = *CD*

Work out the coordinates of *D*.

[3 marks]

A → D 6 × 3 = 18 in x-axis

3 × 3 = 9 in y-axis

(0, 2)

+18 ↓ ↓ +9

D = (18, 11)

Answer (18 , 11)

Turn over for the next question

6

Turn over ►

Do not write outside the box

8 A coin is thrown 50 times.

It lands on heads 31 times.

8 (a) Write down the relative frequency it lands on heads.

[1 mark]

Answer $\frac{31}{50}$

8 (b) Raj says,

"The coin is biased towards heads."

Use the data to give a reason why he might be correct.

[1 mark]

The frequency that the coin lands on tails is $50 - 31 = 19$

$31 > 19$

Alternatively: A fair coin will have 25 heads in 50 throws.

Do not write outside the box

9 The range of a set of numbers is $15\frac{1}{4}$

The smallest number is $-2\frac{7}{8}$

Work out the largest number.

[3 marks]

Range = Largest number - Smallest number.

$15\frac{1}{4} = \text{largest} - \left(-2\frac{7}{8}\right)$

$15\frac{2}{8} = \text{largest} + 2\frac{7}{8}$ $\quad 15\frac{2}{8} = \frac{(15\times 8)+2}{8}$

$15\frac{2}{8} - 2\frac{7}{8} = \text{largest}$

$\frac{122}{8} - \frac{23}{8} = \frac{122-23}{8} = \frac{99}{8}$ $\quad 2\frac{7}{8} = \frac{(2\times 8)+7}{8}$

Answer $\frac{99}{8}$

10 y is inversely proportional to x.

Complete the table.

[2 marks]

x	12	6	3
y	2	4	8

$y \propto \frac{1}{x}$ so $y = \frac{k}{x}$

when $x=6$, $y=4 \Rightarrow 4 = \frac{k}{6} \Rightarrow k = 6\times 4 = 24$

$\therefore y = \frac{24}{x}$

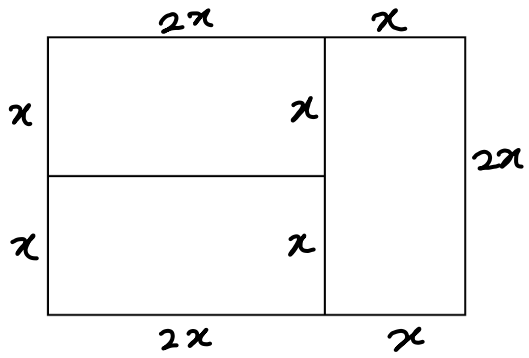
When $x=12$; $y = \frac{24}{12} = 2$ $\quad y=8$; $8 = \frac{24}{x} \Rightarrow x = \frac{24}{8} = 3$

Turn over for the next question

7

Do not write outside the box

11 A large rectangle is made by joining three identical small rectangles as shown.



Not drawn accurately

The perimeter of one small rectangle is 15 cm

Work out the perimeter of the large rectangle.

[4 marks]

Perimeter of small rectangle:

$2x+x+2x+x = 15$

$6x = 15$

$x = \frac{5}{2}$

Perimeter of large rectangle: $x+x+2x+x+2x+x+2x = 10x$

$: 10(\frac{5}{2}) = 25\text{ cm}$

Answer 25 cm

Do not write outside the box

12 Put these numbers in order from smallest to largest.

8×10^{-4} (2) 4×10^{-2} (3) 6×10^{-4} (1) 0.07 (4)

[2 marks]

$8 \times 10^{-4} = 8 \div 10000 = 0.0008$

$4 \times 10^{-2} = 4 \div 100 = 0.04$

$6 \times 10^{-4} = 6 \div 10000 = 0.0006$

Smallest → Largest : 0.0006, 0.0008, 0.04, 0.07

Smallest 6×10^{-4}

8×10^{-4}

4×10^{-2}

Largest 0.07

13 Circle the volume that is the same as 15 cm^3

[1 mark]

