GCSE Maths – Ratio, Proportion and Rates of Change

Direct and Inverse Proportion

Notes

WORKSHEET

Direct and Inverse Proportion

Direct Proportion

Direct proportion describes two amounts where one amount increases at the same **rate** as the other amount.

For example, Dan is paid £11/hour. As the number of hours increases, the amount he is paid increases at the same rate. If he works for 2 hours, he is paid 2 x £11 = £22. If he works 3 hours, he is paid 3 x 11 = £33.

Unitary Method

Simple proportion problems can be solved by working out the amount of **b** for one unit of **a**, and then multiplying accordingly.

Example: If 5 pens cost £5.50, how much will it cost to buy 13 pens?

1. Find the cost of 1 pen.

$$\textit{Cost of 1 pen is } £5.50 \div 5 = £1.10$$

2. Multiply the cost of 1 pen by 13.

$$\textit{Cost of 13 pens is } £1.10 \times 13 = £14.30$$

Algebraic Method

If **a** is directly proportional to **b** we write $a \propto b$. To solve proportion problems, we can introduce a variable **k**:

$a \propto b$ is equivalent to $a = bk$ for some constant k

Example: Pay is directly proportional to hours worked. Laura is paid £30 for 3 hours work. How much is she paid for 100 hours of work?

1. Write an equation involving constant k.

$$P \propto H \textit{ (pay is proportional to hours worked)}$$
$$P = kH \textit{ (introduce } \boldsymbol{k} \textit{ to remove } \propto)$$

2. Substitute known values to find k.

Substitute P = 30, H = 3: $30 = 3k \Rightarrow 10 = k$

$$\boldsymbol{P = 10H}$$

3. Substitute H = 100 to find amount paid.

$$P = 10 \times 100 = \boldsymbol{£1000}$$

Example: T is directly proportional to the square of U, and T is 32 when U is 8. Find T when U is 2.

1. Write an equation involving constant k.

$$T \propto U^2$$

$$\boldsymbol{T = kU^2} \text{ (remove } \propto \text{ and introduce k)}$$

2. Solve for k.

Substitute $T = 32, U = 8$*:*

$$32 = (8)^2 k$$
$$32 = 64k$$
$$0.5 = k$$

Rewrite equation: $\boldsymbol{T = 0.5U^2}$

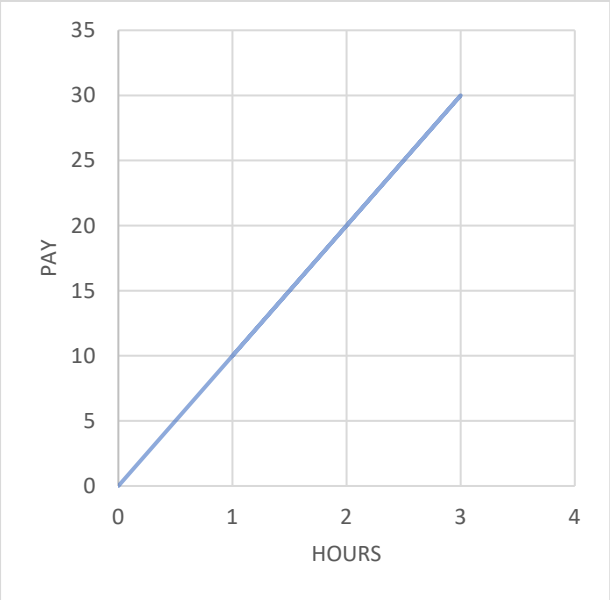
3. Substitute U = 2 to find T.

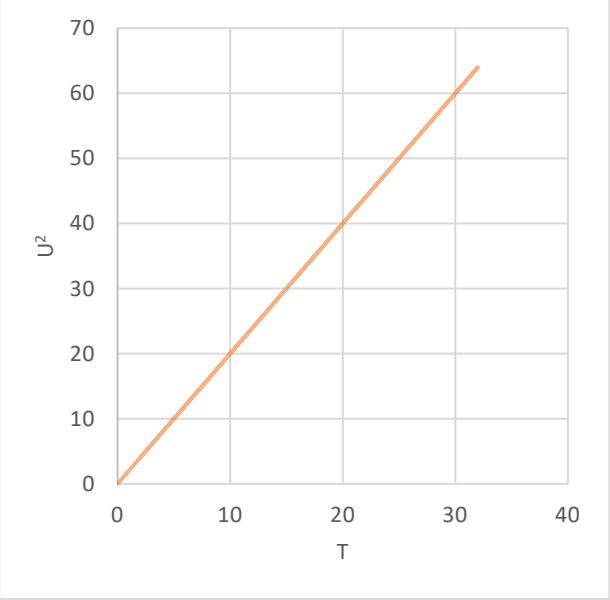
$$T = 0.5(2)^2$$
$$T = 0.5 \times 4 = 2$$
$$T = 2$$

Graphical representation

Direct proportion produces a **straight-line graph that passes through the origin**.

