

GCSE Maths – Ratio, Proportion and Rates of Change

Interpreting Gradients (Higher Only)

Notes

WORKSHEET



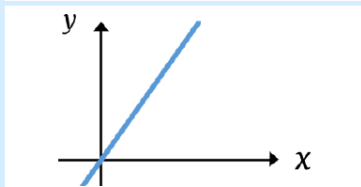
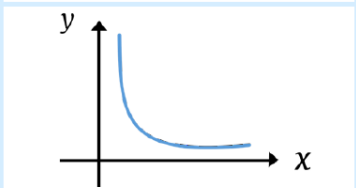
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Interpreting Gradients (Higher)

A gradient of a line measures how **steep** it is. The gradient of a line can be calculated using this formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$. In context, the gradient of a line may denote the **rate of change** between x and y variables and show the **direct or inverse relationship**.

Direct and inverse proportion

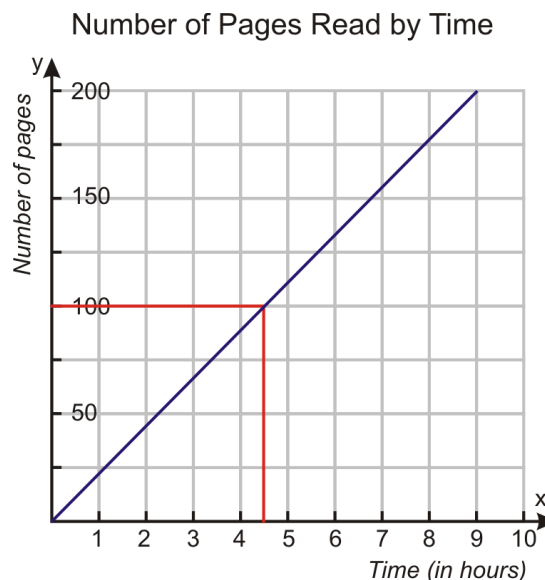
Direct proportion	Inverse proportion
$y \propto x \Rightarrow y = kx$ for some constant k	$y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x}$ for some constant k
	

Straight Line Graphs

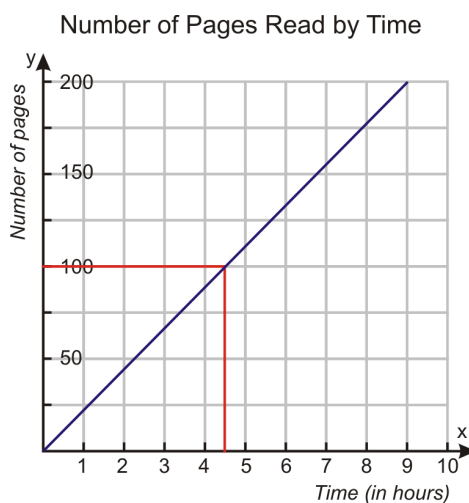
In many real-life graphs, the gradient represents the **rate of change**. As the gradient is defined as “the change in the y variable over the change in the x variable”, graphs represent **the y variable per the x variable**.

For example, this graph represents the number of pages read per hour. The graph also shows there is direct proportion between the number of pages read and the time as the graph is a straight line, intersecting the origin.

In order to calculate the “rate of change” between the variables, pick **two points** on this line and calculate the **gradient**.



Example: Using the graph below, calculate the rate of change between the number of pages read to the time spent reading them.



- Find two co-ordinates' points on this graph.

$$(x_1, y_1) = (9, 200)$$

$$(x_2, y_2) = (0, 0)$$

- Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{200 - 0}{9 - 0}$$

$$m = \frac{200}{9}$$

The rate of change between the number of pages read and the time spent reading them is $\frac{200}{9}$.

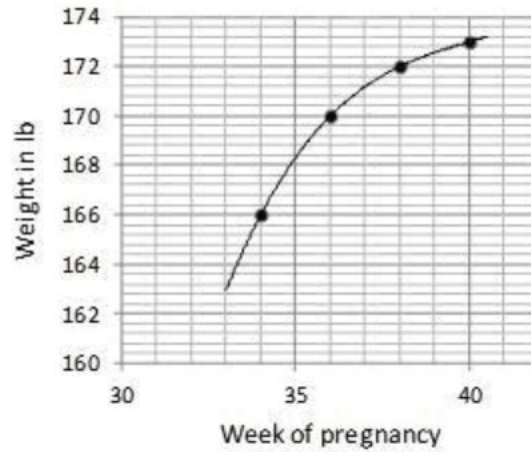
Estimating the Rate of Change

Sometimes the graph given is not a simple straight-line graph and may be a curve. This means that an exact rate of change cannot be calculated. However, we can still **estimate** the rate of change across the **whole graph** or at a **given point**.

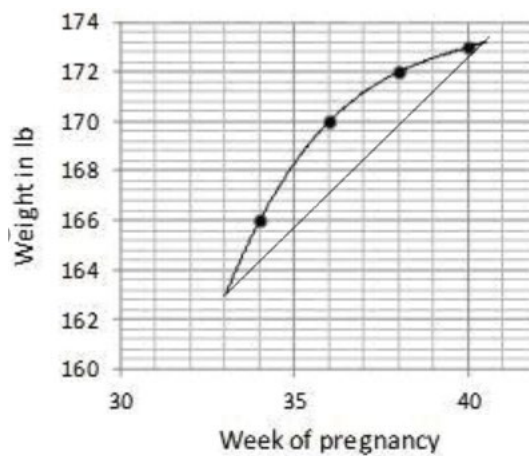
To find the average gradient, a chord must be drawn from the starting coordinate to the last coordinate.



Example: Using the graph below, estimate the average rate of change between the weight during the pregnancy.



