GCSE Maths – Ratio, Proportion and Rates of Change

Ratio and Similar Shapes

Notes

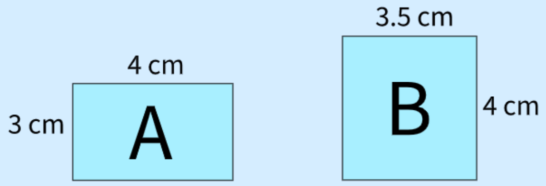
WORKSHEET

Ratio and similar shapes

Using Ratios to Compare

We can use ratio notation to compare the length, area and volume of different shapes.

Example: Find the ratio of the area of A to the area of B.



1. **Find** the **area** of each rectangle.
   The formula for the area of a rectangle is base x height.

   area of shape A: $4 \times 3 = 12\ cm^2$
   area of shape B: $3.5 \times 4 = 14\ cm^2$

2. Note these areas as a **ratio** in the form asked for in the question.

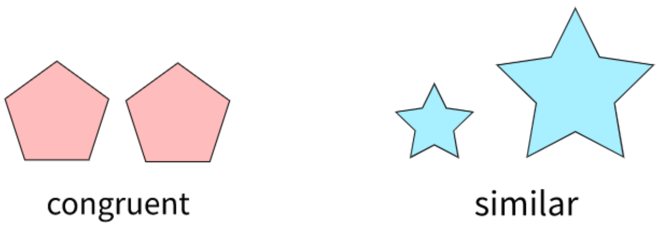
   $$area\ \ shape\ A : area\ shape\ B$$
   $$12 : 14$$

3. **Simplify** the ratio.
   Divide both sides by the same number until the ratio is written as the smallest possible set of integers.

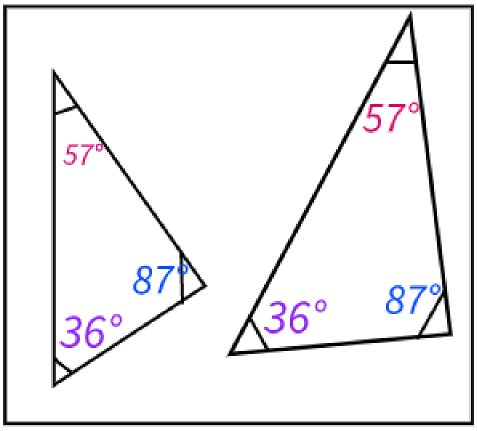
   $$12 : 14$$
   $$6 : 7$$

Conditions for similarity

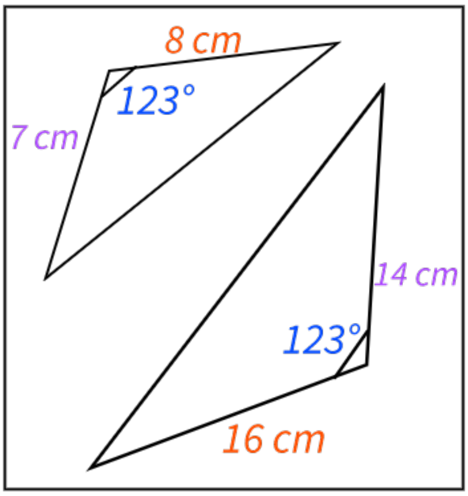
Two shapes are **similar** if one is an **enlargement** of the other.



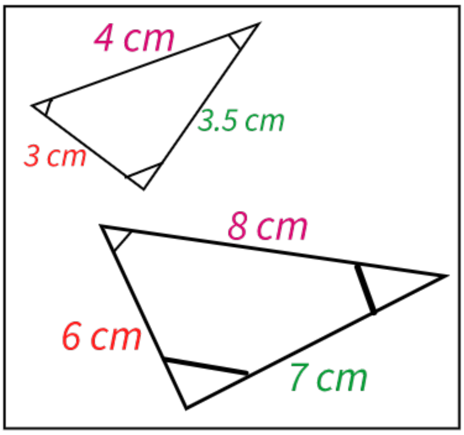
For **two triangles** to be similar, they must satisfy **three conditions**. There are three cases for similar triangles:



AAA
All three pairs of angles are of equal size.



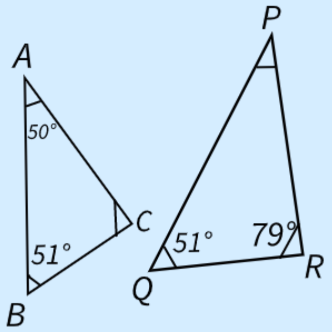
SAS
Two pairs of sides are in the same ratio, and their included angle is equal.



SSS
All three pairs of sides have lengths in the same ratio.

When proving two triangles are similar, you must **show one of these conditions is being met** by analysing the sides and angles.

Example: Are triangle ABC and triangle PQR similar? Explain your answer.



