GCSE Maths – Probability

Tables of Outcomes and Frequency Trees

Notes

WORKSHEET

Combined Events

By **listing**, **displaying** or **counting** the possible outcomes of multiple combined events, the probability of each independent outcome occurring can be calculated.

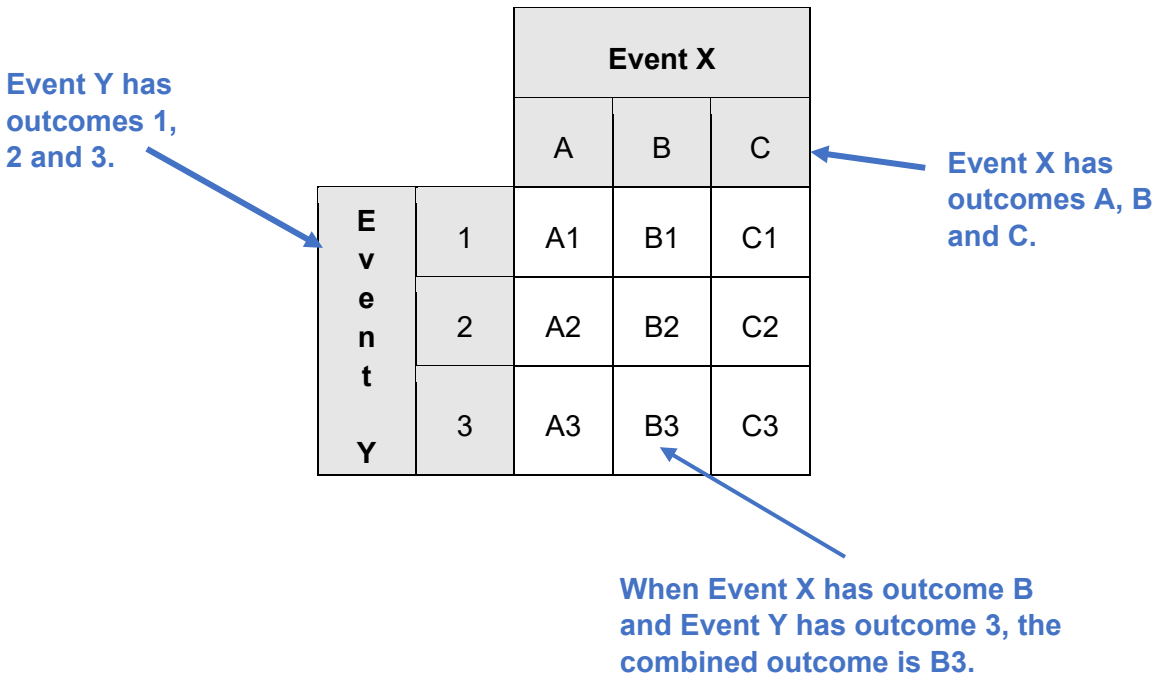
Table of Outcomes

A table of outcomes is a **visual** method of showing the possible results of **two independent events**. The table can then be used to find the probability of each possible outcome occurring.

Events are **independent** if the probability of one event occurring does not affect the probability of the other event occurring.

Constructing a Table of Outcomes

In a table of outcomes one event's outcomes are displayed **vertically**, and the other's displayed **horizontally**. Each box in the table represents one possible combined outcome.



In this table, every outcome is different, so each outcome has a 1 in 9 ($\frac{1}{9}$) probability of occurring.

Example: Two fair, six-sided dice are rolled at the same time. Construct a table of outcomes for the product of the values show on the die.

1. Identify the two events.

   Event 1: *Dice roll 1-6.*
   Event 2: *Dice roll 1-6.*

2. Identify the operation.

   "…for their product." – so the operation is multiplication of their values

3. Draw the table and fill in the combined outcomes.

		Die 1					
		1	2	3	4	5	6
Die 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	16	24	30	36

Calculating Probability

Once a table of outcomes is completed, it can be used to **calculate the probability** of each combined outcome. Firstly, the **target outcome is identified** and the **number of appearances are counted**. The probability of a particular outcome is the number of target outcomes divided by the total number of outcomes.

$$Probability = \frac{Appearances\ of\ required\ outcome}{Total\ number\ of\ possible\ outcomes}$$

Example: Two fair, six-sided dice are rolled at the same time. Calculate the probability that their product is a square number.

1. Construct a **Table of Outcomes** for the problem.

   Events: Dice Roll 1, Dice Roll 2
   Operation: Multiplication

		Die 1					
		1	2	3	4	5	6
Die 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	16	24	30	36

2. Identify the **target outcome**.

   *"…that their product is a square number." – so **square numbers** are the target outcome.*

3. **Count the appearances** of the target outcome.

   Square numbers: 1, 4, 9, 16, 25, 36

		Die 1					
		1	2	3	4	5	6
Die 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

4. Calculate **probability**:
   The diagram shows there are 8 ways of making a square number out of 36 possible outcomes.

$$\text{6 square numbers out of 36 outcomes} = \frac{8}{36} = \frac{2}{9}$$

The probability of getting a square number is $\frac{1}{6}$.

