GCSE Maths – Number

Four Operations

Notes

WORKSHEET

Order Using $<, >, \leq, \geq, =$ and $\neq$

We can use the above symbols to order positive and negative integers, decimals, and fractions.

The symbol $>$ is used to show that the number on the **left is greater than the number on the right,** but **not equal**. For example, $7 > 6$ makes sense since 7 is greater than 6.

The symbol $<$ is used to show that the number on the **left is less than the number on the right**, but **not equal**. For example, $-7 < 6.5$ since -7 is less than 6.5.

We use the symbol $\geq$ when we want to show that the **number on the left is greater than or equal to the number on the right**. For example, both $6 \geq \frac{5}{6}$ and $3 \geq 3$ both make sense.

Similarly, we use $\leq$ when we want to show that the **number on the left is less than or equal to the number on the right**. Examples being $-4 \leq 5$ and $-8 \leq -8$.

The equals sign, =, is used to show the **number on the left is equal to the number on the right**. For instance, we have $5 = 5$ and the more complicated $5 = \frac{2020}{2021} + 10.2 + 5.8 - \frac{24251}{2021}$. On the contrary, we use $\neq$ to show the **left and right are not equal**. $5 \neq 2$, for example.

Place Value

The position of a digit within a number signifies its place value. This can be easily summarised with a table, for example with the number 1253.569:

Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths
1	**2**	**5**	**3**	**.**	**5**	**6**	**9**

The Four Operations

In mathematics, the four operations are **addition, subtraction, multiplication, and division**. You may be asked to use the four operations with integers, decimals, simple fractions (proper and improper), and mixed numbers. All of which can be **positive or negative**.

Integers

Addition and subtraction of integers can be performed using **column methods**, we will have a quick recap of these methods with two examples:

Example: Calculate $456 + 346$

1. *Put the numbers into columns:*

	4	**5**	**6**
+	**3**	**4**	**6**

2. *Starting with the right, add* 6 *onto* 6*. This gives us* 12 *which is equal or above* 10*, meaning we have to carry the* 1 *into the tens column, leaving a* 2 *in the units column:*

	4	**5**	**6**
+	**3**	**4**	**6**
			2

3. *In the tens column, we add* 4 *onto* 5*, giving* 9 *(representing* $40 + 50 = 90$*). But, since we have carried an 'extra'* 10 *from the previous column, we get* 10 *(representing* 100*). Again, we must carry this into the hundreds column and leave a* 0 *in the tens column:*

	4	**5**	**6**
+	**3**	**4**	**6**
		0	**2**

4. *In the leftmost column, add the* 3 *to the* 4 *to get* 7 *(representing* $300 + 400 = 700$*). Since we carried an 'extra'* 100 *from the previous column we get* 8 *(representing* 800*):*

	4	**5**	**6**
+	**3**	**4**	**6**
	8	**0**	**2**

Hence, we have that $456 + 346 = 802$*.*

Example: Calculate $573 - 428$

1. *Put the numbers into columns:*

	5	**7**	**3**
-	**4**	**2**	**8**

2. *Looking at the rightmost column, the unit column, we see that* $3 < 8$ *so we must 'borrow' a ten from the tens column and perform* $13 - 8 = 5$ *instead, whilst changing the* 7 *in the tens column into a* 6*:*

	5	**6**	**13**
-	**4**	**2**	**8**
			5

3. *Next, since for the next columns we have that* $6 > 2$ *and* $5 > 4$*, we don't need to do any borrowing, so we simply proceed with simple subtraction on each column:*

	5	**6**	**13**
-	**4**	**2**	**8**
	1	**4**	**5**

Hence, $573 - 428 = 145$

Multiplying integers can be done using the **grid method**, where you split the numbers into units, tens, hundreds etc. and multiply in steps. The following example will illustrate this:

Example: Calculate 42×56

1. *Put the numbers into a grid:*

×	40	2
50		
6		

2. *Multiple each of the numbers in the top row with each of the numbers in the leftmost column, filling in the grid:*

×	40	2
50	2000	100
6	240	12

3. *Next, sum up all the numbers on the inside of the grid, in this case we want* $2000 + 100 + 240 + 12$. *This can be done mentally or by using the column method.*

$$2000 + 100 + 240 + 12 = 2352$$

Hence, $42 \times 56 = 2352$.