1. **Calculate the missing angles** of the triangles.

$$\angle ACB = 180 - 50 - 51 = 79°$$
$$\angle QPR = 180 - 51 - 79 = 50°$$

2. Look at the information we have about each triangle and draw **similarities** between them.

$$\angle ABC = 51° = \angle PQR$$
$$\angle ACB = 79° = \angle QRP$$
$$\angle BAC = 50° = \angle QPR$$

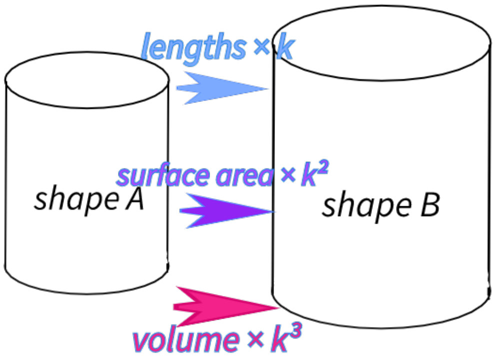
3. **Decide which condition** you can prove with the sides/angles you have.

You know that triangle ABC and PQR have three pairs of equal angles. So, the triangles satisfy the condition AAA, and can be said to be similar.

Scale factors

Since when working with similar shapes **one is an enlargement of the other**, the relationship between the two shapes can be called a **scale factor.**

The relationship between sides is known as the **linear scale factor (k).** If we know the linear scale factor, we can work out the area and volume of enlargements of the shape:



- $Surface\ Area\ of\ B\ =\ Surface\ Area\ of\ A\ \times\ k^2$
- $Volume\ of\ B\ =\ Volume\ of\ A\ \times k^3$

Example: A company is modelling a prototype of its newest candle. The model is 7 cm high and weighs 50 g. If the actual prototype will be 21 cm high, how much can they expect the prototype to weigh?

1. **Calculate the linear scale factor** (k) between the model and the prototype.

$$\frac{Prototype\ Height}{Model\ Height} = k$$

$$21 \div 7 = 3$$

The height of the prototype is three times the height of the model.

2. **Cube** the linear scale factor to find the volume scale factor.

$$k^3 = 3^3 = 27$$

3. **Multiply** the mass of the model **by the volume scale factor**, to find the volume of the prototype.

$$50\ g\ \times k^3 = 50\ g\ \times 27 = 1350\ g$$

The actual prototype weighs 1.35 kg.

Example: A carpenter makes a chest for his daughter, and a miniature version for his daughter's doll house. The version for the doll house takes 100 cm² of wallpaper for decoration, whilst the real-life version takes 400 cm². If the chest for the doll house is 2.5 cm high, how tall is the chest for his daughter?

1. **Calculate the area scale factor** (k^2) between the two versions.

$$\frac{Surface\ Area\ of\ Chest}{Surface\ Area\ of\ Toy} = k^2$$

$$400 \div 100 = 4$$

2. **Find the linear scale** of the chest.

$$Area\ Scale\ Factor = Linear\ Scale\ Factor^2$$

$$Linear\ scale\ factor = k = \sqrt{4} = 2$$

3. **Multiply** the height of the miniature chest **by the linear scale factor**, to find the heights of the life-size chest.

$$2.5\ cm \times k = 2.5\ cm \times 2 = 5\ cm$$

The chest for the daughter is $5\ cm$ *tall.*

Ratio and Similar Shapes - Practice Questions

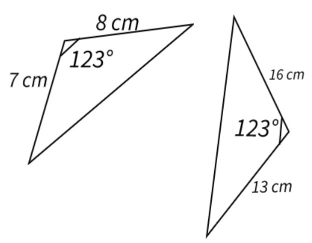
1. The base of cone P has a radius of 4 cm and is 12 cm high. Cone Q is 10 cm high and has a base with radius 5 cm. Work out the ratio of the volume of cone P to cone Q.

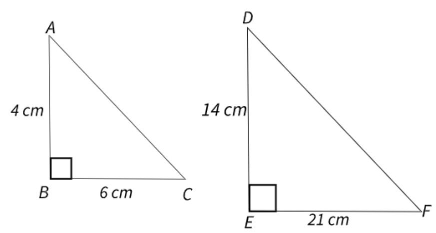
2. The surface area of the moon is 14.6 square miles.

   The surface area of Earth is 196.9 square miles.

   Write the ratio of the surface area of the moon to the surface area of the sun in the form 1:n, where n is an integer rounded to the nearest whole number.

3. Are the following pairs of triangles similar? Explain your answers



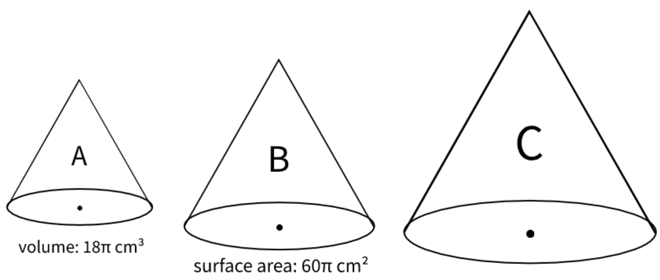


4. A standard teacup holds 150 ml of liquid. An enlarged teacup holds 216 ml of liquid. What is the volume scale factor to go from the standard teacup to the enlarged version?

5. A hexagon has an area of 22.1 cm^2. If it is enlarged to an area of 47.736 cm^2, by what factor has the area, and lengths, increased?

6. Cone A has a height of 6 cm, cone B has a height of 9 cm and cone C is 15 cm high.

   Using the information, what is the volume of cone B and the surface area of cone C?



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.