GCSE Maths – Probability

Independent and Dependent Events

Notes

WORKSHEET

Events and Probability

An event is the **outcome** of an action or experiment. This could be:

- an action that you do, such as flipping a coin - where the event is heads or tails.
- an action performed by someone or something else, such as a football game – where the event is a win or a loss.
- another happening that has an outcome, such as a natural event (rain/storm/etc.).

Events have a **probability** – this is **how likely** they are to occur. Probabilities are often displayed as decimals, percentages or fractions, and must always **add up to 1**.

$$Probability = \frac{Number\ of\ target\ events}{Total\ number\ of\ possible\ events}$$

A probability of **0** means that an event **definitely will not** happen.
Lower probabilities mean that an event is **less likely** to happen.
Higher probabilities mean that an event is **more likely** to happen.
A probability of **1** means that an event **definitely will** happen.

Here are some example events and their probabilities:

Experiment	Event	Probability	Total
Flipping a fair coin	Heads	0.5	0.5 + 0.5 = 1
	Tails	0.5	
Rolling a fair die	1	$\frac{1}{6}$	$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=1$
	2	$\frac{1}{6}$	
	3	$\frac{1}{6}$	
	4	$\frac{1}{6}$	
	5	$\frac{1}{6}$	
	6	$\frac{1}{6}$	
The sun rising tomorrow	The sun rises	1	$1 + 0 = 1$
	The sun doesn't rise	0	

When an experiment happens more than once, we can calculate the probability of **combined events** (multiple outcomes all occurring). For example, if a coin is flipped five times, what is the probability of getting 4 or more heads? The method we use will depends on whether the events are **independent** or **dependent**.

Independent Events

Events are **independent** if the probability of one outcome occurring does **NOT** affect the probability of another outcome occurring. Some examples of independent events include:

- If you roll two dice, getting a 1 on the first die will **not** affect the probability of getting a 1 on the second die. The probability of getting any number on a 6-sided die is fixed at $\frac{1}{6}$, regardless of whatever numbers you have already rolled.
- When you flip a coin, the probability of getting a head is 50% - the probability of getting a tail is also 50%. If you get 5 heads in a row, the probability of getting a tail on your next flip is still 50% - it does **not change**.

Both these types of events are independent because the probabilities of each outcome are **fixed** – they do not change based on other previous outcomes.

Dependent Events

Events are **dependent** if the probability of one outcome occurring **DOES** affect the probability of another outcome occurring. Dependent events could be:

- A bag contains 4 balls – 2 red and 2 blue. Mia takes 1 ball out of the bag at random and does not replace it.

At the start, the probability of getting a red ball is $\frac{2}{4}$, which is equal to $\frac{1}{2}$.
The probability of getting a blue ball is also $\frac{1}{2}$.
But, when Mia takes a ball out of the bag, there will only be 3 balls left in the bag:

If she has chosen a red ball, then the probabilities change to:

$$Probability\ of\ Red = \frac{1}{3}$$

$$Probability\ of\ Blue = \frac{2}{3}$$

If she has chosen a blue ball, the probabilities change to:

$$Probability\ of\ Red = \frac{2}{3}$$

$$Probability\ of\ Blue = \frac{1}{3}$$

The events are dependent because the probabilities of each outcome **can change** depending on the previous outcome.

Example: Decide if the following events are dependant or independent:
a) Flipping a coin 5 times
b) Taking different coloured balls out of a bag at random and replacing them
c) Rolling a 6-sided die 10 times
d) Taking balls out of a bag and not replacing them

a) Independent
The probabilities are fixed at 50%. Any one flip does not affect the outcome of the other flips.
b) Independent
When the balls are replaced, the probability is fixed for any colour. Taking any colour on one pick does not affect the probability of taking another colour on the next pick.
c) Independent
Rolling any number does not affect the probability of rolling any other number. The probabilities are fixed at $\frac{1}{6}$.
d) Dependent
Not replacing the balls changes the total number of balls in the bag – or total possible outcomes. This changes the probability of choosing another colour on the next pick.

The AND Rule

When we are finding the probability of one event **AND** another both occurring, we use:

$$P(A\ AND\ B) = P(A)\ \times P(B)$$

The probability of A and B occurring is equal to the probability of A occurring, multiplied by the probability of B occurring.

If the events are dependent, you must take into account the **change in probability** of event B, when event A has already happened. $P(A\ AND\ B)$ is sometimes written $P(A\ \cap B)$. The AND rule has **higher priority** than the OR rule, so it should always be calculated first – unless brackets are included, then these should be calculated first.

