GCSE Maths – Probability

Sample Spaces

Notes

WORKSHEET

Sample Spaces

The **sample space** is a mathematical term that represents 'all **possible outcomes** of an event'. The sample space **lists the possible outcomes** in a pair of braces: the curly brackets {}.

For example, the sample space for rolling a 6-sided die is $\{1, 2, 3, 4, 5, 6\}$, and the sample space for flipping a coin is $\{\text{Head}, \text{Tail}\}$.

Set Notation

- Z is the set of **integer** numbers.
- N is the set of **natural numbers**. This is the set of positive integers $\{1, 2, 3, 4, \dots\}$.
- ξ is the **universal set**. The universal set is the whole set we are considering in a question or problem. This is used most frequently in Venn diagrams.
- A ⊂ B means that A is a **subset** of B.

Set notation can be used to describe a situation. For example,

$$Z = \{x : x \text{ is a factor of } 20\}$$

means Z is the set of numbers x such that x is a factor of 20, i.e. Z is the set of factors of 20. It could also be written as $Z = \{1, 2, 4, 5, 10, 20\}$.

Sample Space Diagrams

Sample space diagrams show the possible outcomes of **combined events** in a mathematical table. Events are combined when their outcomes are **considered together**, instead of separately.

In a sample space diagram, one event is listed **horizontally** and the other listed **vertically**. Then the table is filled in with **every possible outcome** of the two events. Systematically filling in the outcome of the row first, then the column helps to find all the outcomes without missing any combinations.

We can construct a sample space diagram for a coin flip and a 6-sided die roll.

A coin flip has two possible outcomes: Heads and Tails. These are listed as H and T in the diagram.

A die roll has six possible outcomes. Its sample space is {1, 2, 3, 4, 5, 6}. These outcomes are listed in the diagram.

		DIE					
		1	2	3	4	5	6
COIN	H	H,1	H,2	H,3	H,4	H,5	H,6
	T	T,1	T,2	T,3	T,4	T,5	T,6

This cell shows that the combined outcome was a Tail on the coin and a 4 on the die.

Each cell in the table contains the outcome of the first event (coin flip) and the outcome of the second event (die roll).

From the diagram, we can see that there are 12 possible outcomes. This allows us to calculate the theoretical probability of a **target outcome** (the outcome specified in the question).

Table of Outcomes

Sometimes you will see sample space diagrams called **tables of outcomes**. These refer to the same diagram – the **visual method** of finding all possible outcomes of combined events.

Example: Two fair, 6-sided dice are rolled. Construct a sample space diagram for the outcomes.

1. Identify the **individual sample spaces**:
   First die roll: $\{1, 2, 3, 4, 5, 6\}$
   Second die roll: $\{1, 2, 3, 4, 5, 6\}$

2. List the sample spaces in the **headings** of a **two-way table**.

   The first die roll is listed horizontally, and the second die roll is listed vertically.

		DIE 1 ROLL					
		1	2	3	4	5	6
DIE 2 ROLL	1						
	2						
	3						
	4						
	5						
	6						

3. Fill in the table by **writing the outcome** from each corresponding row and column.

The first square should take a 1 from the row and 1 from the column – so write 1,1 in this space.
The second square takes a 1 from the row and a 2 from the column – so write 1,2 in this space.

		FIRST DIE ROLL					
		1	2	3	4	5	6
DIE 2 ROLL	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,4	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

Theoretical Probability

Theoretical probability is the mathematical **likelihood** of something happening. It is always given as a **decimal number between 0 and 1** (or the equivalent fraction or percentage). We can use the idea of theoretical probability, along with the equation

$$Probability = \frac{Appearences\ of\ target\ outcome}{Total\ number\ of\ possible\ outcomes}$$

to work out how **likely** something is to happen.

- The **target outcome** is the outcome that we want to find the probability of.
- The **possible outcomes** are all other outcomes in the sample space.

Example: Two fair, six-sided dice are rolled at the same time. Calculate the probability that their product is a square number.

1. Construct a **Table of Outcomes** for the problem.
   Look back at the previous example if you get stuck.

2. Identify the **target outcome**.
   *"…that their product is a square number." – so **square numbers** are the target outcome.*

3. **Count the appearances** of the target outcome.
   Square numbers: 1, 4, 9, 16, 25, 36. Circle or highlight them in the table to count.

		DIE 1					
		1	2	3	4	5	6
DIE 2	1	**1**	2	3	**4**	5	6
	2	2	**4**	6	8	10	12
	3	3	6	**9**	12	15	18
	4	**4**	8	12	**16**	20	24
	5	5	10	15	20	**25**	30
	6	6	12	16	24	30	**36**

4. Calculate the required **probability**:

   The diagram shows there are 8 ways of making a square number out of 36 possible outcomes.

   8 square numbers out of 36 outcomes → $\mathrm{P(Product\ is\ a\ square\ number)} = \frac{8}{36} = \frac{2}{9}$

   Probability of getting a square number is $\frac{2}{9}$.

Sample Spaces – Practice Questions

1. Write down the sample space for each of these events.

   a) Flipping a coin.

   b) Rolling an 8-sided die.

   c) Running a random number generator numbered 1-10.

2. Miles has organised a game to raise money for the local rugby club. Players must roll a 6-sided die and flip a coin.

   If the coin lands on heads, the score is doubled.
   If the coin lands on tails, the score is halved.

   People pay 50p to play.
   If they score more 10 or more, they win £1.
   If they score more than 5 but less than 10, they win 75p.

   800 people play his game.

   Use probability to estimate how much money he raises.

3. A six-sided die is rolled twice.

   a) Construct a sample space diagram to show the possible outcomes.

   b) Work out the probability that the second roll is the same as the first roll.

   c) State the probability that the second roll is NOT the same as the first roll.

   d) Work out the probability that five sequential rolls are all the same.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.