GCSE Maths – Probability

Expected Outcomes, Frequency and Theoretical Probability

Notes

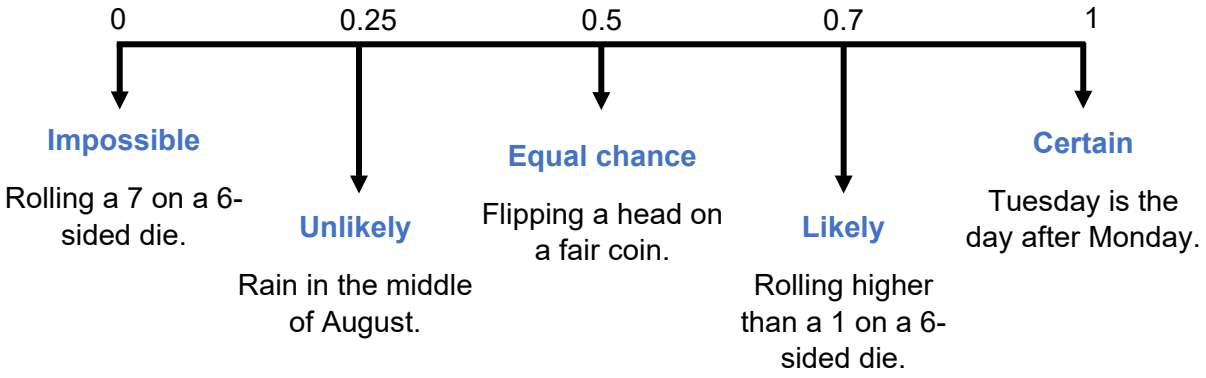
WORKSHEET

Theoretical Probability

Probability represents the **chance**, or likelihood, of something happening. Different words can be used to describe probability, such as 'likely', 'even chance' or 'certain'.

In mathematics, probability is given a value on the scale from 0 to 1:

- A probability of 0 means something **will definitely not** happen.
- A probability of 1 means something **will definitely** happen.



Finding Probability

The probability of an outcome is the number of ways it can happen out of the total number of possible outcomes.

$$Probability = \frac{Number\ of\ ways\ target\ outcome\ can\ occur}{Total\ number\ of\ possible\ outcomes}$$

For example, on a 6-sided die, there is one way of rolling a 2 and there are 6 possible numbers we could roll. The probability of rolling a 2 is $\frac{1}{6}$.

$$Probability\ of\ rolling\ 2 = \frac{Number\ of\ ways\ to\ roll\ a\ 2}{Total\ number\ of\ possible\ outcomes} = \frac{1}{6}$$

To find the outcomes of an experiment, **multiply** the number of times the experiment is carried out by the probability of the outcome. For example, if you rolled a 6-sided die 24 times, you would expect $\mathbf{24} \times \frac{\mathbf{1}}{\mathbf{6}} = \mathbf{4}$ of the outcomes to be a 2.

Events that cannot both occur at the same time are called **mutually exclusive**. A basketball team can win, draw or lose a game but they **cannot** both win and lose a game. If events are mutually exclusive, then their probabilities add up to 1, as one of the outcomes **must** happen.

$$\textbf{Probability of event not occurring} = 1 - \textbf{probability of event occurring}$$

Calculating Probability

You may be asked to calculate probability from **tables, graphs, diagrams and trees**. These topics are covered in the following resources:

- Maths GCSE Revision Notes - Probability – Enumeration, Venn Diagrams, Tree Diagrams and Tables
- Maths GCSE Revision Notes - Probability – Independent and Dependent Events
- Maths GCSE Revision Notes - Probability – Sample Spaces
- Maths GCSE Revision Notes - Probability – Table of Outcomes and Frequency Trees

Example: What is the theoretical probability of rolling a 5 on an unbiased 10-sided die?

1. Identify the target outcome and the number of possible outcomes.

The target outcome is a 5. There is one 5 on a 10-sided die.
There are 10 possible outcomes. These are the numbers 1-10 on the faces of the die.

2. Apply the formula for probability:

$$Probability = \frac{Number\ of\ ways\ target\ outcome\ can\ occur}{Total\ number\ of\ possible\ outcomes}$$

$$Probability\ of\ rolling\ a\ 5 = \frac{Number\ of\ ways\ to\ obtain\ a\ 5}{Total\ number\ of\ possible\ outcomes} = \frac{1}{10}$$

Example: A bag contains 10 red, 6 blue and some green counters. The probability of picking a red counter from the bag is 0.25. How many counters are in the bag?

1. Identify the target outcome and probability.

The target outcome is a red counter, and there are 10 ways in which this can happen. The probability of this happening is 0.25.

2. Apply the formula for probability:

$$Probability = \frac{Number\ of\ ways\ target\ outcome\ can\ occur}{Total\ number\ of\ possible\ outcomes}$$

$$0.25 = \frac{10}{Total\ number\ of\ possible\ outcomes}$$

$$Total\ number\ of\ possible\ outcomes \times 0.25 = 10$$

$$Total\ number\ of\ possible\ outcomes = \frac{10}{0.25} = 40$$

The total number of possible outcomes is 40 so there are 40 counters in the bag.

Relative Frequency

Relative frequency is an estimate of outcomes using probability and is worked out from trials in an experiment. It can be used to estimate the theoretical probability of an outcome.