15 000 mm^3 1.5 mm^3 0.0015 mm^3 150 mm^3

$1\ cm = 10\ mm$

$1\ cm^3 = 10^3\ mm^3$

Turn over for the next question

So $15 cm^3 = 15000 mm^3$

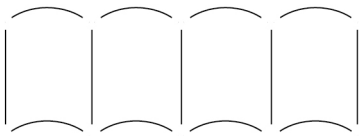
7

Do not write outside the box

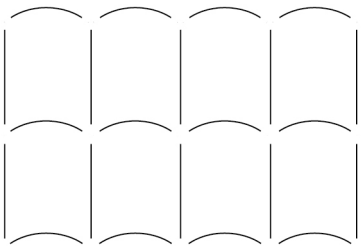
14 Patterns are made using straight lines and arcs.

14 (a)

Pattern A (one row)



Pattern B (two rows)



More rows are added to **Pattern B** so that

number of straight lines : number of arcs = 10 : 9

How many rows are added?

[2 marks]

Pattern A : 5 Straight lines : 8 arcs
↓+5 ↓+4
Pattern B : 10 Straight lines : 12 arcs

No. of rows added	Straight	:	arcs
1)	15	:	16
2)	20	:	20
3)	25	:	24
4)	30	:	28
5)	35	:	32
6)	40	:	36

+5, +4, ÷4 → 10 : 9

Answer 6

Do not write outside the box

14 (b) A different pattern is made using 20 straight lines and 16 arcs.

The straight lines and arcs are made from metal.

20 straight lines cost £12

cost of one straight line : cost of one arc = 2 : 3

Work out the **total** cost of the metal in the pattern.

[3 marks]

cost of one straight line: $\frac{£12}{20} \overset{\div 2}{=} \frac{£6}{10} = £0.60$ ($\div 2$)

Straight line : arc

2 : 3

$\times 0.30$ → £0.60 : £0.90 ← $\times 0.30$

Total cost = £12 + (£0.90) × 16 = £12 + £14.40 = £26.40

Answer £ 26.40

16.0 × 0.9 = 5.4 + 9.0 = 14.4

Turn over for the next question

5

Do not write outside the box

15 A biased dice is thrown.

Here are the probabilities of each score.

Score	1	2	3	4	5	6
Probability	0.25	0.05	0.15	0.05	0.3	0.2

The dice is thrown 200 times.

Work out the expected number of times the score will be odd.

[3 marks]

$P(\text{odd}) = P(1) + P(3) + P(5)$

$= 0.25 + 0.15 + 0.3$

$= 0.7$

expected = $P(\text{odd}) \times$ no. of throws

$= 0.7 \times 200$

Answer 140

Do not write outside the box

16 The value of y is 20% more than the value of x.

Circle the ratio $x : y$

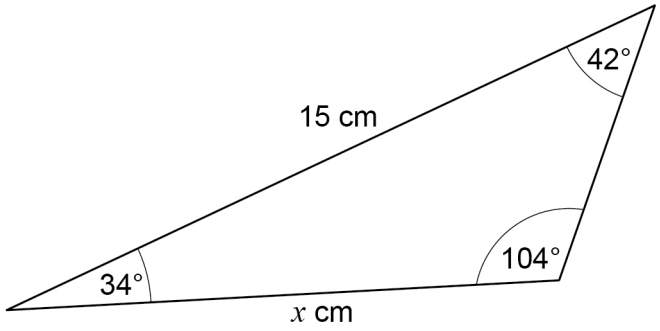
[1 mark]

5 : 6 6 : 5 4 : 5 5 : 4

$x : y$
$x : 1.2x$
$1 : 1.2$ ×10
$10 : 12$ ÷2
$5 : 6$

17 Here is a triangle.

Not drawn accurately



Circle the correct equation.