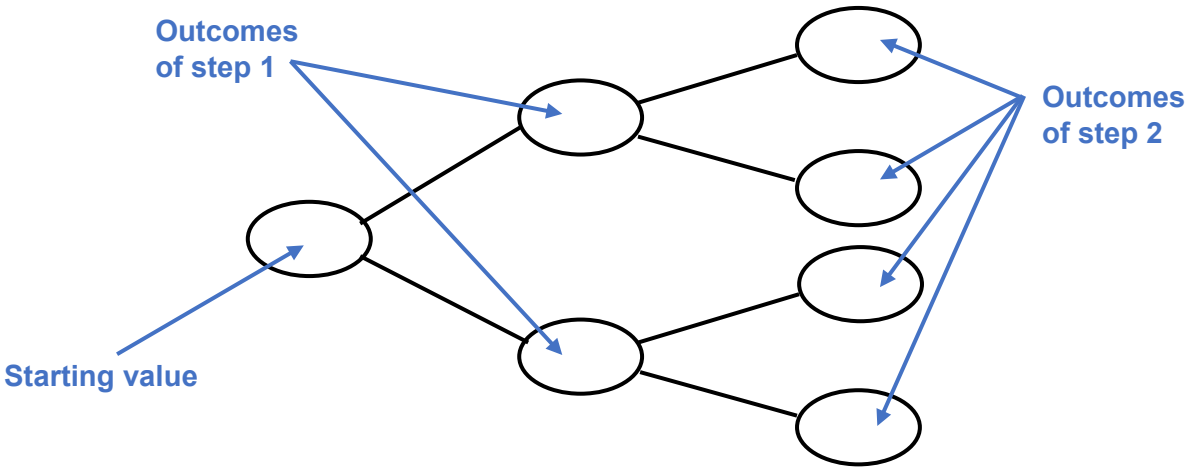
Frequency Trees

Frequency trees are a visual method of recording the possible outcomes of an event with **multiple steps**. You can use the tree to find the probability of any outcome occurring. Events can only be recorded if:

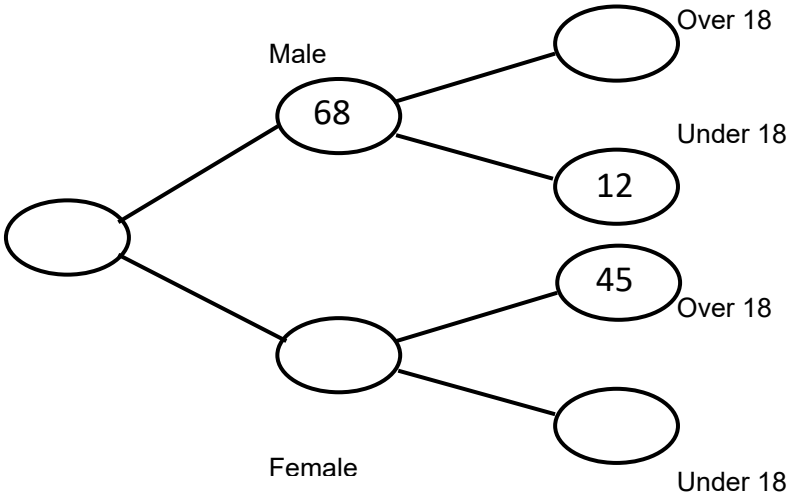
- they have 2 or more **steps** (such as being a square AND being blue).
- they have a **definite starting value** (such as the fixed number of balls in a bag).

Constructing a Frequency Tree

In a frequency tree, each outcome is displayed as a **branch** of the tree, spreading horizontally across the page.



Example: 140 people are members of a running club. Complete the frequency tree below.



1. Find the **starting value**. Write this in the first circle on the diagram.
   "140 people…" – so the starting value is 140.

2. Work out the **missing values**. Fill in the corresponding circles.

   If 68 of the runners are male, then the rest are female.

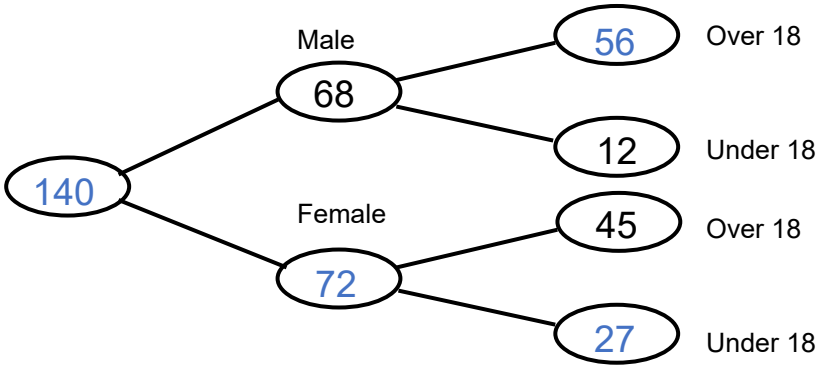
   140 – 68 = 72 female runners

   12 of the 68 male runners are under 18, so the rest are over 18.

   68 – 12 = 56 male over 18 runners

   45 of the 72 female runners are over 18, so the rest are under 18.