Dividing integers is slightly trickier, it can be done using **short division** (also known as the bus stop method). Again, this will be illustrated by way of example:

Example: Calculate $168 \div 8$

1. *Place the number to be divided under the bus stop, and the number dividing it to the left:*

8	1	6	8

2. *Now, compare* 8 *with the* 1 *in the hundreds column.* 8 *does not go into 1 any times so we carry this* 1 *into the next column to create* 16, *whilst writing* 0 *above the* 1:

	0		
8	1	16	8

3. *Comparing* 8 *with the next column, we see that* 8 *goes into* 16 *twice, so we write* 2 *above it:*

	0	2	
8	1	16	8

4. *Lastly,* 8 *goes into* 8 *one time, so we write* 1 *above the* 8:

	0	2	1
8	1	16	8

5. *Read the number of at the top, ignoring any preceding zeroes, to get the answer. Hence,* $168 \div 8 = 21$.

Fractions

Addition and Subtraction of Fractions

Now we move onto the addition and subtraction of fractions. The key thing to remember when adding/subtracting fractions together is that they must have the **same denominator** for it to work and to convert mixed fractions into improper ones before addition.

To make two fractions have the same denominator we need to find a multiple of both denominators and multiply the numerators accordingly. Once they have the same denominator, we simply add/subtract the numerators. The following example will show this:

Example: Calculate $\frac{4}{3}+\frac{7}{4}$

1. *Looking at the denominators,* 3 *and* 4*, we see that they share the same multiple of* 12 *(*$3 \times 4 = 12$ *and* $4 \times 3 = 12$*).*

2. *With* $\frac{4}{3}$*, if we wish to change the denominator into 12, we multiply the denominator by* 4*. This means we must also multiply the numerator by* 4 *otherwise the fraction will not be equivalent to the original one. Hence,*

$$\frac{4}{3}=\frac{4}{3}\times\frac{4}{4}=\frac{16}{12}$$

3. *Similarly, with* $\frac{7}{4}$*, we multiply the top and bottom by* 3*:*

$$\frac{7}{4}=\frac{7}{4}\times\frac{3}{3}=\frac{21}{12}$$

4. *Now we simply add the fractions:*

$$\frac{4}{3}+\frac{7}{4}=\frac{16}{12}+\frac{21}{12}=\frac{16+21}{12}=\frac{37}{12}$$

Example: Calculate $\frac{7}{13}-\frac{5}{7}$

1. *To find a shared multiple of* 13 *and* 7 *we can simply multiply them together to obtain* 91*.*

2. *Much like the previous example, we convert* $\frac{7}{13}$ *into a fraction with a denominator of* 91*:*

$$\frac{7}{13}=\frac{7}{13}\times\frac{7}{7}=\frac{49}{91}$$

3. *For* $\frac{5}{7}$*:*

$$\frac{5}{7}=\frac{5}{7}\times\frac{13}{13}=\frac{65}{91}$$

4. *Now that they have a common denominator, we subtract the fractions:*

$$\frac{7}{13}-\frac{5}{7}=\frac{49}{91}-\frac{65}{91}=\frac{49-65}{91}=\frac{-16}{91}$$

Multiplication of Fractions

The **multiplication of fractions** simply involves **multiplying the numerators together and the denominators together**.

Example: Calculate $\frac{5}{6} \times \frac{7}{3}$

Multiply the top and bottom parts of the fractions together:

$$\frac{5}{6} \times \frac{7}{3} = \frac{5 \times 7}{6 \times 3} = \frac{35}{18}$$

Division of Fractions

When **dividing two fractions, flip the second fraction and then multiply** the two resulting fractions together. The following examples will demonstrate this.

Example: Calculate $\frac{8}{3} \div \frac{1}{2}$

1. *The second fraction is* $\frac{1}{2}$, *flipping this gives* $\frac{2}{1} = 2$.
2. *Now, multiply the first fraction by this new fraction:*

$$\frac{8}{3} \div \frac{1}{2} = \frac{8}{3} \times \frac{2}{1} = \frac{8 \times 2}{3 \times 1} = \frac{16}{3}$$

Example: Calculate $\frac{4}{5} \div \frac{9}{10}$

1. *The second fraction is* $\frac{9}{10}$, *flipping this gives* $\frac{10}{9}$.
2. *Now, multiply the first fraction by this new fraction:*

$$\frac{4}{5} \div \frac{9}{10} = \frac{4}{5} \times \frac{10}{9} = \frac{4 \times 10}{5 \times 9} = \frac{40}{45} = \frac{8}{9}$$

Decimals

Addition and Subtraction of Decimals

When adding/subtracting decimals, you add/subtract them as if they were integers, and simply put a decimal place in a column when using the various column methods. Alternatively, you can ignore the decimal place and then add it back in after completing the sum.