[1 mark]

$\frac{\sin x}{42} = \frac{\sin 15^\circ}{104}$

$\frac{x}{\sin 42^\circ} = \frac{15}{\sin 104^\circ}$

$\frac{\sin x}{34} = \frac{\sin 15^\circ}{104}$

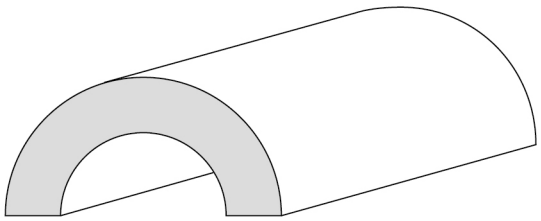
$\frac{x}{\sin 42^\circ} = \frac{15}{\sin 34^\circ}$

$\frac{x}{\sin 42} = \frac{15}{\sin 104}$ ← Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

5

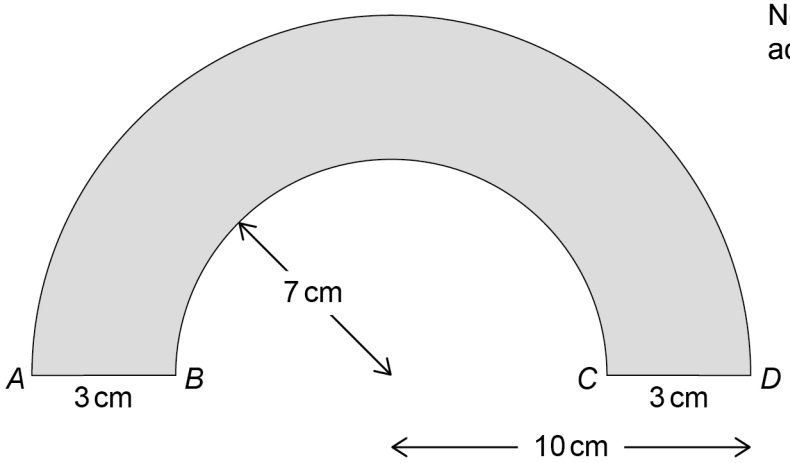
Do not write outside the box

18 Here is a tunnel for a toy train.



The diagram below shows the cross section of the tunnel.

Not drawn accurately



AD is a semicircular arc of radius 10 cm
BC is a semicircular arc of radius 7 cm
The length of the tunnel is 30 cm

Work out the total area of all **six** faces of the tunnel.

Give your answer in terms of π.

[5 marks]

area of 1 cross section $= \frac{1}{2} \times \pi \times (10)^2 - \frac{1}{2} \times \pi \times (7)^2 = 50\pi - \frac{49}{2}\pi = \frac{100\pi - 49\pi}{2} = \frac{51\pi}{2}\text{ cm}^2$

area in contact with ground $= 2 \times 3 \times 30 = 6 \times 30 = 180\text{ cm}^2$ (2 sides)

area of inner curved surface $= \frac{1}{2} \times$ circumference $\times$ length $= \frac{1}{2} \times (2\pi \times 7) \times 30 = 210\pi$ ($2\pi r$)

area of outer curved surface $= \frac{1}{2} \times (2\pi \times 10) \times 30 = 300\pi$

Total area: $300\pi + 210\pi + 2\left(\frac{51}{2}\pi\right) + 180 = (561\pi + 180)\text{cm}^2$