The graph will have a **positive gradient** if constant '*k*' is positive, and a **negative gradient** if constant '*k*' is negative.





Inverse Proportion

Inverse proportion describes two amounts where if one amount decreases, the other increases.

An example of this is speed and time. As speed increases, it takes less time to cover the same distance. As speed decreases, it takes more time to cover the same distance.

Algebraic Method

If **a** and **b** are **inversely proportional,** we can write $a \propto \frac{1}{b}$.

To solve **indirect proportion** problems, we also introduce a variable **k**.

Example: 1 builder takes 6 days to build a wall. How many days would it take 3 builders to build the same wall?

1. Write an equation involving constant k.

   This is indirect proportion because as the number of builders increases the number of days will decrease.

   $$B \propto \frac{1}{D}$$

   $B = \frac{k}{D}$ *(replace* $\propto$ *and introduce k)*

2. Solve for k.

   Substitute $B = 1$ *and* $D = 6$ *to find k:*

   $$1 = \frac{k}{6}$$
   $$6 = k$$

   Rewrite equation: $B = \frac{6}{D}$

3. Substitute $B = 3$ to find D.

   $$3 = \frac{6}{D}$$
   $$3D = 6$$
   $$D = 2$$

   It takes 3 builders ***2 days*** *to build the wall.*

As with **direct proportion**, questions may state that **A is inversely proportional to the cube of B**. In which case, you would write the expression $A \propto \frac{1}{B^3}$ and proceed as usual.

Graphical Representation

Indirect proportion produces a **reciprocal** graph. If X is inversely proportional to Y, then as the X value increases towards infinity, the Y value tends towards 0, and vice versa.

Example: A is indirectly proportional to B. When A = 1, B = 20. Draw a graph to show this relationship for values of A ranging from 1 to 4.

1. Write equation involving k

This is indirect proportion so we can write:

$$A \propto \frac{1}{B}$$

$A = \frac{k}{B}$ *(replace* $\propto$ *and introduce k)*

2. Solve for k

Substitute $A = 1$ *and* $B = 20$ *to find k:*

$$1 = \frac{k}{20}$$

$$20 = k$$

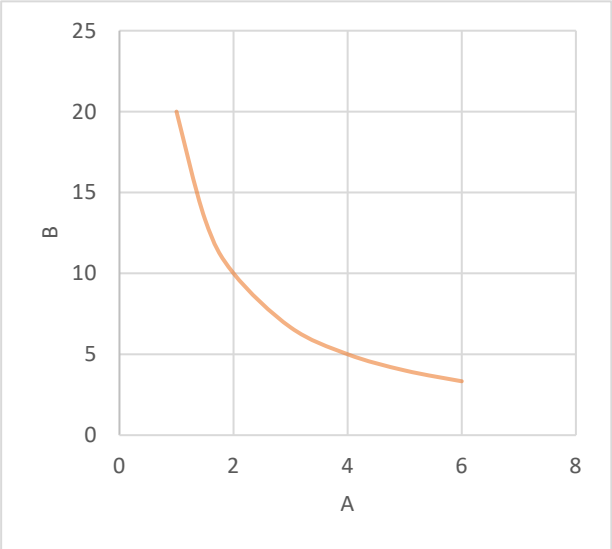
Rewrite equation: $A = \frac{20}{B}$

3. Substitute in A = 2,3,4 and form a table of values for each B.

A	1	2	3	4
B	20	10	20/3	5

4. Plot a graph.

Since A and B are indirectly proportional, we know the graph should be a reciprocal graph.



Direct and Inverse Proportion - Practice Questions

1. 6 pens cost £2.16. Calculate the cost of 12 pens.

2. 8 water bottles cost £20. Calculate the cost of 13 water bottles.

3. X is directly proportional to the square of Y. When $X = 50$, $Y = 5$.
   Find X when $Y = 3$.

4. C is directly proportional to the cube root of D. When $C = 32$, $D = 8$.
   Find D when $C = 16$

5. X is inversely proportional to the square of Y. When $X = 2$, $Y = 5$.
   Find X when $Y = \sqrt{10}$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.