   72 – 45 = 27 female under 18 runners



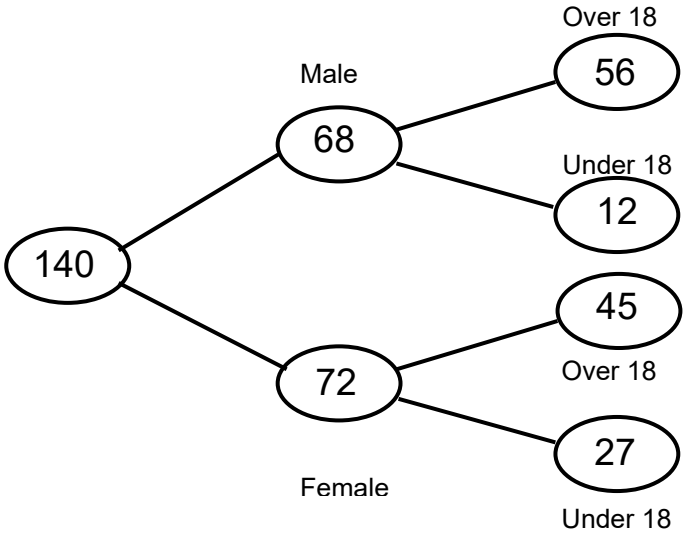
*From the tree, we can see that 56 of the runners are male **and** over 18.*

Calculating Probability

A frequency tree separates the starting value into a number of **sub-groups**. The probability of **randomly** choosing an item from one of the sub-groups can be calculated in a similar way to using the table of outcomes:

$$Probability = \frac{Number\ in\ required\ sub-group}{Total\ number\ in\ starting\ value}$$

Example: Using the frequency tree below, find the probability of randomly choosing:
a) A female runner.



1. Identify the sub-group in the question.
   *"A **female** runner." – so the sub-group is **female.***

2. Find the corresponding circle in the tree.

3. Calculate probability:

$$Probability = \frac{Number\ in\ required\ sub-group}{Total\ number\ in\ starting\ value} = \frac{72}{140}$$

The probability of randomly choosing a runner who is female is $\frac{72}{140}$.

b) A runner who is male and over 18.

Use the same steps as above to find the probability.

$$Probability = \frac{Number\ in\ required\ sub-group}{Total\ number\ in\ starting\ value} = \frac{56}{140}$$

Table of Outcomes and Frequency Trees - Practice Questions

1. Construct a table of outcomes for each of the following scenarios:

a) A fair coin is flipped and a 10-sided die is rolled. Record the possible combinations.

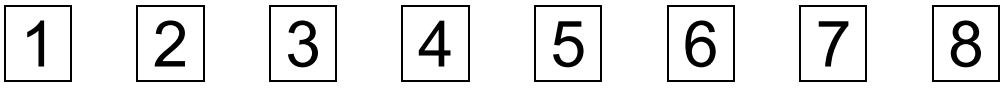
b) A restaurant serves four main meals and four desserts, the prices shown in the tables below. Latisha chooses a main meal and a dessert at random. Record the possible prices of her meal.

Main	£
Stir Fry	9.50
Beef Soup	8.75
Roast Lamb	14.00
Carbonara	9.20

Dessert	£
Brownie	4.50
Apple Pie	4.20
Tiramisu	3.25
Trifle	3.90

2. A fair coin is flipped and a 10-sided die is rolled. Find the probability of getting a head and an even number.

3. Alex has 8 cards:



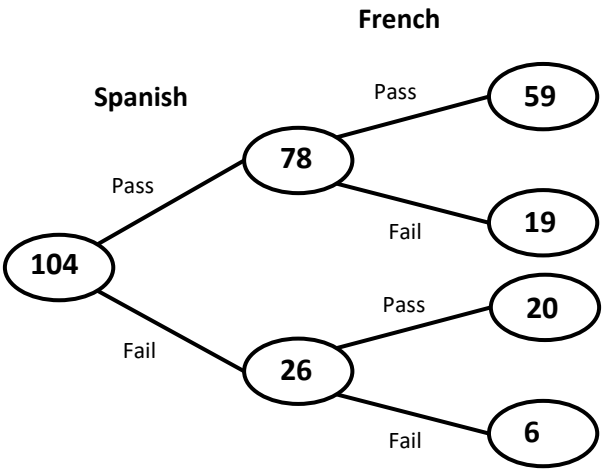
She takes one card followed by another to make a two-digit number. E.g.

Find the probability that:

a) The number is more than 50.
b) The number is even.
c) The number is a multiple of 7.

4. 104 students took their Spanish and French speaking exams. Write down the number of students who:

   a) Passed Spanish
   b) Passed both
   c) Passed neither



5. 60 people are asked if they prefer strawberry or vanilla ice cream.
   - 24 of them are children.
   - One third of adults prefer vanilla.
   - 30 people in total prefer strawberry.

   A person is chosen at random to win a free ice cream. Find the probability that they are:

   a) A child.

   b) An adult who prefers strawberry.

   c) A child who prefers strawberry OR an adult who prefers vanilla.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.