2 cross sections

Do not write outside the box

Answer $561\pi + 180$ cm^2

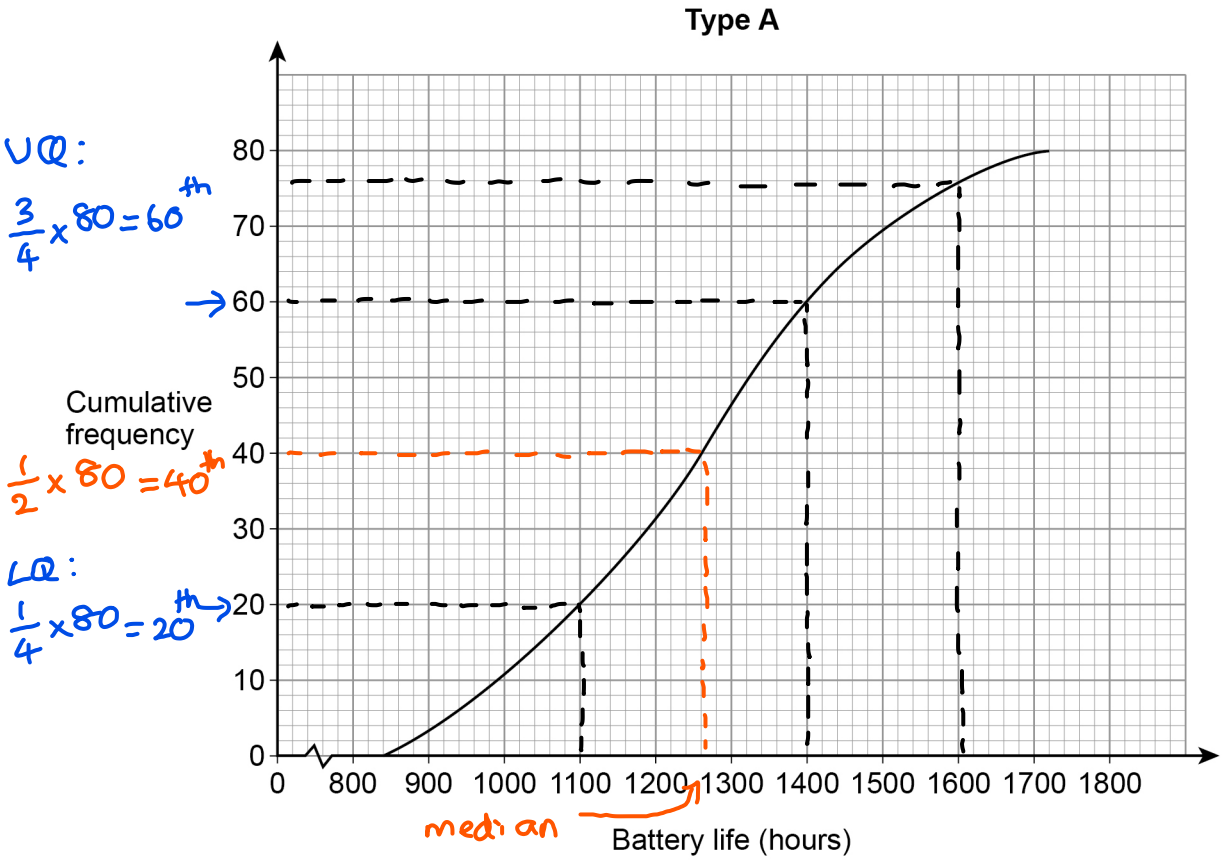
5

Turn over ►

Do not write outside the box

19 Type A batteries and type B batteries were tested.

The cumulative frequency diagram shows information about the battery life of type A.



19 (a) Estimate the interquartile range for type A.

[2 marks]

$1400 - 1100 = 300$

Answer 300 hours

Do not write outside the box

19 (b) Estimate the number of type A batteries that had a battery life of more than 1600 hours.