The OR Rule

When we are finding the probability of one event **OR** another event occurring, we use:

$$P(A\ OR\ B) = P(A) + P(B) - P(A\ AND\ B)$$

$P(A\ OR\ B)$ is sometimes written $P(A \cup B)$.

Mutually Exclusive Events

Events are **mutually exclusive** if they cannot **both** happen. In this case, $P(A\ AND\ B) = 0$ so the formula becomes:

$$P(A\ OR\ B) = P(A) + P(B)$$

Example: Janae flips a coin and rolls a 6-sided die. Find the probability of her getting:

a) A head and a 2

b) A tail or a 6

c) A head and a 4 or 5

a) Find the probability of each individual event:

$Probability\ of\ getting\ a\ head = \frac{1}{2}$

$Probability\ of\ getting\ a\ 2 = \frac{1}{6}$

$$P(Getting\ a\ head\ AND\ Getting\ a\ 2) = P(Getting\ a\ head)\ \times P(Getting\ a\ 2) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

The probability of getting a head and rolling a 2 is $\mathbf{\frac{1}{12}}$.

b) Find the probability of each individual event:

$Probability\ of\ getting\ a\ tail = \frac{1}{2}$

$Probability\ of\ getting\ a\ 6 = \frac{1}{6}$

We use this formula because the events are not mutually exclusive:

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$P(Getting\ a\ tail\ OR\ Getting\ a\ 6) = \frac{1}{2} + \frac{1}{6} - \left(\frac{1}{2} \times \frac{1}{6}\right) = \frac{6}{12} + \frac{2}{12} - \left(\frac{1}{12}\right) = \frac{8}{12} - \frac{1}{12} = \frac{7}{12}$$

The probability of getting a tail and rolling a 6 is $\mathbf{\frac{7}{12}}$.

c) Find the probability of each individual event. Here, ‘getting a 4 or a 5’ has ‘brackets’ around it as these events occur from the same action, so should be calculated first.

$Probability\ of\ getting\ a\ head = \frac{1}{2}$

$Probability\ of\ getting\ a\ 4 = \frac{1}{6}$

$Probability\ of\ getting\ a\ 5 = \frac{1}{6}$

$$P(Getting\ a\ 4\ OR\ Getting\ a\ 5) = \frac{1}{6} + \frac{1}{6} - \left(\frac{1}{6} \times \frac{1}{6}\right) = \frac{2}{6} - \frac{1}{36} = \frac{11}{36}$$

$$P(Getting\ a\ head\ AND\ (Getting\ a\ 4\ OR\ a\ 5)) = \frac{1}{2} \times \frac{11}{36} = \frac{11}{72}$$

The probability of getting a head and rolling either a 4 or 5 is $\mathbf{\frac{11}{72}}$.

Visual Methods

Frequency trees, tables of outcomes and other **visual methods** are useful for working out the possible outcomes of an event. They can then be used to calculate the probability of specific outcomes. For further information and practice questions using visual methods, refer to:

- Maths GCSE Revision Notes – Probability – Table of Outcomes and Frequency Trees
- Maths GCSE Revision Notes – Probability – Enumeration
- Maths GCSE Revision Notes – Probability – Possibility Spaces

Independent and Dependent Events - Practice Questions

1. Find the following when $P(A) = 0.35$ and $P(B) = 0.65$:

   a) $P(A \cap B)$ when A and B are independent.
   b) $P(A \cup B)$ when A and B are mutually exclusive.

2. Find $P\big(P(A \cap B) \ \cup \ P(A \cup B)\big)$ when $P(A) = 0.2$ and $P(B) = 0.4$:

3. Marley rolls two 6-sided dice. Find the probability that he rolls the following combinations:

   a) A 2 on both dice.
   b) A 1 on only one of the dice.

4. Emma has a bag containing twenty coloured balls: 3 of the balls are red, 5 of the balls are blue and 12 of the balls are yellow.

   a) She pulls a ball from the bag and then replaces it. What is the probability that the ball was red or blue?

   b) She pulls a green ball from the bag and doesn't replace it. What is the probability that the next ball will also be green?

   c) She replaces all the balls and then adds four red balls to the bag. What is the probability that the next four balls she pulls from the bag without replacing will all be red?

   d) She replaces all the balls and then pulls another ball from the bag. What is the probability that it is not green?

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.