Theoretical

Probability is **theoretical** because you do not actually conduct a series of experiments and record the outcomes – instead, you apply **mathematical theories and logic** to calculate the probability of an outcome. For this reason, theoretical probability may not always give the same outcomes as an experiment which involves **random processes**.

For example, if you flipped a coin 10 times, you would **expect** it to land on heads 5 times, and tails 5 times, because the theoretical probability of each outcome is **0.5**. However, this is not always the case.

Also, recognise that random events do **not have a memory** – so landing heads 7 times on a coin flip does **not increase** the probability of the next flip being a tail.

Experimental

Relative frequency shows how often an outcome actually happened and is calculated from the actual outcomes of an experiment (rather than the theoretical probability of an outcome).

$$Relative\ frequency\ = \frac{Number\ of\ times\ outcome\ occurred}{Number\ of\ times\ experiment\ was\ carried\ out}$$

Consider the following example:

Mateo rolls an unbiased 6-sided die and records the number of times he scores a 1.

Number of rolls	10	20	50	100	500
Number of times 1 is scored	1	4	9	16	84
Relative frequency	0.1	0.2	0.18	0.16	0.168

Mateo sees that as the number of rolls increases, the relative frequency gets **closer** to the theoretical probability ($\frac{1}{6}$ or 0.167) of rolling a 1. This means that the more **reliable estimate** of probability is found when the **number of trials is increased**.

Example: A biased coin is flipped 500 times. The relative frequency of landing on heads is calculated to be 0.65. How many times did the coin land on heads?

Apply the formula for relative frequency, identifying the relative frequency and number of trials from the question:

$$0.65 = \frac{Number\ of\ times\ outcome\ occurred}{500}$$

$$0.65 \times 500 = Number\ of\ times\ outcome\ occurred = 325$$

The coin landed on heads 325 times.

Expected Outcomes

When anything happens, it has a result, or an **outcome**. Some things will have lots of outcomes that could possibly occur. Outcomes are **expected** when we know that one of them **must** happen, because there are no alternative options.

Probability of Outcomes

Some outcomes are more **likely** to occur than others. We can use the concept of theoretical probability to describe the likelihood of expected outcomes.

Dinosaurs coming to your school tomorrow is **impossible**. It will definitely not happen, so has a probability of 0.

Rolling a number bigger than 2 on a 6-sided die is **possible**. It has a probability of more than 0.5, so is a **likely outcome**.

The sun rising tomorrow is a **certain** event. It will happen, so has a probability of 1.

Bias

In certain cases, one outcome is **more likely** to happen than another.

- Something has **bias** when one outcome is **more or less** likely to happen than another.
- Something is **unbiased** when each outcome is **equally** likely to happen.
- Something is **fair** if it is unbiased.

For example, we know that a coin has two flat sides – heads and tails. When the coin is flipped, it **must** land on one of the two sides, so heads and tails are our **expected outcomes**. The coin **will** land on heads or tails. As long as the coin is not biased, each outcome is **equally likely**.

Similarly, a **fair** 6-sided die can land on 1, 2, 3, 4, 5, or 6. It is **equally likely** to land on each of the numbers, so is **unbiased**. Therefore, when it is rolled there is an **even chance** of it landing on each number.

Example: Decide whether the following events are impossible, unlikely, even chance, likely or certain:

a) Snow in winter
b) Flipping a fair coin and getting heads
c) Saturday happening after Friday

a) Likely – we cannot say that it will definitely snow in winter, but it is likely to happen.
b) Even chance – the probability of landing a head or tail on a fair coin flip is 0.5, so it is an even chance.
c) Certain – Saturday will always be the day after Friday.

Expected Outcomes, Frequency and Theoretical Probability - Practice Questions

1. Anderson LTD makes 10,000 doors a day. Each door costs £5.10 to make and sells for £13.99. The probability of a door being faulty is 0.0045.

    a) Estimate the number of faulty doors made in one day.
    b) Calculate an estimate for the profit that Anderson LTD makes in one working week (5 days).
    c) On Wednesday, a machine breaks down. Anderson LTD makes 138 faulty doors. Calculate the theoretical probability of a door being faulty on Wednesday.
    d) Anderson LTD wants to start making 500 kettles a day. Each kettle costs £4.99 to make and sells for £7.10. The probability of a kettle being faulty is in the range $0.25 \leq P(Faulty) \leq 0.3$. The new item will only be approved if it will definitely make the company a daily profit. Should Anderson LTD start selling kettles?

2. Lanais asks some people in her village what their favourite sport is.
    22 people said hockey.
    56 people said football.
    14 people said rugby.

    a) Work out the relative frequency of someone in the town liking hockey.
    b) There are 2000 people living in Lanais' village. Using your answer to part a), estimate the number of people whose favourite sport is hockey.

3. A bag contains only red, green and yellow counters. The relative frequency of each counter is shown in the table.

Colour	Red	Green	Yellow
Relative Frequency	0.35	0.6	

    a) Find the relative frequency of a yellow counter.
    b) Jolene takes a counter from the bag. What is the probability that it is not red?
    c) There are 5 yellow counters in the bag. How many green counters are in the bag?

4. An unbiased 10-sided die is rolled. Complete the sentences below using the words:

Likely Unlikely Even Chance Certain Impossible

a) It is ..... that the die will land on 1.
b) It is ..... that the die will land on an even number.
c) It is ..... that the die will land on 11.
d) It is ..... that the die will land on a number bigger than 1.
e) It is ..... that the die will land on a number between 1 and 10.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.