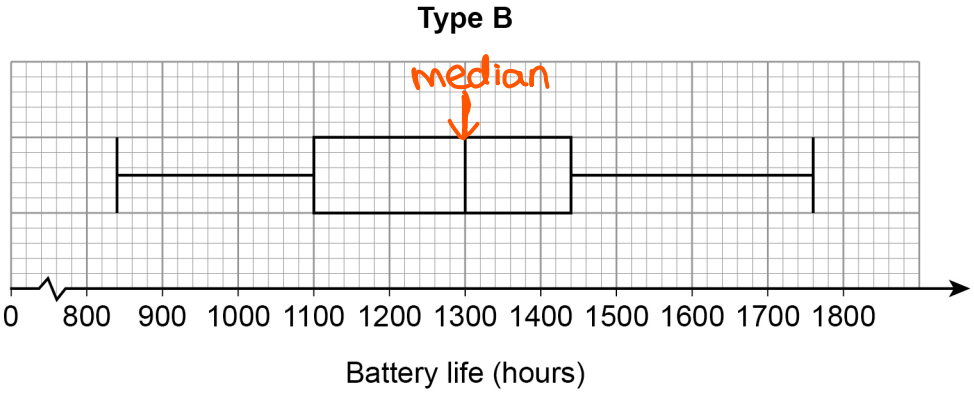
[1 mark]

1600 ← 76 batteries 80 - 76 = 4

or less

Answer 4

19 (c) The box plot shows information about the battery life of type B.



On average, which type had the greater battery life?

Tick a box.

☐ type A ☑ type B

Using data from **both** diagrams, state how you chose your answer.

[2 marks]

Type B, because the median for A is 1260 while the median for B is 1300, which is higher.

5

Turn over ►

Do not write outside the box

20 A linear sequence starts

$a + 2b$, $a + 6b$, $a + 10b$, ... , ...

+4b +4b +4b +4b

The 2nd term has value 8

The 5th term has value 44

Work out the values of a and b.

[4 marks]

2nd term $\Rightarrow$ $a+6b = 8$ — ①

5th term $\Rightarrow$ $a+18b = 44$ — ②

② − ① $\div 12$ $\Rightarrow$ $12b = 36$ $\div 12$

$b = \frac{36}{12} = 3$

Subs. $b=3$ in ① $\quad a + 6(3) = 8$

$a + 18 = 8$ (−18 / −18)