Example: Calculate $1.234 + 4.321$

1. *Put the numbers, and decimal place, into columns:*

	1	**.**	**2**	**3**	**4**
+	**4**	**.**	**3**	**2**	**1**

2. *Proceed as normal with column addition:*

	1	**.**	**2**	**3**	**4**
+	**4**	**.**	**3**	**2**	**1**
	5	**.**	**5**	**5**	**5**

Hence, $1.234 + 4.321 = 5.555$.

Example: Calculate $9.540 - 4.348$

1. *Ignore the decimal place and proceed with* $9540 - 4348$.

2. *Using any method of subtraction, such as column subtraction, we get* $9540 - 4348 = 5192$.

3. *Put the decimal back in to obtain* $9.540 - 4.348 = 5.192$.

Multiplication of Decimals

When **multiplying** decimals, first count **how many numbers appear after the decimal place** in both numbers and add them together – this tells you how many decimals places should be seen in the answer. For example, 4.5×8.9 would have **two decimal places** in the answer and 2.47×3.4 would have **three**. Once we know how many decimal places the answer should have, we can proceed by ignoring the decimal place, multiplying the numbers together and adding the decimal place in after.

Example: Calculate 4.5×8.9

1. *Combined, there are two decimal places in these numbers, therefore the answer will have two.*

2. *Using the grid method, or otherwise, we get that* $45 \times 89 = 4005$.

3. *We know the answer will have two decimal places, so we put the decimal place such that two of the numbers in* 4005 *are after it:* 40.05

   Hence, $4.5 \times 8.9 = 40.05$.

Example: Calculate 23×5.2

1. *Here, we have a combined one decimal place, so the answer will only have one too.*

2. *Using the grid method, or otherwise, we get that* $23 \times 52 = 1196$.

3. *Putting the decimal place in the correct place yields* $23 \times 5.2 = 119.6$

Division of Decimals

When dividing decimals, it is easier to write them in **fractional form** and then multiply the denominator (and numerator) to make it an integer and then use division of integer methods. This will be shown in the following example:

Example: Calculate $14.8 \div 0.4$

1. *Writing this as a fraction we get* $\frac{14.8}{0.4}$.

2. *Since* 0.4 *has one decimal place, we need to multiply it by* 10 *to make an integer. So, we multiply the numerator and denominator by* 10*:*

$$\frac{14.8}{0.4} = \frac{14.8}{0.4} \times \frac{10}{10} = \frac{148}{4}$$

3. *We then simplify the fraction:*

$$\frac{148}{4} = \frac{74}{2} = 37$$

So, $14.8 \div 0.4 = 37$.

Inverse Operations

Each of the four operations has an inverse, which is another one of the operations.

- The **inverse of addition is subtraction**, and vice versa
- The **inverse of multiplication is division** and vice versa.

Performing an operation and then performing its inverse (with the same number) is the same as doing nothing. For example:

$$(4 \times 9) \div 9 = (36) \div 9 = 4$$

and,

$$(400 + 19) - 19 = (419) - 19 = 400.$$

We can use this fact to **simplify expressions**. The most prominent way we can use it is cancellation in fractions - if a factor appears in both the numerator and denominator of a fraction, we can cancel it. This is because having said factor in the numerator is essentially multiplication by that factor whereas having it in the denominator is division by it, and we know that performing these two operations **cancel each other out**. For example:

$$\frac{56}{4} = \frac{4 \times 14}{4 \times 1} = \frac{\cancel{4} \times 14}{\cancel{4} \times 1} = 14$$

Or

$$\frac{2x + 4x^2}{2x} = \frac{2x \times (1 + 2x)}{2x \times 1} = \frac{\cancel{2x} \times (1 + 2x)}{\cancel{2x} \times 1} = 1 + 2x.$$

BIDMAS

In mathematics, the **order** in which you apply the operations **matters**. The convention is to follow the order of BIDMAS which stands for:

- **B**rackets
- **I**ndices
- **D**ivision
- **M**ultiplication
- **A**ddition
- **S**ubtraction

This means that you first simplify anything contained in brackets, before raising anything to any powers in the expression, and then dividing etc.

For example, $5 \times (4 + 1) = 5 \times (5) = 25$ and not $5 \times 4 + 1 = 20 + 1 = 21$.

The Four Operations - Practice Questions

1. Calculate:

a) $78 + 341$

b) $590 - 233$

c) 12×45

d) $564 \div 3$

2. Calculate

a) $\frac{9}{3} \div \frac{3}{2}$

b) $16 \times (24 \div 4)$

c) $\frac{8}{5} \times \frac{3}{9}$

d) $42 + (54 \div 9) - (2 \times 7)$

e) $51.2 \div 6.4$

f) 9.8×6

g) 4.2×9.5

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.