GCSE Maths – Ratio, Proportion and Rates of Change

Scale Factors and Scale Diagrams

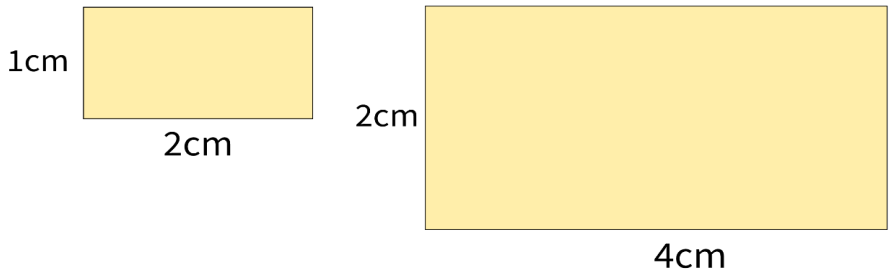
Notes

WORKSHEET

Scale Factors

Scale factors are numbers that tell us how much **larger** or **smaller** one shape is to another. A scale factor of 4 means that one shape is 4 times larger than another, and a scale factor of $\frac{1}{4}$ means that one shape is 4 times smaller than another.

To work out how much larger one shape is to another, we need to compare the **corresponding sides**. For example, consider the shapes below:

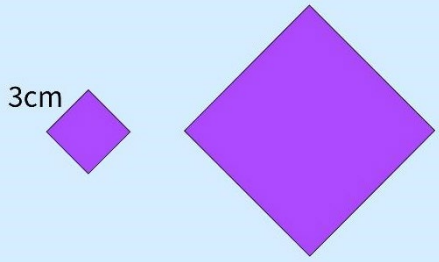


The rectangle on the right is an enlargement of the rectangle on the left, but we need to work out the scale factor by comparing the same side of each rectangle. Looking at the bottom side, we see that the smaller rectangle has a width of 2 cm, and the larger rectangle has a width of 4 cm.

The width of the larger rectangle is twice that of the smaller rectangle, so the scale factor here is 2.

Alternatively, if we know the scale factor that describes the enlargement or reduction of one shape to another, we can calculate the **dimensions** of the second shape.

Example:



The scale factor that describes the enlargement of the small diamond to fit the large diamond is 3. Calculate the length of one of the sides of the large diamond.

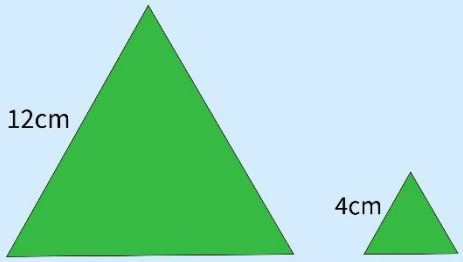
The scale factor tells us that the larger shape is 3 times larger. Therefore, each side is 3 times longer.

$$3 \times 3 \text{ cm} = 9 \text{ cm}$$

Each side of the large diamond is 9 cm.

If we are calculating an enlargement, we work out $\frac{larger}{smaller}$, but if we are calculating a reduction, then we work out $\frac{smaller}{larger}$. Because of this, enlargements will have a scale factor greater than 1, whereas reductions will have a scale factor less than 1. However, even if the scale factor is a fraction **less than 1**, we still call it an **enlargement**.

Example:



Calculate the scale factor for the reduction from the large triangle to the small triangle.

To calculate the scale factor, we look at corresponding sides. Since it is a reduction, we will divide the smaller side by the larger side.

$$\frac{4\text{ cm}}{12\text{ cm}} = \frac{1}{3}$$

The scale factor is $\frac{1}{3}$.

Scale Diagrams

Scale factors are often used for **maps** and floor plans of buildings, because the objects are **too large to draw to scale**. On a map or floor plan, we are told the scale factor. For example, a map may have a scale factor of 1:25000. This means that 1 cm on the map corresponds to 25000 cm (or 250 m) in real life.

Example: On a map, the distance between points A and B is 5 cm. The map has a scale factor of 1:50000. What is the distance between A and B in real life?

To calculate this, we need to multiply the distance on the map by the scale factor. The scale factor is $50\,000$.

$$5 \times 50\,000 = 250\,000\text{ cm}$$

This could be simplified to 2500 m *or* 2.5 km.

If we are calculating the real-life distance from the distance on a map or plan, we **multiply** the map distance by the scale factor.

Alternatively, if we are calculating the distance on the map or plan from the real-life distance, then we **divide** the real-life distance by the scale factor.

Example: The distance between London and Paris is approximately 344 km. On a map with scale factor 1:200000, what is the distance between London and Paris?

First, we need to calculate the distance in cm. We will multiply 344 by 1000 to obtain the distance in metres, then by 100 to obtain the distance in cm.

$$344 \text{ km} = 344\,000 \text{ m} = \mathbf{34\,400\,000 \text{ cm}}.$$

To work out the distance on the map, divide the real-life distance by the scale factor.

$$34\,400\,000 \div 200\,000 = 17.2$$

The distance on the map would be $\mathbf{17.2 \text{ cm}}$.

We can calculate the scale factor if we know the real-life distance and the distance on the map or plan. All we need to do is **divide** the real-life distance by the map/plan distance. This tells us **how many times bigger** the real-life object is compared to its size on the map.

Example: The dimensions of a garden are 10 m by 8 m. On a plan, it is drawn as 4 cm by 3.2 cm. What scale factor should be written on the plan?

First, we should convert to the units used on the plan.

$$10 \text{ m} = 1000 \text{ cm}$$
$$8 \text{ m} = 800 \text{ cm}$$

Now we divide one of the sides by its corresponding size on paper:

$$1000 \div 4 = 250$$

This means that the scale factor is 1:250, as 1 cm *on paper is equal to* 250 cm *(*2.5 m*) in real life.*

Scale Factor and Scale Diagrams - Practice Questions

1. A rectangle, *A,* has a length of 8 cm and width of 5 cm. Calculate the length and width of the second rectangle, *B,* if *A* is enlarged by scale factor 4.

2. A square, *E*, with sides of length 10 cm are enlarged by a scale factor less than 1. The resulting shape, *F*, has sides of length 3 cm. Calculate the scale factor of enlargement.

3. On a map, the distance between points *C* and *D* is 5 cm. The scale factor of this map is 1:10 000. Calculate the real-life distance between points *C* and *D*.

4. A house has a kitchen with length 4.5 m and width 3 m. Calculate the length and width of this kitchen on a floor plan with scale factor 1:150.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.