$a = -10$

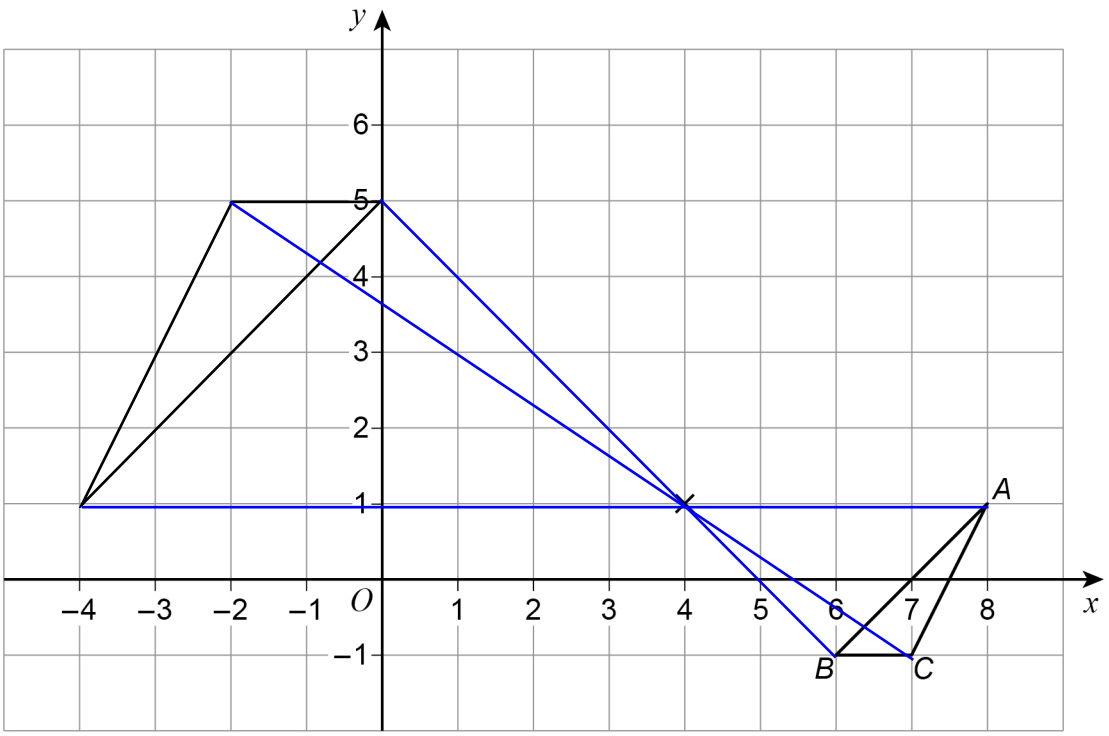
$a =$ −10

$b =$ 3

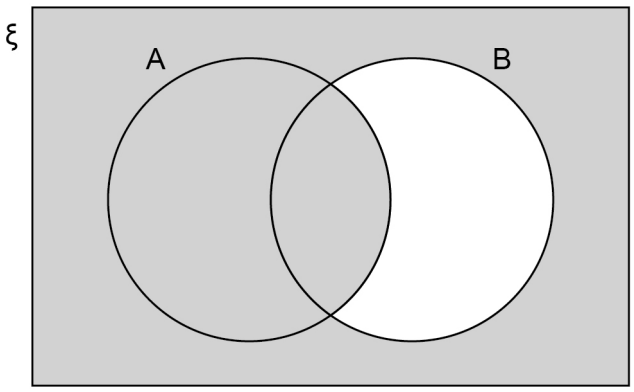
Do not write outside the box

21 Enlarge triangle *ABC* by scale factor –2, centre (4, 1)

[2 marks]



22



Which of these represents the shaded region?

Circle your answer.

[1 mark]

$A \cap B'$ B' $A \cup B'$ $A' \cup B'$

A or not B

7

Turn over ►

Do not write outside the box

23 A shopkeeper compares the income from sales of a laptop in March and April.

April

Price	$\frac{1}{5}$ more than March
Number sold	$\frac{1}{4}$ less than March

By what fraction does the income from these sales decrease in April?

[3 marks]

	March	April
Price	p	$\frac{6}{5}p$
nos.	n	$\frac{3}{4}n$
Income	pn	$\frac{6}{5}p \times \frac{3}{4}n = \frac{18}{20}pn$

change: $\frac{2}{20}$

decreases by: $\frac{2}{20} = \frac{1}{10}$

Answer $\frac{1}{10}$

Do not write outside the box

24 (a) Work out the value of $2^{14} \div \left(2^9\right)^2$

Give your answer as a fraction in its simplest form.

[3 marks]

$\frac{2^{14}}{(2^9)^2} = \frac{2^{14}}{2^{18}} = 2^{14-18} = 2^{-4}$

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

Answer $\frac{1}{16}$

24 (b) Work out the value of $25^{\frac{3}{2}}$

[2 marks]