1. Draw a chord using a ruler and pencil, connecting the starting and ending point.



2. Find any two co-ordinates' points on this chord:

$$(x_1, y_1) = (33, 163)$$

$$(x_2, y_2) = (40, 173)$$

3. Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{173 - 163}{40.5 - 33}$$

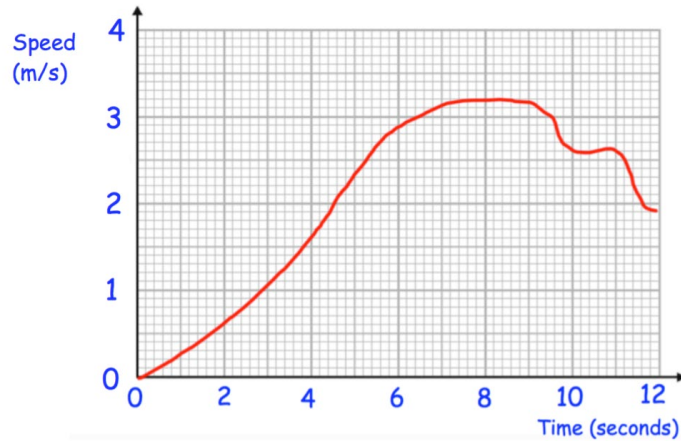
$$m = \frac{10}{7.5} = \frac{4}{3}$$

An estimation for the rate of change is $\frac{4}{3}$ lbs per week.



To find the instantaneous rate of change at a given point, a tangent must be drawn at that point and the gradient of the tangent should be calculated.

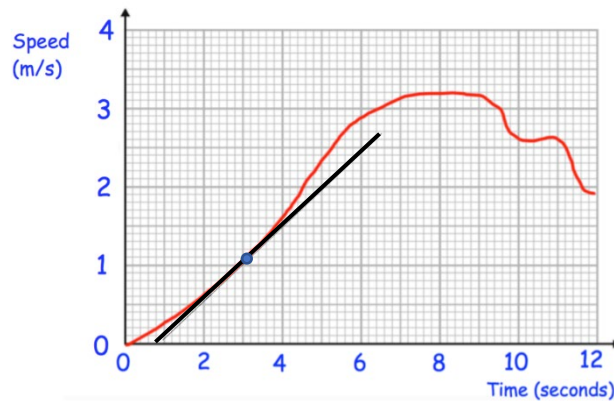
Example: Using the graph below, calculate the acceleration at 3 seconds.



1. Identify what the question is asking.

Acceleration is the rate of change between speed and time. Therefore, we have to find the rate of change at 3 seconds.

2. Draw a tangent at 3 seconds.



3. Find any two co-ordinates' points on this tangent.

$$(x_1, y_1) = (3, 1.1)$$

$$(x_2, y_2) = (4, 1.5)$$

4. Calculate the gradient using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$.

$$m = \frac{1.5 - 1.1}{4 - 3}$$

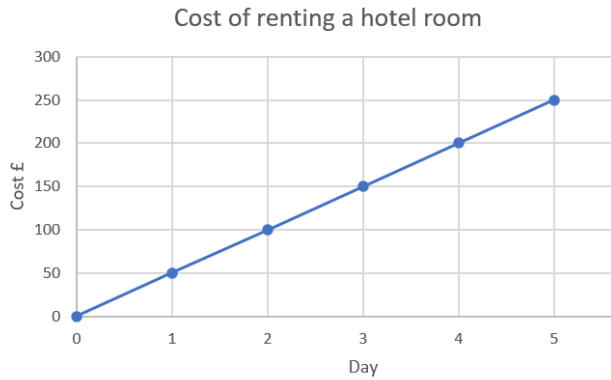
$$m = \frac{0.4}{1} = 0.4$$

The acceleration at 3 seconds is 0.4 m/s²

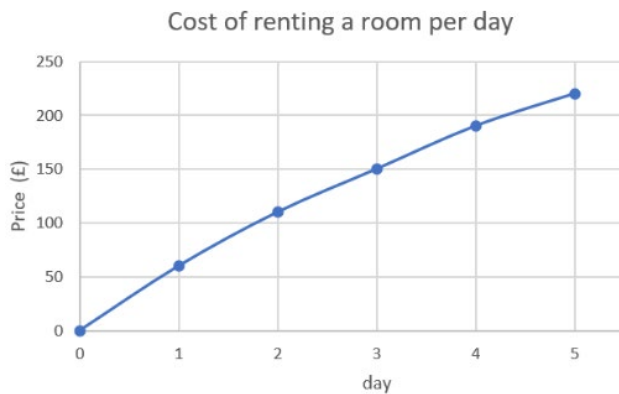


Interpreting Gradients (Higher Only) - Practice Questions

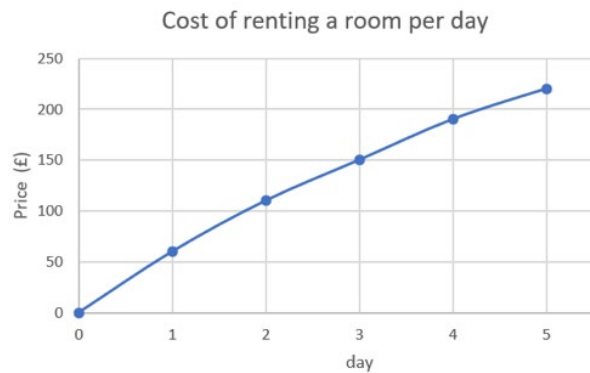
1. Calculate the cost per day of renting a hotel room.



2. Estimate the cost per day for renting a hotel room.



3. Estimate the cost per day for renting a hotel room on day 3.



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.