$25^{3/2} = (25^{1/2})^3 = (\sqrt{25})^3 = 5^3$

$5^3 = 5 \times 5 \times 5 = 125$

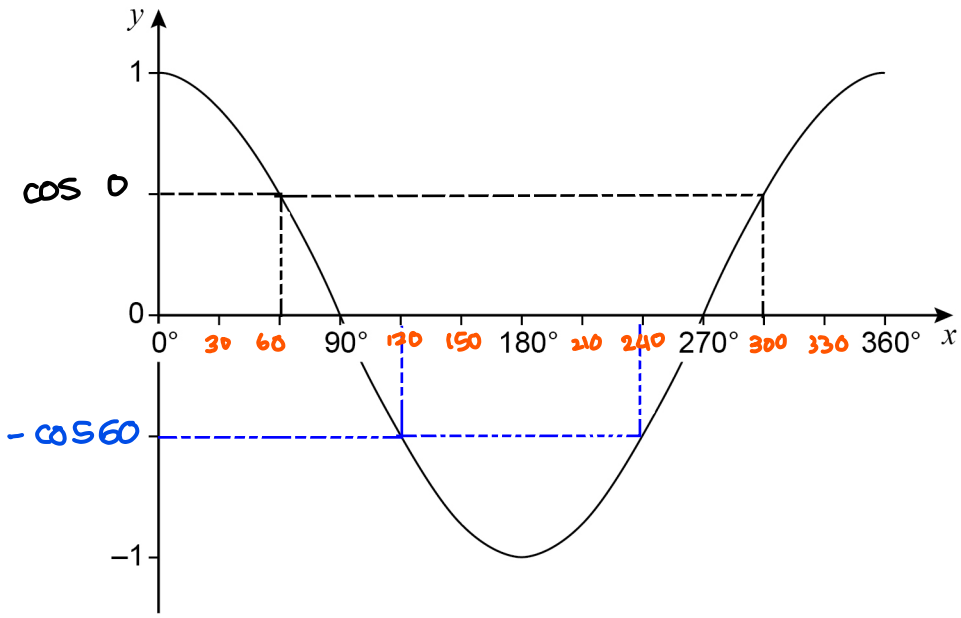
Answer 125

Turn over for the next question

8

Do not write outside the box

25 Here is a sketch of the graph of $y = \cos x$ for values of x from 0° to 360°



25 (a) cos x = cos 60°

Work out the value of x when $90° \leqslant x \leqslant 360°$

[1 mark]

Answer 300 degrees

25 (b) cos x = – cos 60°

Work out the value of x when $180° \leqslant x \leqslant 360°$

[1 mark]

Answer 240 degrees

Do not write outside the box

26 b is two thirds of c.

$5a = 4c$

Work out the ratio $a : b : c$

Give your answer in its simplest form where a, b and c are integers.

[3 marks]

$b = \frac{2}{3}c$ $a = \frac{4}{5}c$

$a : b : c$

$\frac{4}{5}c : \frac{2}{3}c : c$

$\frac{4}{5} : \frac{2}{3} : 1$ $\times 15$

$12 : 10 : 15$

Answer 12 : 10 : 15

Turn over for the next question

5

Do not write outside the box

27 (a) Jo wants to work out the solutions of $x^2 + 3x - 5 = 0$

She says,

"The solutions **cannot** be worked out because
$x^2 + 3x - 5$ does **not** factorise to $(x + a)(x + b)$ where a and b are integers."

Is Jo correct?

Tick a box.

☐ Yes ☑ No

Give a reason for your answer.

[1 mark]

No, She could work out the soutions using the quadratic formula.

27 (b) **Without** expanding any brackets,

show how to work out the **exact** solutions of $9(x + 3)^2 = 4$

Give the solutions.

[3 marks]

$(x+3)^2 = \frac{4}{9}$ ✓

$x+3 = \pm\frac{2}{3}$ ✓

$3 = \frac{9}{3}$

$x+3 = \frac{2}{3}$ OR $x+3 = -\frac{2}{3}$

-3

$x = \frac{2}{3} - \frac{9}{3} = \frac{-7}{3}$ $\quad x = -\frac{2}{3} - \frac{9}{3} = \frac{-11}{3}$

Do not write outside the box

28 Simplify $\sqrt{80} + \sqrt{2\frac{2}{9}}$

Give your answer in the form $\frac{a\sqrt{5}}{b}$ where a and b are integers.

[3 marks]

$\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

$\sqrt{2\frac{2}{9}} = \sqrt{\frac{20}{9}} = \sqrt{\frac{4 \times 5}{9}} = \sqrt{\frac{1}{9} \times 5} = \frac{2}{3}\sqrt{5}$

$\therefore 4\sqrt{5} + \frac{2}{3}\sqrt{5} = \frac{12}{3}\sqrt{5} + \frac{2}{3}\sqrt{5} = \frac{14\sqrt{5}}{3}$

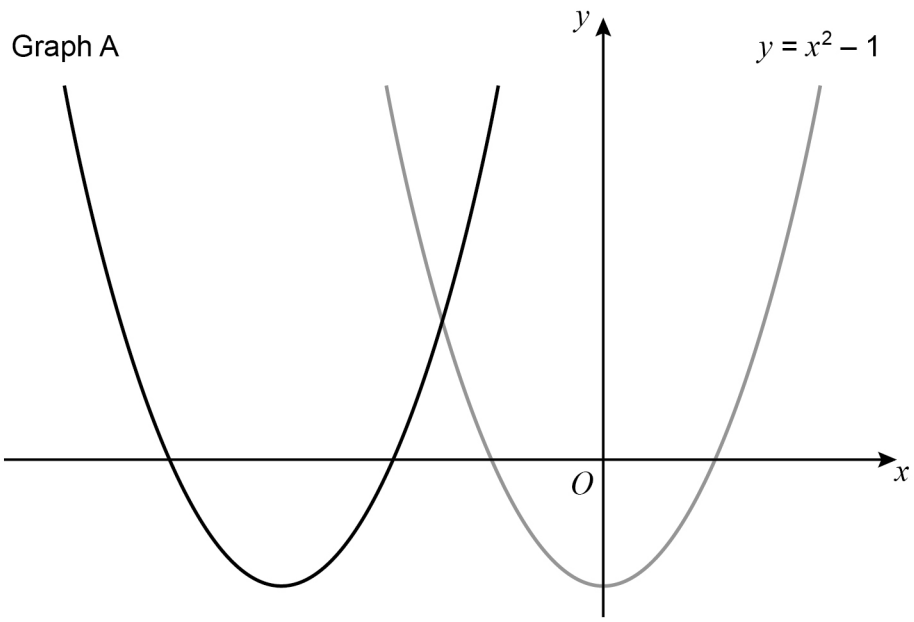
Answer $\frac{14\sqrt{5}}{3}$

Turn over for the next question

7

Do not write outside the box

29 Here are sketches of two graphs.



The graph of $y = x^2 - 1$ is translated 3 units to the left to give graph A.

29 (a) The equation of graph A can be written in the form $y = x^2 + bx + c$

Work out the values of b and c.

[3 marks]

Translated by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ ← (−) translation in x

New eqⁿ → $y = (x+3)^2 - 1$ ∴ $(x-(-3)) = (x+3)$

$= (x+3)(x+3) - 1$

$= x^2 + 3x + x + 9 - 1 = x^2 + 6x + 8$

$b =$ 6

$c =$ 8

Do not write outside the box

29 (b) The graph of $y = x^2 - 1$ is reflected in the x-axis to give graph B.

Work out the equation of graph B.

[1 mark]

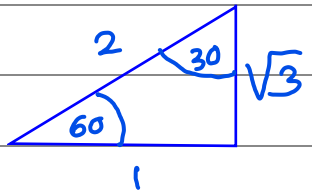
$y = -(x^2 - 1) = -x^2 + 1$

$= 1 - x^2$

Answer $1 - x^2$

30 Show that the value of $\cos 30° \times \tan 60° + \sin 30°$ is an integer.

[3 marks]



$\cos 30 = \frac{A}{H} = \frac{\sqrt{3}}{2}$

$\sin 30 = \frac{O}{H} = \frac{1}{2}$

$\tan 60 = \frac{O}{A} = \frac{\sqrt{3}}{1}$

$\cos 30 \times \tan 60 + \sin 30$

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{1}\right) + \frac{1}{2} = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$ ← an integer is a whole number.

END OF QUESTIONS

7

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ANSWER IN THE SPACES PROVIDED